

**Math233 Calculus 3 Spring 12 Sample midterm 1**

**Problem 1.** Let  $\vec{u} = \langle 4, -3, 5 \rangle$  and  $\vec{v} = \langle 1, 2, -1 \rangle$ .

- (a) Find a unit vector in the direction of  $\vec{v}$ .
- (b) Find  $\|\text{proj}_{\vec{v}}\vec{u}\|$ .
- (c) Express  $\vec{u}$  as the sum of  $\vec{m} = \vec{u}_{\parallel}$  parallel to  $\vec{v}$ , and  $\vec{n} = \vec{u}_{\perp}$  orthogonal to  $\vec{v}$ .

**Problem 2.** Consider three points  $A(-2, 1, -2)$ ,  $B(1, 2, -1)$ ,  $C(1, 2, 5)$ .

- (a) Are the points  $A, B, C$  collinear? Justify your answer using the cross-product.
- (b) Find the area of the triangle formed by  $A, B, C$ .
- (c) Find the equation of the plane that contains  $A, B, C$ .

**Problem 3.** Consider two points  $E(1, 0, 1)$ ,  $F(-3, 2, 3)$ . Find a parametric equation of the line through  $E$  and  $F$ .

**Problem 4.**

- (a) Find the angle between the planes  $x - y = 1$  and  $y - z = 2$ , and the line of intersection.  
(Hint: Angle between the planes is the angle between their normal vectors.)
- (b) Find the equation of the plane that passes through the point  $(2, -1, -1)$  and is perpendicular to the line  $\langle 0, 1, 2 \rangle + t\langle 2, -1, -1 \rangle$ .

**Problem 5.** For each equation below, sketch the surface in  $\mathbb{R}^3$  that it describes.

(a)  $4x^2 + 4y^2 + z^2 = 16$

(b)  $4x^2 + y^2 + 4z^2 = 16$

(c)  $z = 4x^2 + 9y^2$

(d)  $z = 4x^2 - 9y^2$

(e)  $4x^2 + 9y^2 = 2z^2 + 72$

(f)  $9x^2 + 4z^2 = 2y^2 - 72$

(g)  $9x^2 + 4y^2 = 2z^2$

(h)  $4x^2 - 9y^2 = 72$

**Problem 6.** A particle starts at location  $\mathbf{i} - 2\mathbf{j} + \mathbf{k}$  with initial velocity  $2\mathbf{i} - \mathbf{j} + 3\mathbf{k}$ . Its acceleration is  $\mathbf{a}(t) = 3t\mathbf{i} + 6t^2\mathbf{j} - 2\mathbf{k}$ . Find the location of the particle at  $t = 2$ .

**Problem 7.**

A string in the shape of a helix has a height of 16 cm and makes three full rotations over a circle of radius 2 cm.

1. Find a parametrization  $\mathbf{r}(t)$  for the string.
2. Compute the length of the string.

**Problem 8.**

Show that if  $\|\mathbf{r}(t)\| = c$  then  $\mathbf{r}(t) \perp \mathbf{r}'(t)$ .

**Problem 9.**

The position of a particle is  $\mathbf{r}(t) = e^t\mathbf{i} + \sqrt{2}t\mathbf{j} + e^{-t}\mathbf{k}$ .

1. Show that the speed of the particle at time  $t$  is  $e^t + e^{-t}$ .
2. Find the unit tangent vector  $\mathbf{T}(t)$ .
3. Find the total distance travelled by the particle for  $1 \leq t \leq 4$ .