Math 231 Calculus 1 Spring 12 Midterm 3a

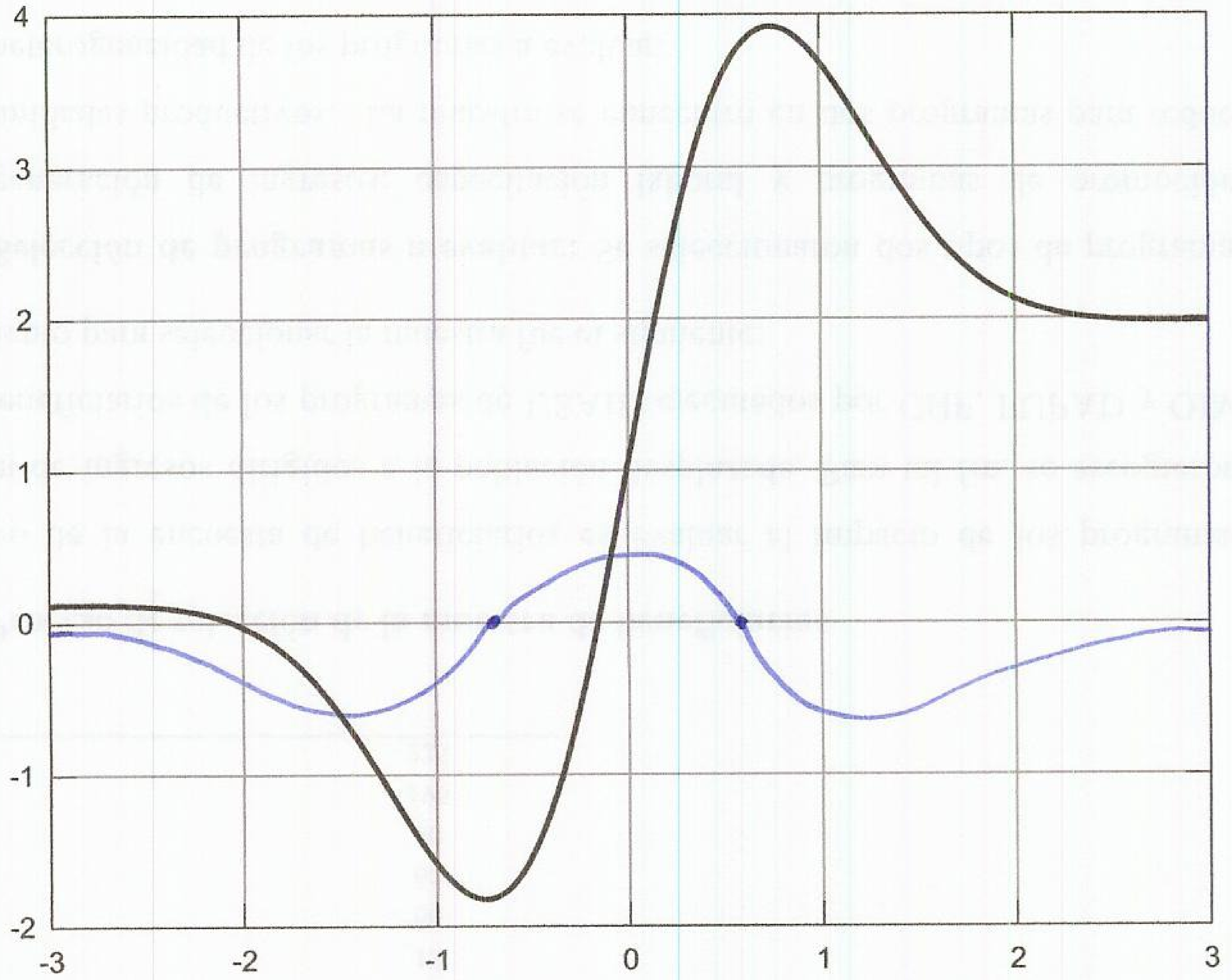
Name: Solutions

- Do any 8 of the following 10 questions.
- You may use a calculator, but no notes.

1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
	80	

Midterm 3	
Overall	

(1) (10 points) Consider the function $f(x)$ defined by the following graph.



(a) Label all regions where $f(x) < 0$. $(-2, 0.2)$

(b) Label all regions where $f'(x) > 0$. $(-0.7, 0.7)$.

(c) What is $\lim_{x\to\infty} f(x)$? 2

(d) What is $\lim_{x\to\infty} f'(x)$? 0

(e) Sketch a graph of $f'(x)$ on the figure.

(2) (10 points) Consider the function $f(x) = \frac{1}{4-x^2}$.

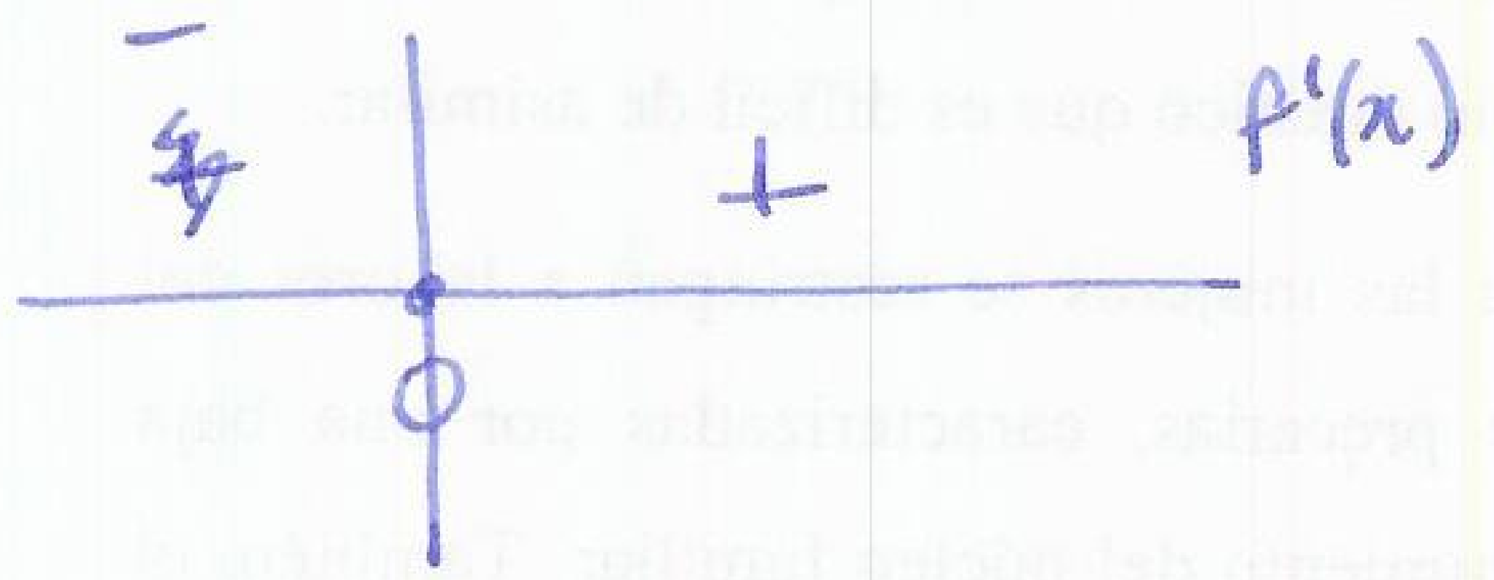
(a) Find all vertical and horizontal asymptotes of the function.
(b) Find all critical points of the function.
(c) Determine the intervals where $f(x)$ is increasing and decreasing.

a) vertical asymptotes: $4-x^2=0 \Rightarrow x = \pm 2$

horizontal asymptotes: $\lim_{x\to\infty} \frac{1}{4-x^2} = 0$ $\lim_{x\to-\infty} \frac{1}{4-x^2} = 0$

b) $f'(x) = -(4-x^2)^{-2} \cdot -2x = 0 \Rightarrow x = 0$

c)



increasing $(0,\infty)$

decreasing $(-\infty,0)$

(3) (10 points) Consider the function $f(x) = xe^{2x}$.

(a) Find all critical points of the function.

(b) Use the second derivative test to attempt to classify them.

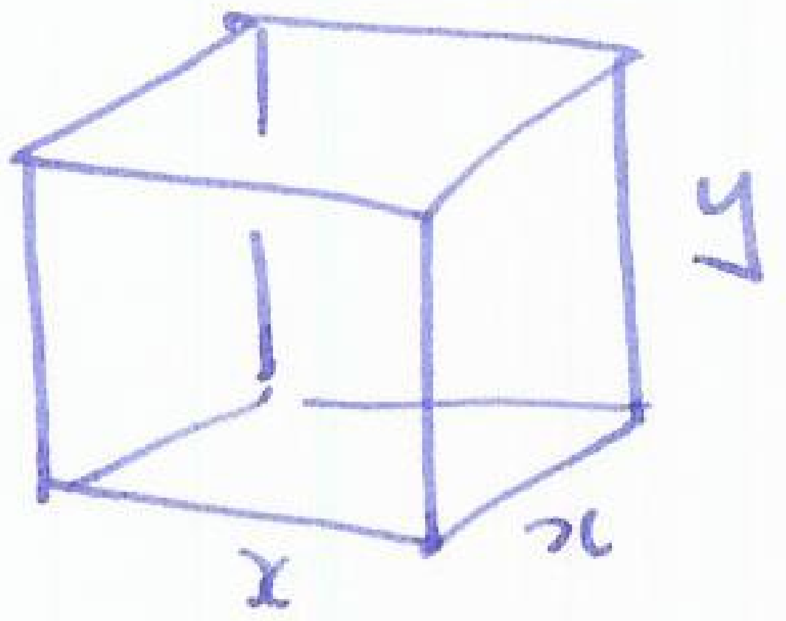
a) $f'(x) = e^{2x} + x(2e^{2x})$

solve $f'(x) = 0$: $e^{2x}(1+2x) = 0 \Rightarrow x = -1/2$

$f''(x) = 2e^{2x} + 2e^{2x} + 4xe^{2x} = 4e^{2x}(1+x)$

$f''(-\frac{1}{2}) > 0 \Rightarrow$ local min

(4) (10 points) A cardboard box has a square base with sides of length x, and four vertical sides of height y, and no top. Find the dimensions of the box of volume 1m^3 with smallest surface area.



$$V = x^2 y = 1 \quad \Rightarrow \quad y = \frac{1}{x^2}$$

$$A = x^2 + 4xy$$

$$A = x^2 + \frac{4}{x}$$

$$\frac{dA}{dx} = 2x - \frac{4}{x^2} = 0$$

$$x^3 = 2$$

$$x = \sqrt[3]{2}, \quad y = \frac{1}{\sqrt[3]{4}}$$

(5) (10 points) Find

$$\lim_{x\to 0} \frac{e^{2x}-1}{\sin x}.$$

L'Hôpital $= \lim_{x\to 0} \frac{2e^{2x}}{\cos x} = 2$

(6) (10 points) Find

$$\lim_{x\to 0+} x^{4x}. = \lim_{x\to 0+} e^{4x\ln(x)}$$

$$\lim_{x\to 0+} 4x\ln(x) = \lim_{x\to 0+} \frac{4\ln(x)}{1/x} \underset{\text{l'Hôpital}}{=} \lim_{x\to 0} \frac{4/x}{-x^{-2}} = \lim_{x\to 0} -4x = 0$$

$$\text{so } \lim_{x\to 0+} e^{4x\ln(x)} = e^{0} = 1$$

(7) (10 points) Which function grows faster, x or $e^{\sqrt{x}}$? Justify your answer. (Hint: take a limit.)

$$\lim_{x\to\infty} \frac{x}{e^{x^{1/2}}} \underset{\text{l'Hôpital}}{=} \lim_{x\to\infty} \frac{1}{e^{\sqrt{x}} \cdot \frac{1}{2}x^{-1/2}} = \lim_{x\to\infty} \frac{2\sqrt{x}}{e^{\sqrt{x}}}$$

$$\underset{\text{l'Hôpital}}{=} \lim_{x\to\infty} \frac{x^{-1/2}}{e^{\sqrt{x}} \frac{1}{2}x^{-1/2}} = \lim_{x\to\infty} \frac{2}{e^{\sqrt{x}}} = 0$$

so $e^{\sqrt{x}}$ grows faster.

(8) (10 points) Find the indefinite integral

$$\int e^x - 4\sin(x)\ dx.$$

$e^{x} + 4\cos(x) + c$

(9) (10 points) Evaluate the definite integral

$$\int_1^2 \frac{\sqrt{x}+1}{x}\,dx. = \int_1^2 x^{-1/2} + x^{-1}\,dx$$

$$= \left[2x^{1/2} + \ln x \right]_1^2 = 2\sqrt{2} + \ln 2 - 2$$

(10) Find the area under the graph $y = 2x^2 + x$ between $x = 0$ and $x = 1$.

$$\int_0^1 2x^2 + x \, dx = \left[\frac{2}{3}x^3 + \frac{1}{2}x^2\right]_0^1 = \frac{2}{3} + \frac{1}{2} = \frac{7}{6}$$