Math 231 Calculus 1 Spring 12 Midterm 3b

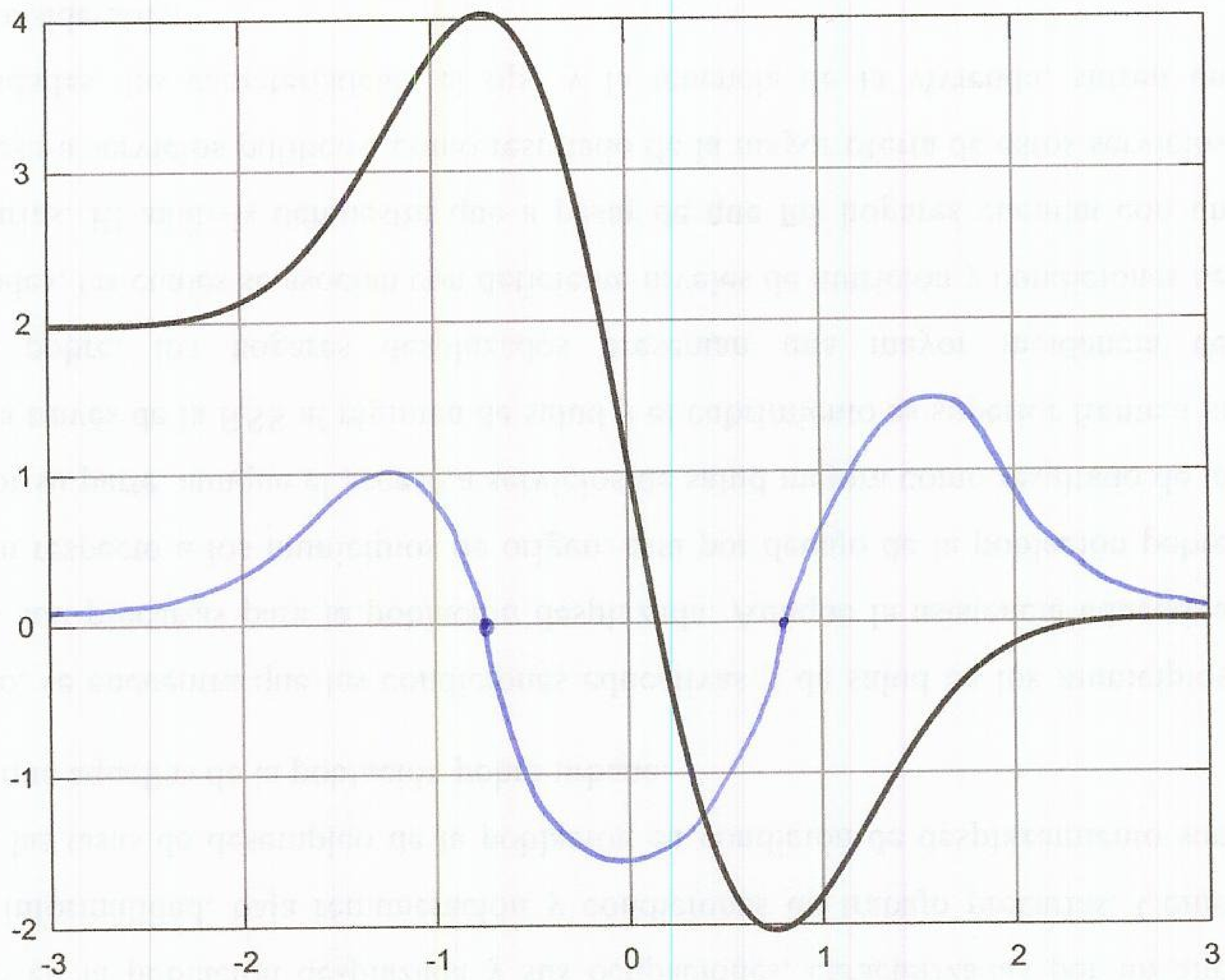
Name: ______________________

- Do any 8 of the following 10 questions.
- You may use a calculator, but no notes.

1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
	80	

Midterm 3	
Overall	

(1) (10 points) Consider the function $f(x)$ defined by the following graph.



(a) Label all regions where $f(x) > 0$. $(-3, 0.2)$

(b) Label all regions where $f'(x) < 0$. $(-0.7, 0.7)$

(c) What is $\lim_{x\to-\infty} f(x)$? 2

(d) What is $\lim_{x\to-\infty} f'(x)$? 0

(e) Sketch a graph of $f'(x)$ on the figure.

(2) (10 points) Consider the function $f(x) = \dfrac{1}{9 - x^2}$.

(a) Find all vertical and horizontal asymptotes of the function.
(b) Find all critical points of the function.
(c) Determine the intervals where $f(x)$ is increasing and decreasing.

a) vertical asymptotes: $9 = x^2 \Rightarrow x = \pm 3$.

horizontal asymptotes: $\lim\limits_{x \to \infty} \dfrac{1}{9-x^2} = 0 \quad \lim\limits_{x \to -\infty} \dfrac{1}{9-x^2} = 0$.

b) $f'(x) = \dfrac{2x}{(9-x^2)^2} = 0 \Rightarrow x = 0$.

f': $-$ $+$ $+$

$-3 \quad 0 \quad 3$

so increasing on $(0, \infty)$
decreasing on $(-\infty, 0)$

(3) (10 points) Consider the function $f(x) = xe^{3x}$.

(a) Find all critical points of the function.

(b) Use the second derivative test to attempt to classify them.

a)

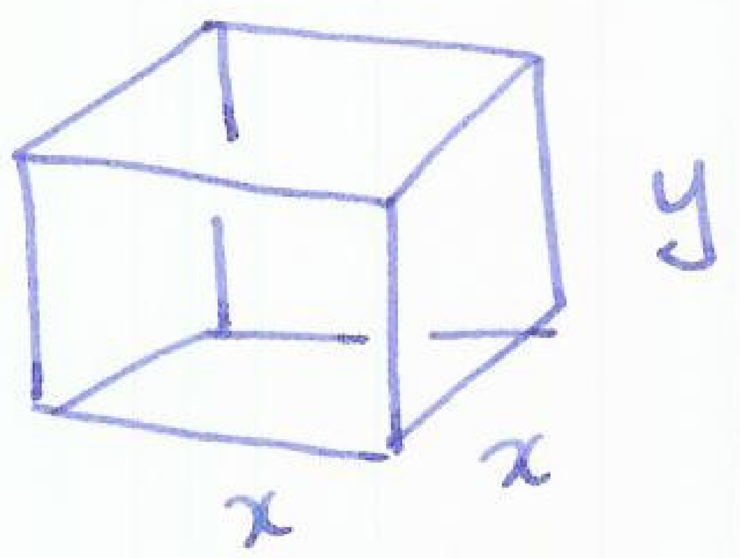
$$f'(x) = e^{3x} + 3xe^{3x}$$

$$f'(x) = 0 \ : \quad e^{3x}(1+3x) = 0 \quad x = -\frac{1}{3}$$

b) $$f''(x) = e^{3x} + 3e^{3x} + 9xe^{3x} = e^{3x}(4 + 9x\cancel{e^{3x}})$$

$$f''(-1/3) > 0 \Rightarrow \text{local min.}$$

(4) (10 points) A cardboard box has a square base with sides of length x, and four vertical sides of height y, and no top. Find the dimensions of the box of volume 1m^3 with smallest surface area.



$$V = x^2 y = 1 \Rightarrow y = \frac{1}{x^2}.$$

$$A = x^2 + 4xy = x^2 + \frac{4}{x}$$

$$\frac{dA}{dx} = 2x - \frac{4}{x^2} = 0 \Rightarrow x^3 = 2$$

$$x = \sqrt[3]{2}.$$

$$y = \frac{1}{\sqrt[3]{4}}.$$

(5) (10 points) Find

$$\lim_{x\to 0}\frac{e^{3x}-1}{\sin x}.$$

$= \text{l'Hospital} = \lim_{x\to 0}\frac{3e^{3x}}{\cos x} = 3$

(6) (10 points) Find

$$\lim_{x\to 0+} x^{2x}. = \lim_{x\to 0+} e^{2x\ln(x)}$$

$$\lim_{x\to 0+} 2x\ln(x) = \lim_{x\to 0+} \frac{2\ln(x)}{1/x} = \lim_{x\to 0+} \frac{2/x}{-x^{-2}} = \lim_{x\to 0+} -2x = 0$$

l'Hôpital

so $\lim_{x\to 0+} x^{2x} = e^0 = 1$

(7) (10 points) Which function grows faster, x or $e^{\sqrt{x}}$? Justify your answer. (Hint: take a limit.)

$$\lim_{x\to\infty} \frac{e^{\sqrt{x}}}{x} \underset{\text{l'Hôpital}}{=} \lim_{x\to\infty} \frac{e^{\sqrt{x}}\cdot\frac{1}{2}x^{-1/2}}{1} = \lim_{x\to\infty} \frac{e^{\sqrt{x}}}{2\sqrt{x}}$$

$$\underset{\text{l'Hôpital}}{=} \lim_{x\to\infty} \frac{e^{\sqrt{x}}\cdot\frac{1}{2}x^{-1/2}}{x^{-1/2}} = \lim_{x\to\infty} \frac{1}{2}e^{\sqrt{x}} \longrightarrow \infty \text{ as } x\to\infty$$

~~as~~ so $e^{\sqrt{x}}$ grows faster.

(8) (10 points) Find the indefinite integral

$$\int e^x - 2\sin(x)\ dx.$$

$e^x + 2\cos(x) + C$

(9) (10 points) Evaluate the definite integral

$$\int_1^2 \frac{1+\sqrt{x}}{x}\,dx. = \int_1^2 x^{-1} + x^{-1/2}\,dx$$

$$= \left[\ln x + 2x^{1/2} \right]_1^2 = \ln 2 + 2\sqrt{2} - 2$$

(10) Find the area under the graph $y = 3x^2 + x$ between $x = 0$ and $x = 1$.

$$\int_0^1 3x^2 + x \, dx = \left[x^3 + \frac{1}{2}x^2 \right]_0^1 = 1 + \frac{1}{2} = \frac{3}{2}$$