Review question solutions

Q1 $c(t) = \langle 1+3t, 1+2t, 1+t \rangle$, $0 \le t \le 1$ $\qquad c'(t) = \langle 3, 2, 1 \rangle$

$$\int_C f\,ds = \int_0^1 \left((1+t)^2 + (1+3t)(1+2t)\right)\sqrt{14}\,dt$$

$$= \int_0^1 7t^2 + 7t + 2\,dt = \left[\frac{7}{3}t^3 + \frac{7}{2}t^2 + 2t\right]_0^1 = \frac{7}{3} + \frac{7}{2} + 2$$

Q2 $\nabla \times \underline{F} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ y & -x & z \end{vmatrix} = \langle 0, 0, -2 \rangle \neq \underline{0}$ so not conservative.

$c(t) = \langle 3\cos t, 3\sin t, 1 \rangle$ $0 \le t \le 2\pi$ $\qquad c'(t) = \langle -3\sin t, 3\cos t, 0 \rangle$

$$\int_0^{2\pi} -9\sin^2 t - 9\cos^2 t\,dt = -18\pi$$

Q3 $\nabla \times \underline{F} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ ye^{xy} & xe^{xy} + ze^{yz} & ye^{yz} \end{vmatrix} = \langle \underbrace{e^{yz} + zye^{yz} - e^{yz} - yze^{yz}}_{=0}, 0, e^{xy} + xye^{xy} - e^{xy} - xye^{xy} \rangle = \underline{0}$

$\Rightarrow \underline{F}$ is conservative.

$$\int_C \underline{F}\cdot d\underline{s} = f(-2,1,1) - f(1,-1,2)$$

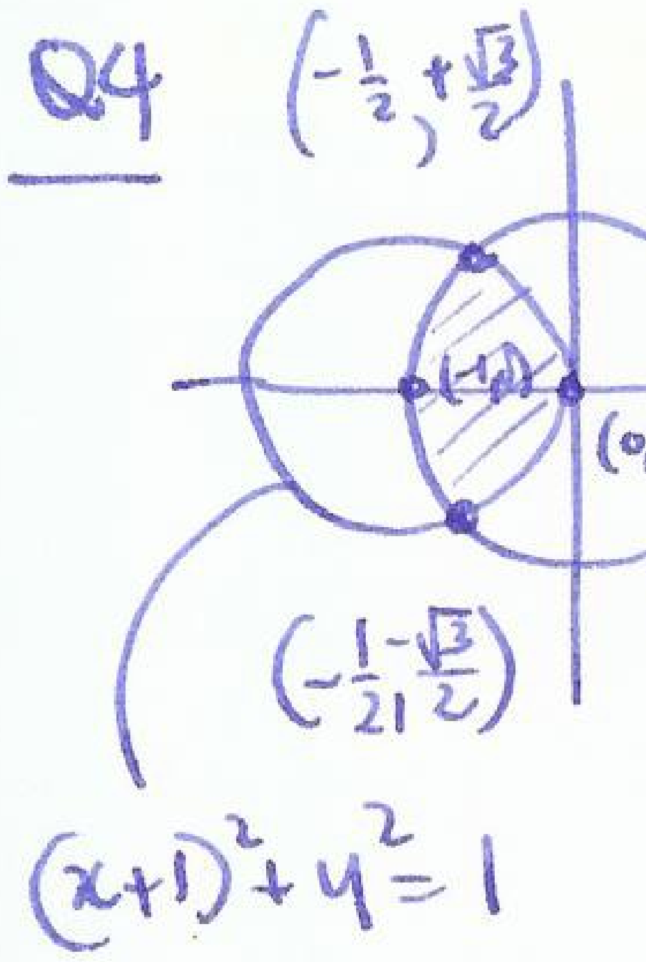
find f:

$$\left.\begin{aligned} \int ye^{xy}\,dx &= e^{xy} + f_1(y,z) \\ \int xe^{xy} + ze^{yz}\,dy &= e^{xy} + e^{yz} + f_2(x,z) \\ \int ye^{yz}\,dz &= e^{yz} + f_3(x,y) \end{aligned}\right\}$$

choose $f(x,y,z) = e^{xy} + e^{yz}$

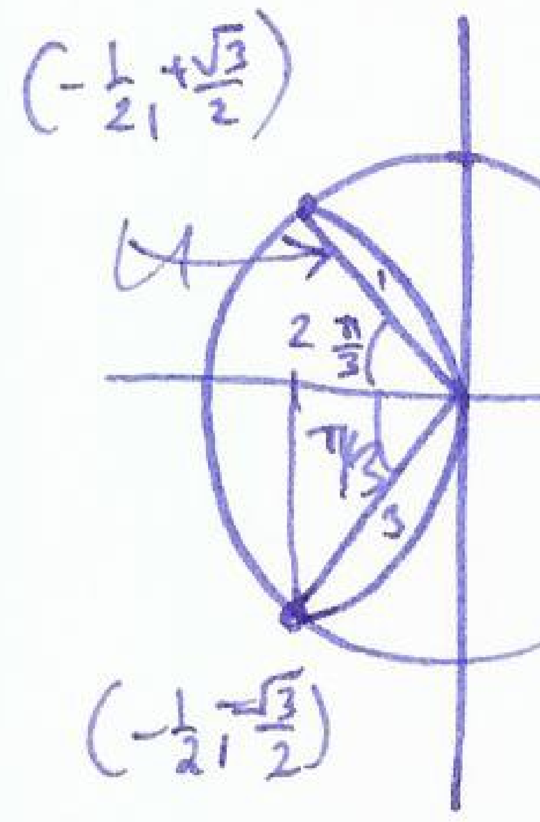
so $\int_C \underline{F}\cdot d\underline{s} = e^{-2} + e - e^{-1} - e^{-2}$
$= e - e^{-1}$.

Q4



$$\int_{-\frac{\sqrt3}{2}}^{+\sqrt3/2} \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}-1} f(x,y)\,dx\,dy$$

$$(x+1)^2+y^2=1$$
$$\Updownarrow$$
$$(r\cos\theta+1)^2+r^2\sin^2\theta=1$$
$$\Updownarrow$$
$$r^2+2r\cos\theta=0$$
$$\Updownarrow$$
$$r=-2\cos\theta.$$

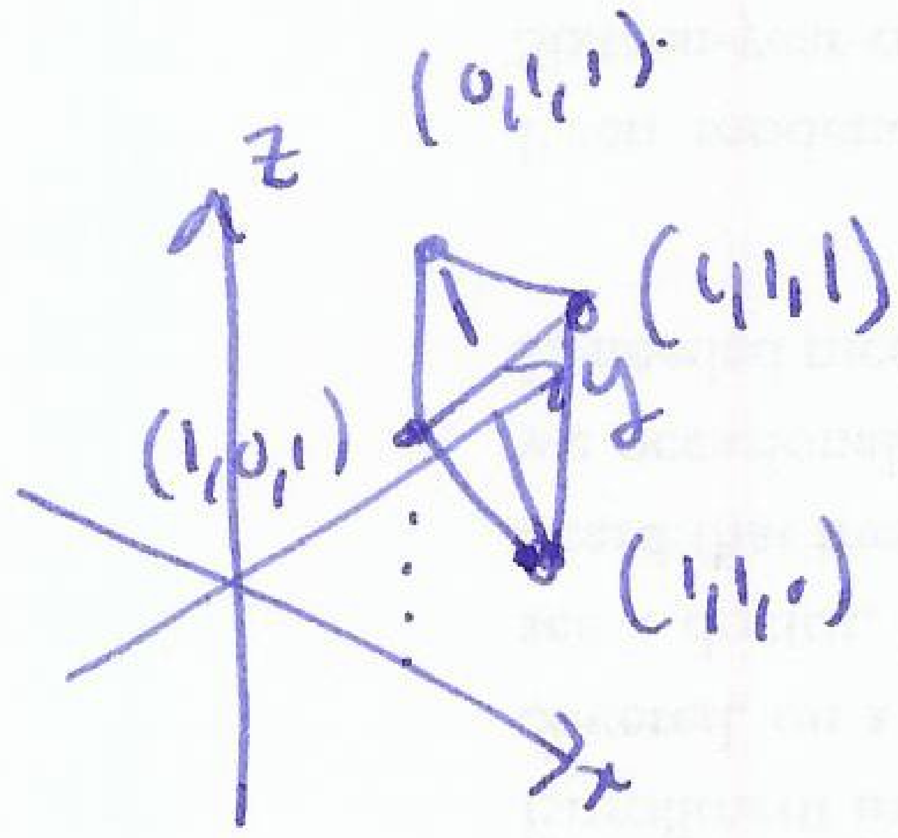


$$1: \int_{\pi/2}^{5\pi/6} \int_0^{-2\cos\theta} f(x(r,\theta),y(r,\theta))\, r\,dr\,d\theta$$

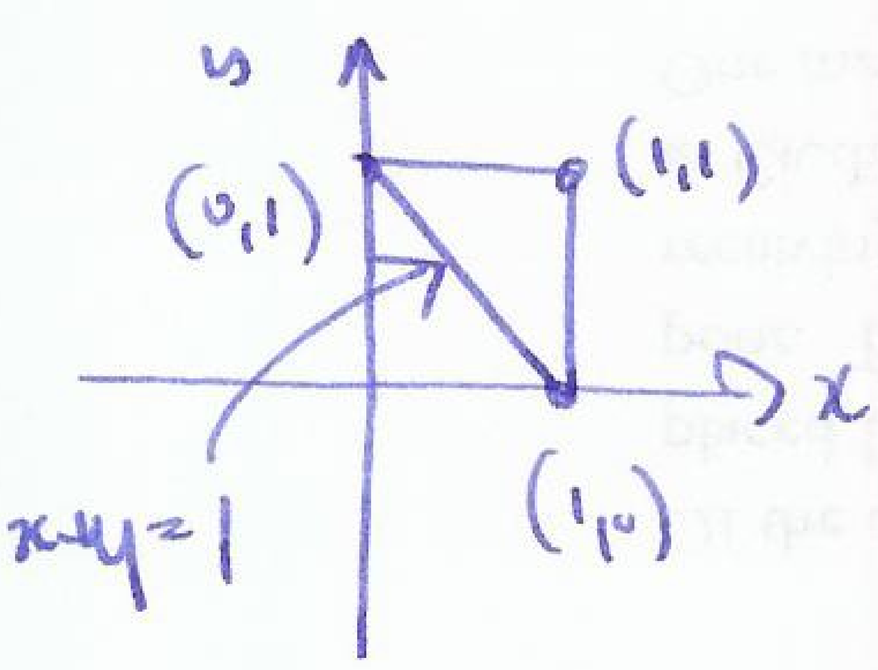
$$+\ 2: \int_{5\pi/6}^{7\pi/6} \int_0^1 \text{---}\ r\,dr\,d\theta$$

$$+\ 3: \int_{7\pi/6}^{3\pi/2} \int_0^{-2\cos\theta} \text{---}\ r\,dr\,d\theta$$

Q5



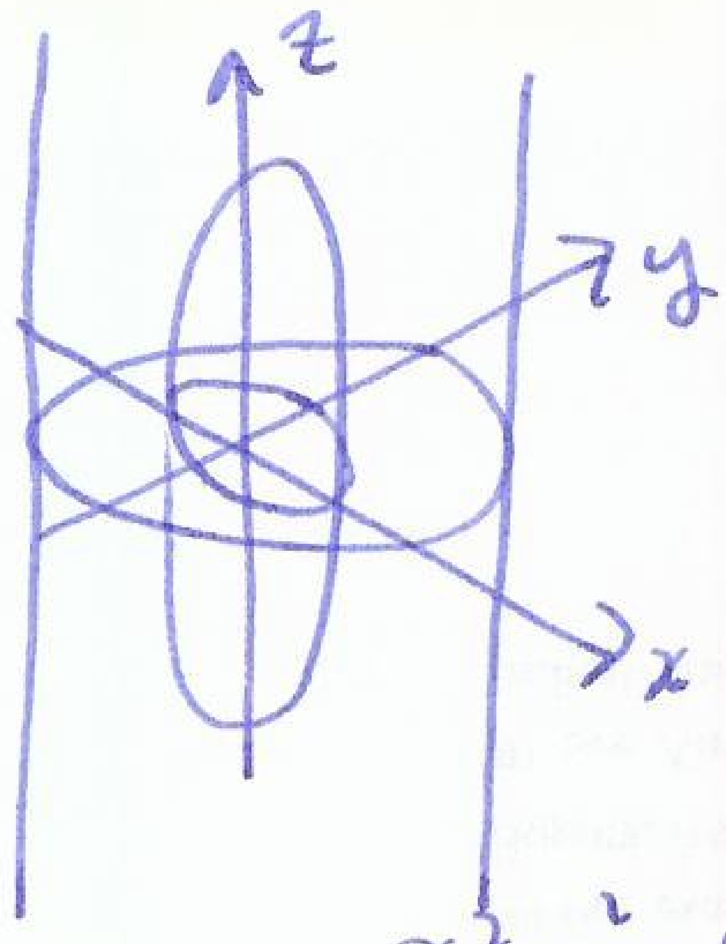
$$\int_0^1 \int_{1-x}^1 \int_{1-x-y}^1 f(x,y,z)\,dz\,dy\,dx$$



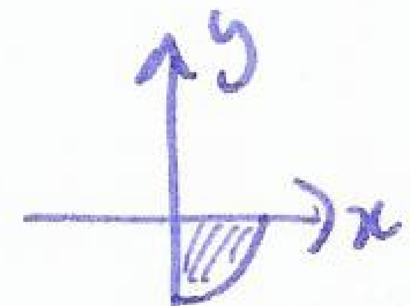
plane through $(0,1,1)$, $(1,0,1)$, $(1,1,0)$

is $x+y+z=1$

Q6



use cylindricals.



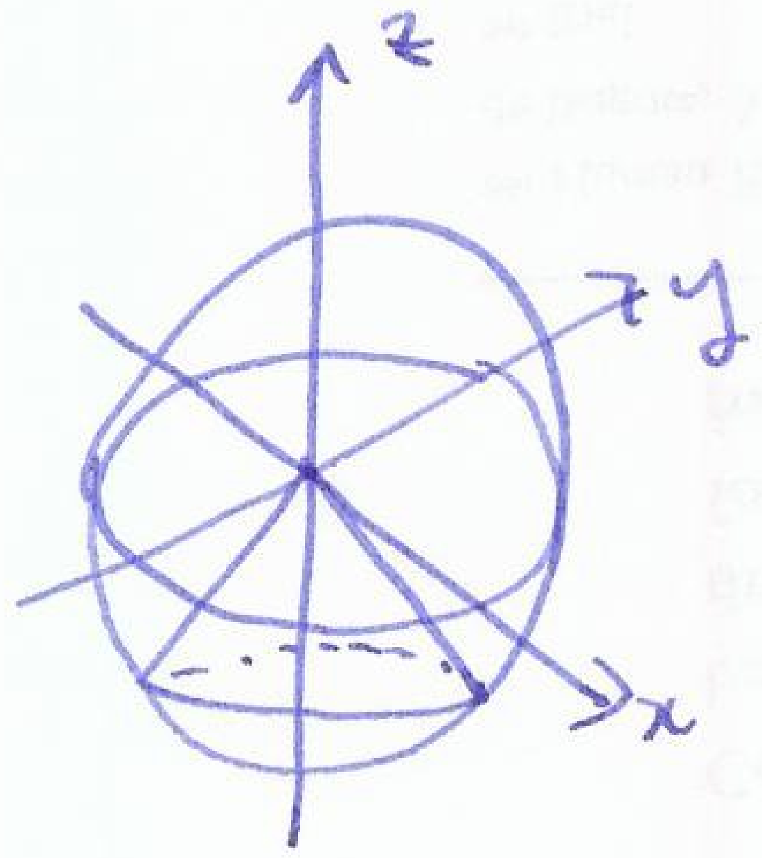
$2x^2+2y^2+z^2=4$

$\iff \quad 2r^2+z^2=4.$

$x^2+y^2=4 \iff r=2.$

$$\int_{\frac{3\pi}{2}}^{2\pi}\int_{0}^{\sqrt{2}}\int_{-\sqrt{4-2r^2}}^{0} f(-)\; r\,dz\,dr\,d\theta$$

Q7

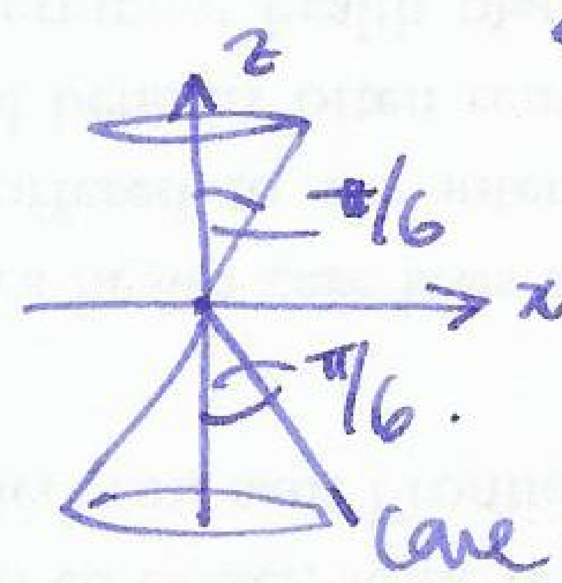


use sphericals.

(meant $z=-\sqrt{3x^2+3y^2}$.

$z^2=+3x^2+3y^2$

$\iff \quad \rho^2\cos^2\phi = +3\rho^2\cos^2\theta\sin^2\phi+3\rho^2\sin^2\theta\sin^2\phi.$

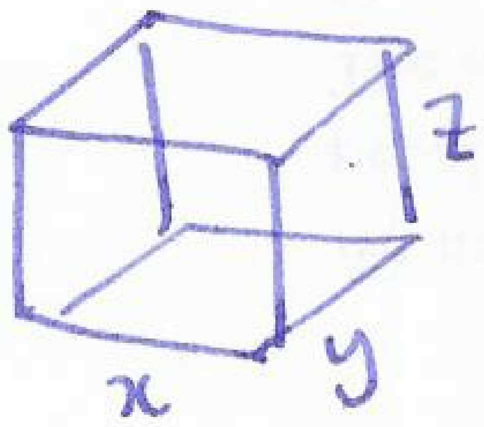


$\iff \quad \cos^2\phi=+3\sin^2\phi \iff \tan\phi=1/\sqrt{3}.$

$\phi=\frac{\pi}{6}.$

$$\int_{\frac{5\pi}{6}}^{\pi}\int_{0}^{2\pi}\int_{0}^{5} f(-)\;\rho^2\sin\phi\;d\rho\,d\theta\,d\phi.$$

Q8



$V=xyz=5000$

$H=4xy+xy+2yz+2xz=5xy+2yz+2xz.$

$\nabla H=\langle 5y+2z,\ 5x+2z,\ 2y+2x\rangle$

$\nabla V=\langle yz,\ xz,\ xy\rangle$

$\nabla H=\lambda\nabla V$:

$V=5000$

$$\left.\begin{array}{l}5y+2z=\lambda yz\\5x+2z=\lambda xz\\2y+2x=\lambda xy\end{array}\right\}\qquad \begin{array}{l}5xy+2xz=\lambda xyz\\5xy+2yz=\lambda xyz\\2yz+2xz=\lambda xyz\end{array}\quad \begin{array}{l}\}\ x=y\\ \}\ y=\cancel{\tfrac{5}{3}}\,\tfrac{2}{5}z.\end{array}$$

$xyz=5000$

$xyz=5000$: $\quad y^2\frac{5}{2}y=5000 \qquad y^3=2000 \qquad y=\sqrt[3]{2}\cdot 10=x$

$z=\frac{5}{2}\sqrt[3]{2}\cdot 10.$