Calc III Sample midterm 1

Q1 a) $\|\underline{v}\| = \sqrt{1+4+1} = \sqrt{6}$ so $\|\hat{\underline{v}}\| = \frac{1}{\sqrt{6}}\langle 1,2,-1\rangle$

b) $\text{proj}_{\underline{v}}\,\underline{u} = \frac{\underline{u}.\underline{v}}{\underline{v}.\underline{v}}\,\underline{v} = \frac{4-6-5}{6}\langle 1,2,-1\rangle = \frac{-7}{6}\langle 1,2,-1\rangle$

so $\|\text{proj}_{\underline{v}}\,\underline{u}\| = \frac{-7}{6}\frac{1}{\sqrt{6}} = \frac{-7\sqrt{6}}{36}$.

c) $\underline{u}_{\parallel} = \|\text{proj}_{\underline{v}}\,\underline{u}\| = -\frac{7}{6}\langle 1,2,-1\rangle$

$\underline{u}_{\perp} = \underline{u} - \underline{u}_{\parallel} = \langle 4,-3,5\rangle + \frac{7}{6}\langle 1,2,-1\rangle$ (check $u_{\perp}.\underline{v} = 0$!)

Q2 a) $\overrightarrow{AB} = \langle 3,1,1\rangle$ $\overrightarrow{AC} = \langle 3,1,7\rangle$

$$\overrightarrow{AB}\times\overrightarrow{AC} = \begin{vmatrix} i & j & k \\ 3 & 1 & 1 \\ 3 & 1 & 7 \end{vmatrix} = \langle 7-1, -(21-3), 3-3\rangle = \langle 6,-18,0\rangle$$

$\overrightarrow{AB}, \overrightarrow{AC}$ not parallel as $\overrightarrow{AB}\times\overrightarrow{AC} \neq \underline{0}$ so A, B, C not colinear.

b) area $= \frac{1}{2}\|\overrightarrow{AB}\times\overrightarrow{AC}\| = \cancel{\frac{1}{2}(36+} \frac{1}{2}.6\,\|\langle 1,-3,0\rangle\| = 3\sqrt{10}$

c) normal vector $\underline{n} = \langle 6,-18,0\rangle$ or $\langle 1,-3,0\rangle$

$\underline{n}.(\underline{x} - \langle \cancel{3,1,7}\ 1,2,5\rangle) = 0 \quad \leftrightarrow \quad \langle 1,-3,0\rangle(\langle x,y,z\rangle - \langle \cancel{3,1,7}\ 1,2,5\rangle) = 0$

$x - 3y = -5$

Q3 $\underline{r}(t) = \langle 1,0,1\rangle + t\langle -4,2,2\rangle$

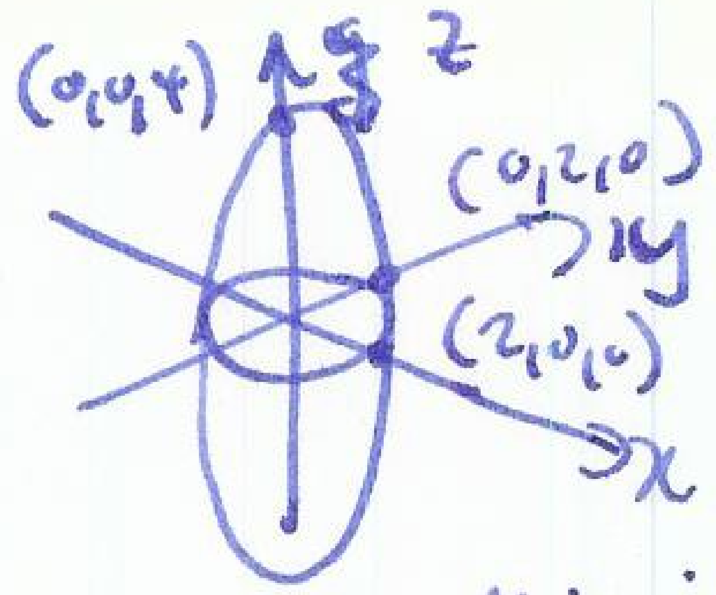
Q4 a) $\underline{n}_1 = \langle 1,-1,0\rangle$ $\underline{n}_2 = \langle 0,1,-1\rangle$

angle: $\underline{n}_1.\underline{n}_2 = \|n_1\|\,\|n_2\|\cos\theta$ $\quad \cos\theta = \frac{-1}{2}$ $\quad \theta = \pi/3$.

b) $\langle 2,-1,-1\rangle(\underline{x} - \langle 2,-1,-1\rangle) = 0$ $\quad 2x - y - z = 6$

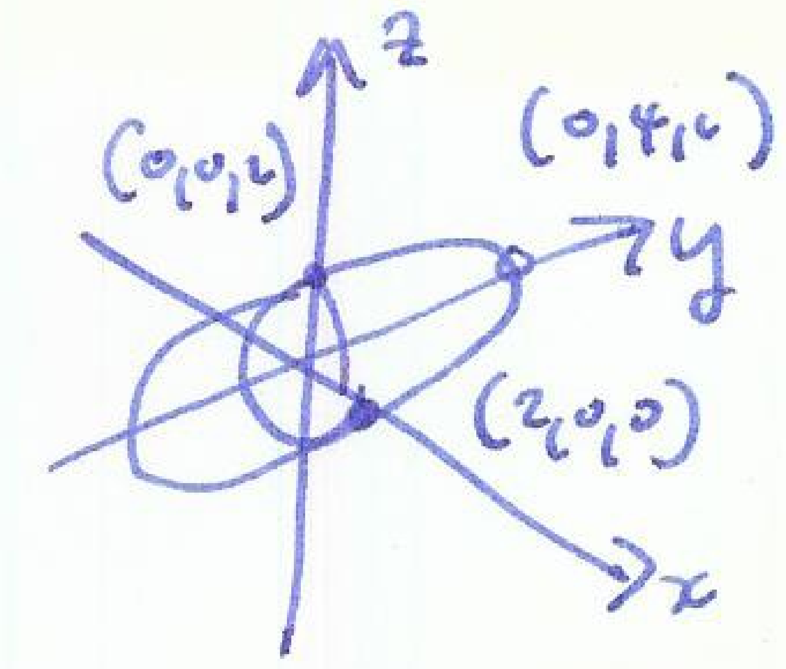
Q5

a)



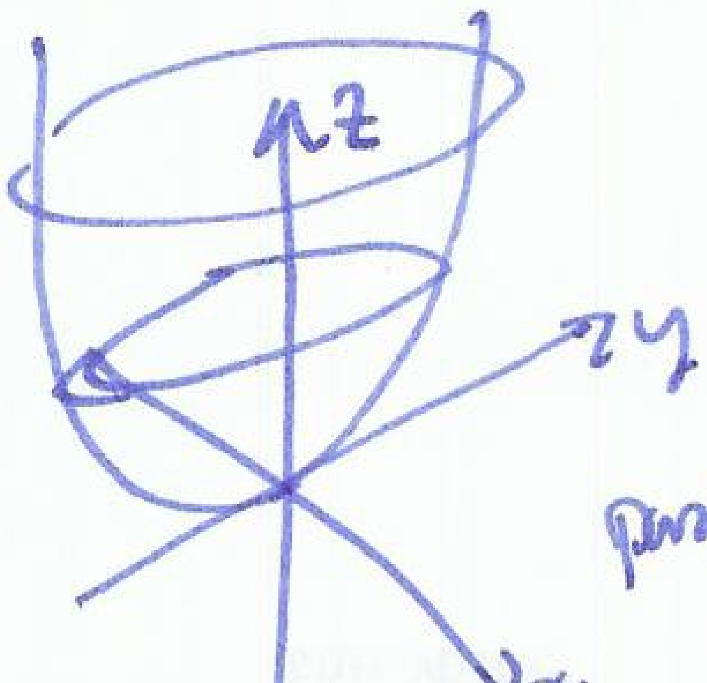
ellipsoid

b)



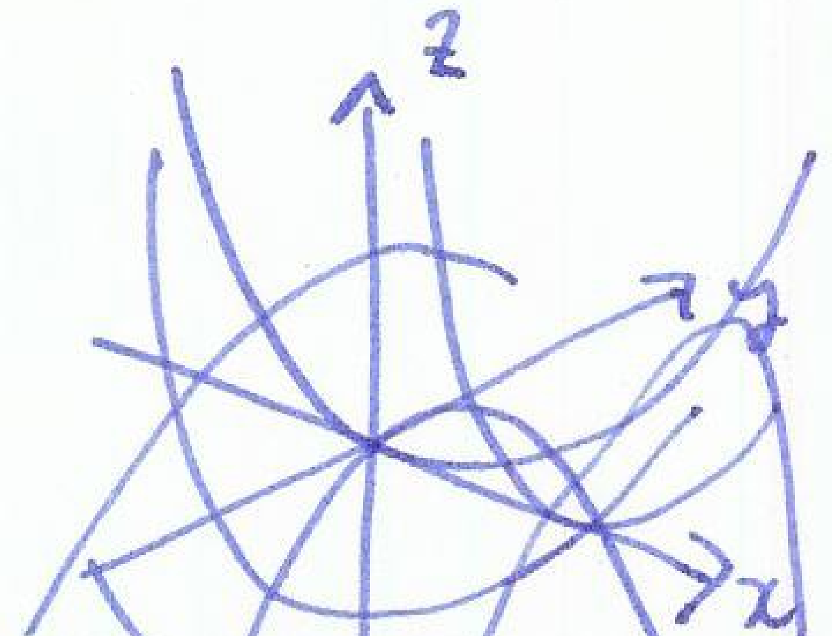
ellipsoid

c)



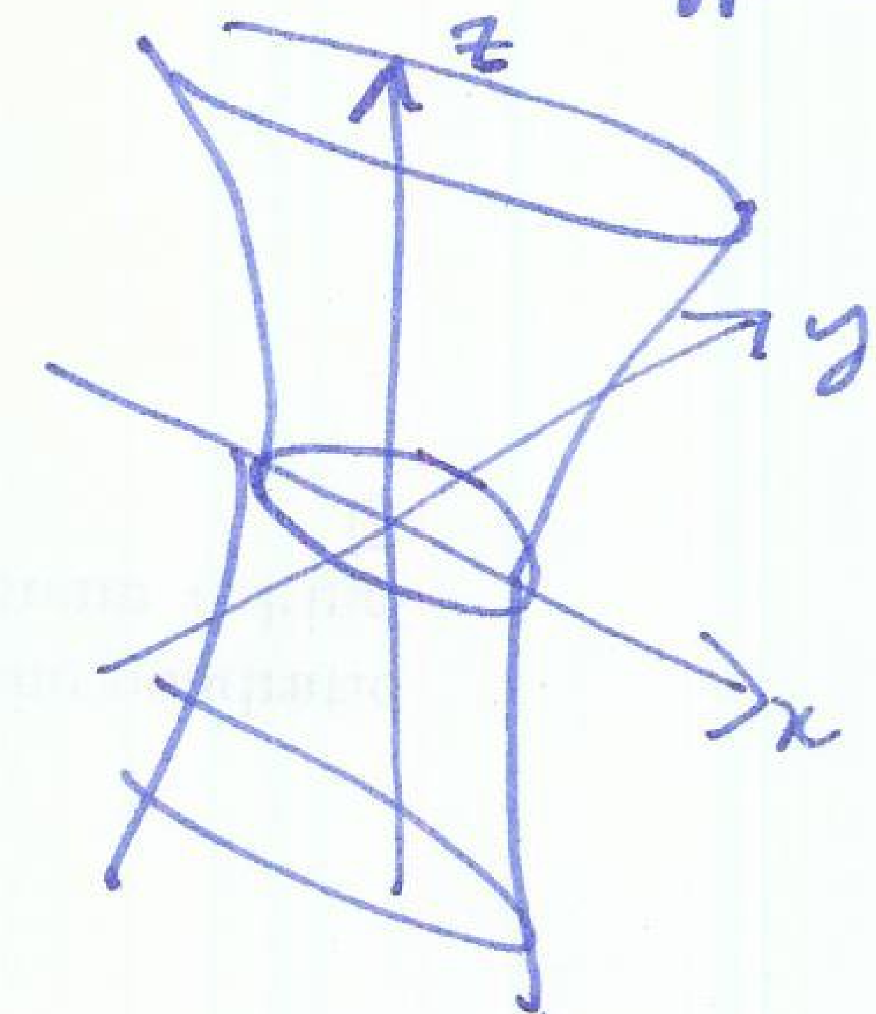
paraboloid

d)

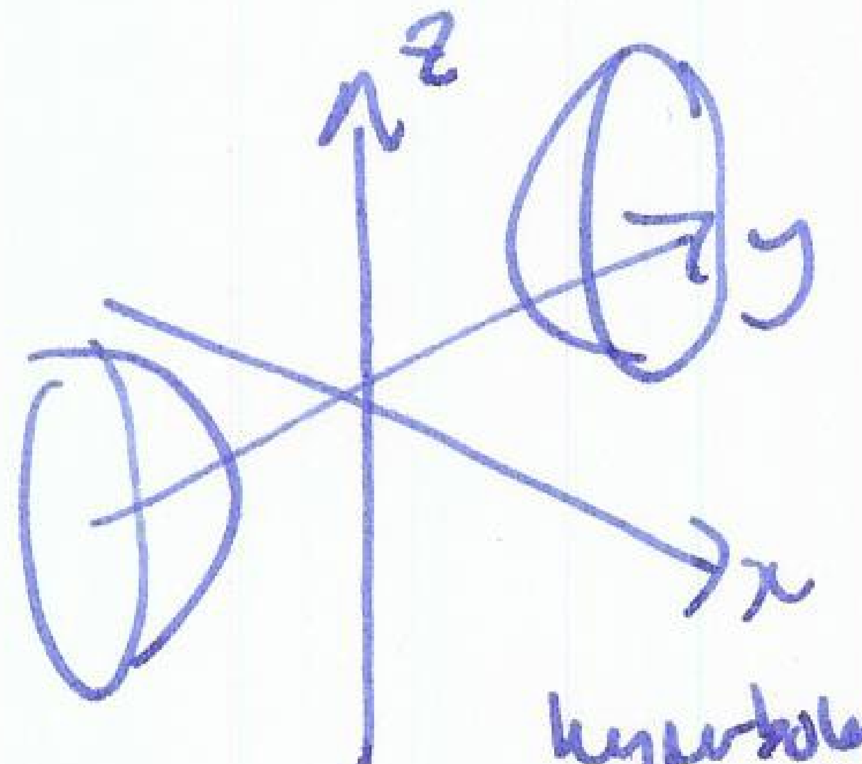


e)

1-sheet hyperboloid

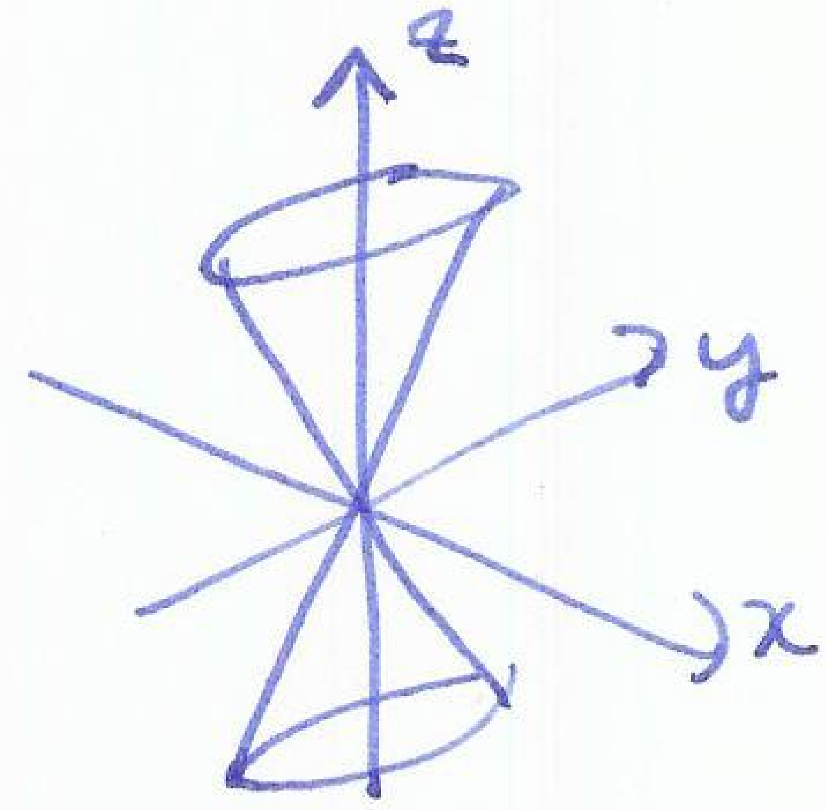


f)



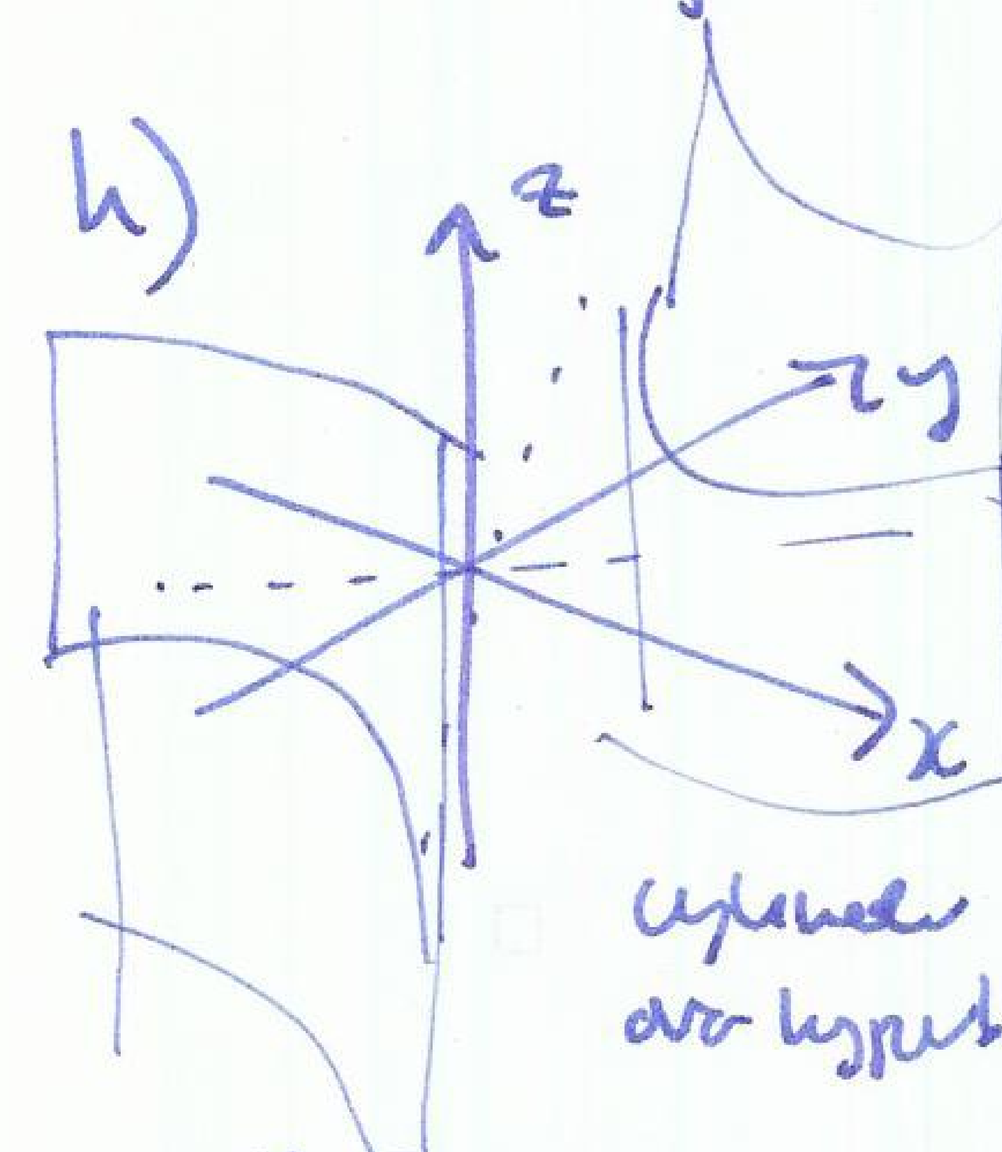
hyperboloid of 2 sheets

g)



cone

h)



cylinder over hyperbola.

Q6 $\underline{r}''(t) = \langle 3t, 6t^2, -2 \rangle$

$\underline{r}'(t) = \langle \frac{3}{2}t^2, 2t^3 - 2t \rangle + \underline{v}_0 = \langle \frac{3}{2}t^2 + 2, 2t^3 - 1, -2t + 3 \rangle$

$\underline{r}(t) = \langle \frac{1}{2}t^3 + 2t, \frac{1}{2}t^4 - t, -t^2 + 3t \rangle + \underline{r}_0 = \langle \frac{1}{2}t^3 + 2t + 1, \frac{1}{2}t^4 - t - 2, -t^2 + 3t + 1 \rangle.$

Q7 1) $\underline{r}(t) = \langle 2\cos t, 2\sin t, \frac{16t}{6\pi} \rangle \qquad 0 \le t \le 6\pi$

2) $\int_0^{6\pi} \|\underline{r}'(t)\| dt = \int_0^{6\pi} \| \langle -2\sin t, 2\cos t, \frac{16}{6\pi} \rangle \| dt$

$= \int_0^{6\pi} \sqrt{4 + \frac{16^2}{36\pi^2}}\, dt = 6\pi \sqrt{4 + \frac{16^2}{36\pi^2}}.$

Q8 $\|\underline{r}(t)\| = c \Rightarrow \underline{r}(t) \cdot \underline{r}(t) = c^2$ differentiate wrt t.

$2\underline{r}(t) \cdot \underline{r}'(t) = 0 \Rightarrow \underline{r}(t), \underline{r}'(t)$ perpendicular.

Q9
4)

$\underline{r}(t) = \langle e^t, \sqrt{2}t, e^{-t} \rangle$

$\underline{r}'(t) = \langle e^t, \sqrt{2}, e^{-t} \rangle$

$\|\underline{r}'(t)\| = \sqrt{e^{2t} + 2 + e^{-2t}} = \sqrt{(e^t + e^{-t})^2} = e^t + e^{-t}$

2) $T(t) = \dfrac{\underline{r}'(t)}{\|\underline{r}'(t)\|} = \dfrac{1}{e^t + e^{-t}} \langle e^t, \sqrt{2}, e^{-t} \rangle$

3) $\int_1^4 \|\underline{r}'(t)\|\, dt = \int_1^4 e^t + e^{-t}\, dt = \left[e^t - e^{-t} \right]_1^4 = e^4 - e^{-4} - e^1 + e^{-1}.$