

Math 231 Calculus 1 Spring 12 Midterm 2a/b

Name: Solutions

- Do any 8 of the following 10 questions.
- You may use a calculator, but no notes.

1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
	80	

Midterm 2	
Overall	

(1) (10 points) Find the derivative of $f(x) = e^{-2x^2} \cos(x)$.

$$f'(x) = e^{-2x^2} \cdot -4x \cos(x) + e^{-2x^2} \cdot -\sin(x)$$

(2) (10 points) Find the derivative of

$$f(x) = \frac{\sin(x)}{x^2 + 1}.$$

$$f'(x) = \frac{(x^2 + 1) \cos(x) - 2x \sin(x)}{(x^2 + 1)^2}$$

(3) (10 points) Find the derivative of $f(x) = \tan^{-1}(1/x)$.

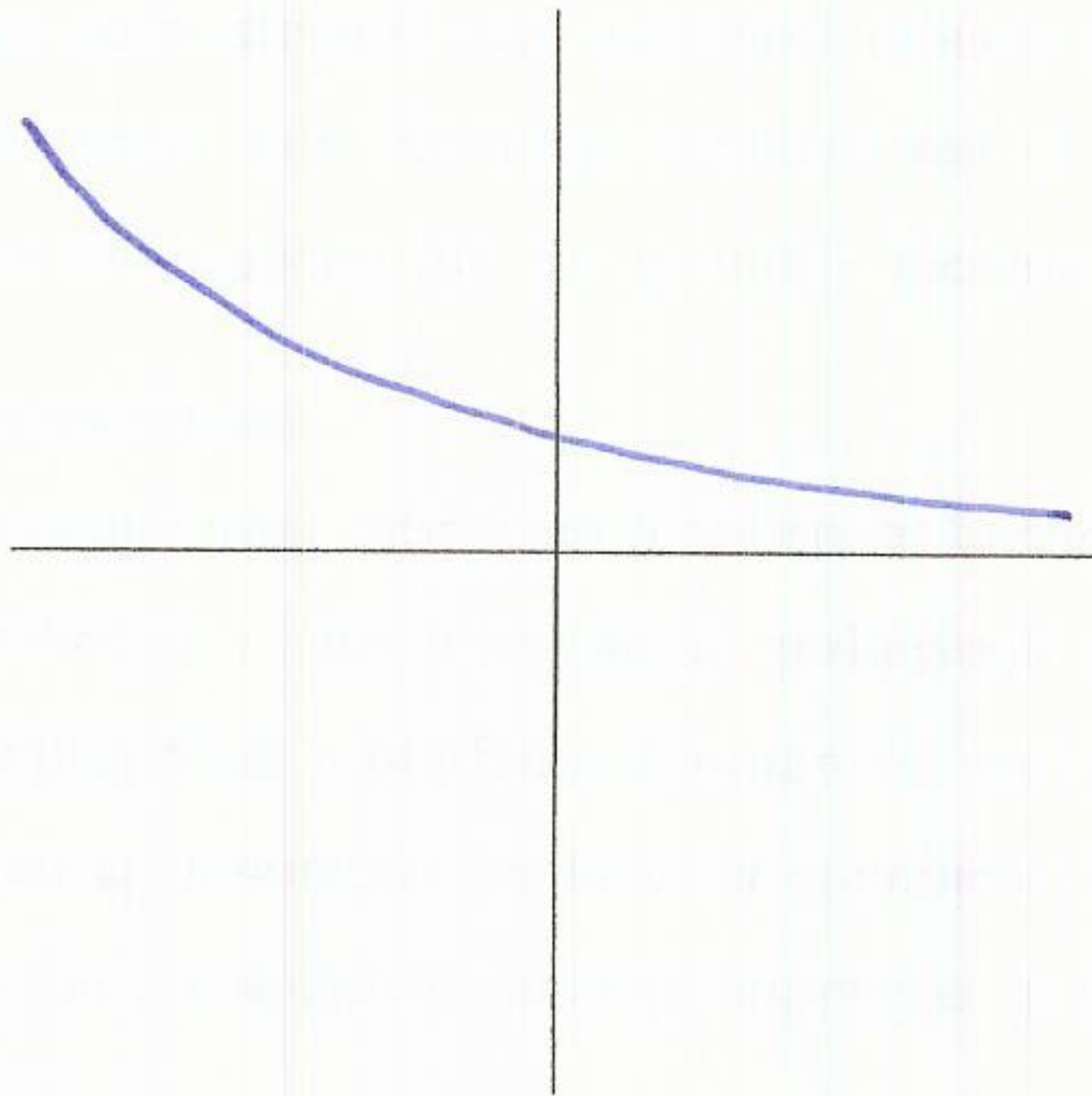
$$f'(x) = \frac{1}{1 + (1/x)^2} \cdot -x^{-2} = \frac{-1/x^2}{1 + 1/x^2} = \frac{-1}{x^2 + 1}$$

(4) (10 points) Find the second derivative of $f(x) = \sqrt{x^2 - 1} = (x^2 - 1)^{1/2}$

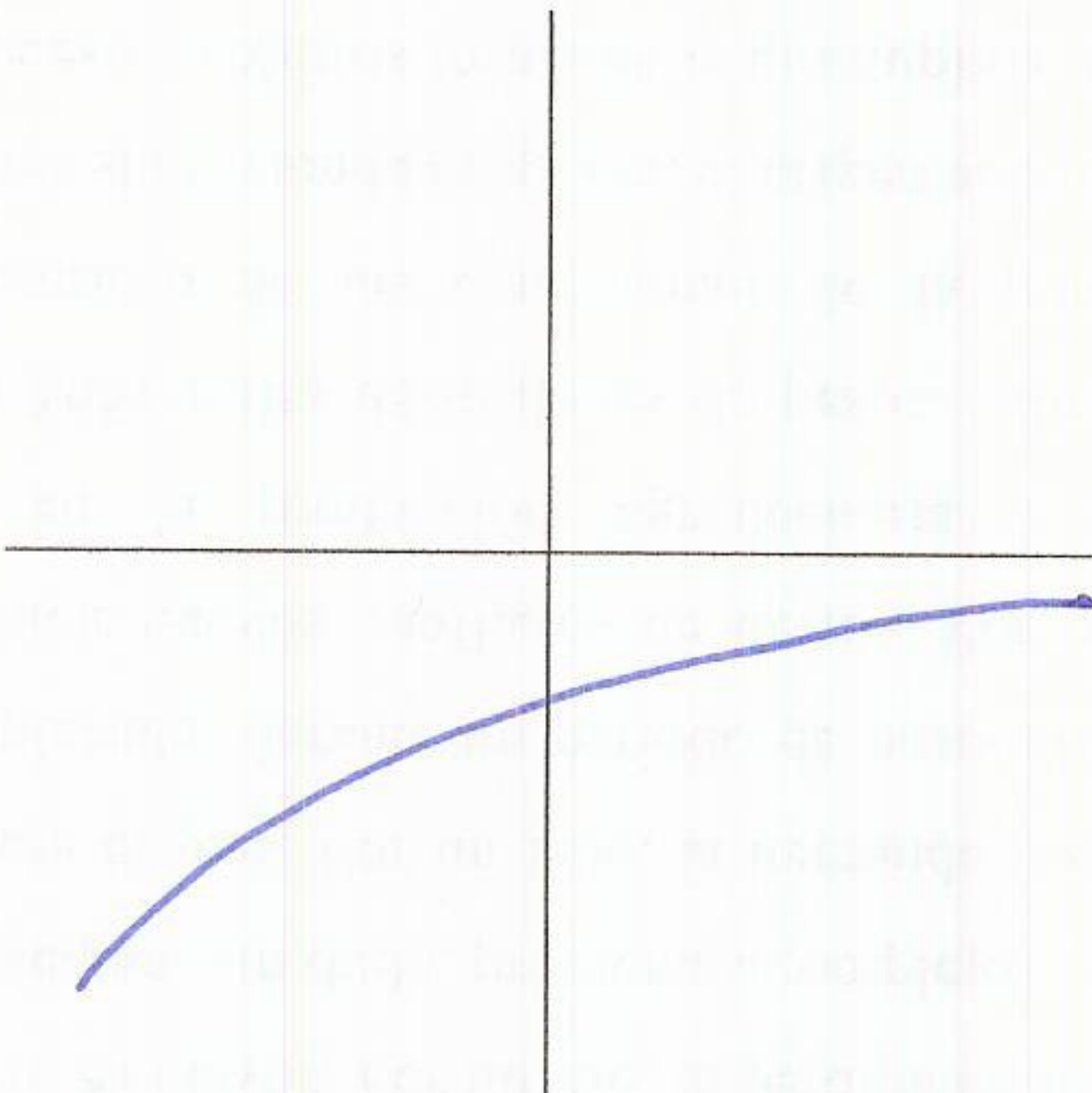
$$f'(x) = \frac{1}{2} (x^2 - 1)^{-1/2} \cdot 2x = x (x^2 - 1)^{-1/2}$$

$$\begin{aligned} f''(x) &= (x^2 - 1)^{-1/2} + x \cdot -\frac{1}{2} (x^2 - 1)^{-3/2} \cdot 2x \\ &= (x^2 - 1)^{-1/2} - x^2 (x^2 - 1)^{-3/2} \end{aligned}$$

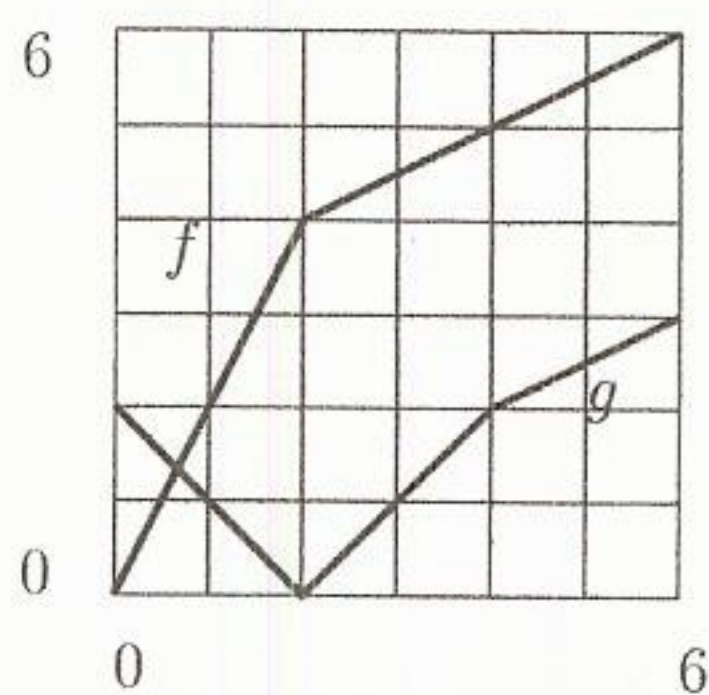
(5) (10 points) Draw the graph of a function $f(x)$ with $f(x) > 0$ and $f'(x) < 0$.



Sketch $f'(x)$ for the graph you drew above.



(6) (10 points) The graphs of the functions $f(x)$ and $g(x)$ are shown below.



- (a) If $h(x) = g(f(x))$, what is $h'(3)$?
 (b) If $h(x) = f(x)/g(x)$, what is $h'(3)$?

$$a) \quad h'(x) = g'(f(x)) \cdot f'(x)$$

$$h'(3) = g'(f(3)) \cdot f'(3) = g'(4\frac{1}{2}) \cdot \frac{1}{2} = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$$

$$b) \quad h'(x) = \frac{g(x)f'(x) - g'(x)f(x)}{(g(x))^2}$$

$$h'(3) = \frac{g(3)f'(3) - g'(3)f(3)}{(g(3))^2} = \frac{1 \cdot \frac{1}{2} - 1 \cdot 4\frac{1}{2}}{1^2} = -4$$

- (7) (10 points) Find the tangent line to the curve given by $x^3 + xy^2 + 2xy = 4$ at the point $(1, 1)$.

$$3x^2 + y^2 + x2y \frac{dy}{dx} + 2y + 2x \frac{dy}{dx} = 0$$

at $(1, 1)$: $6 + 4 \frac{dy}{dx} = 0$ $\frac{dy}{dx} = -\frac{3}{2}$

tangent line : $y - 1 = -\frac{3}{2}(x - 1)$

- (8) (10 points) Use linear approximation to estimate $\sqrt{50}$. What is the percentage error?

$$f(x) = \sqrt{x} \quad 7^2 = 49 \quad 50 = 49 + 1$$

$$f'(x) = \frac{1}{2}x^{-1/2}$$

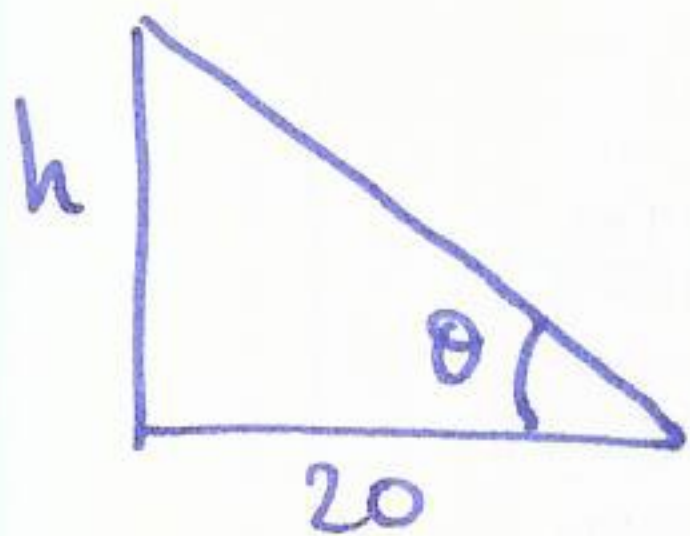
$$f\left(\frac{a+x}{x+a}\right) \approx f(a) + f'(a)(x-a)$$

$$7 + \frac{1}{2} \frac{1}{7} \cdot 1 = 7\frac{1}{14} \approx 7.071429$$

$$\text{absolute error} = \left| \sqrt{\frac{50}{50}} - 7.071429 \right| = 0.000361$$

$$\text{percentage error} = \frac{100 \times 0.000361}{\sqrt{50}} = 0.0051020$$

- (9) (10 points) A rocket is launched vertically upwards from a point 20 km away. When you see the rocket at an angle of $\pi/6$ radians, the angle is increasing at a rate of 0.2 radians per second. How fast is the rocket moving?

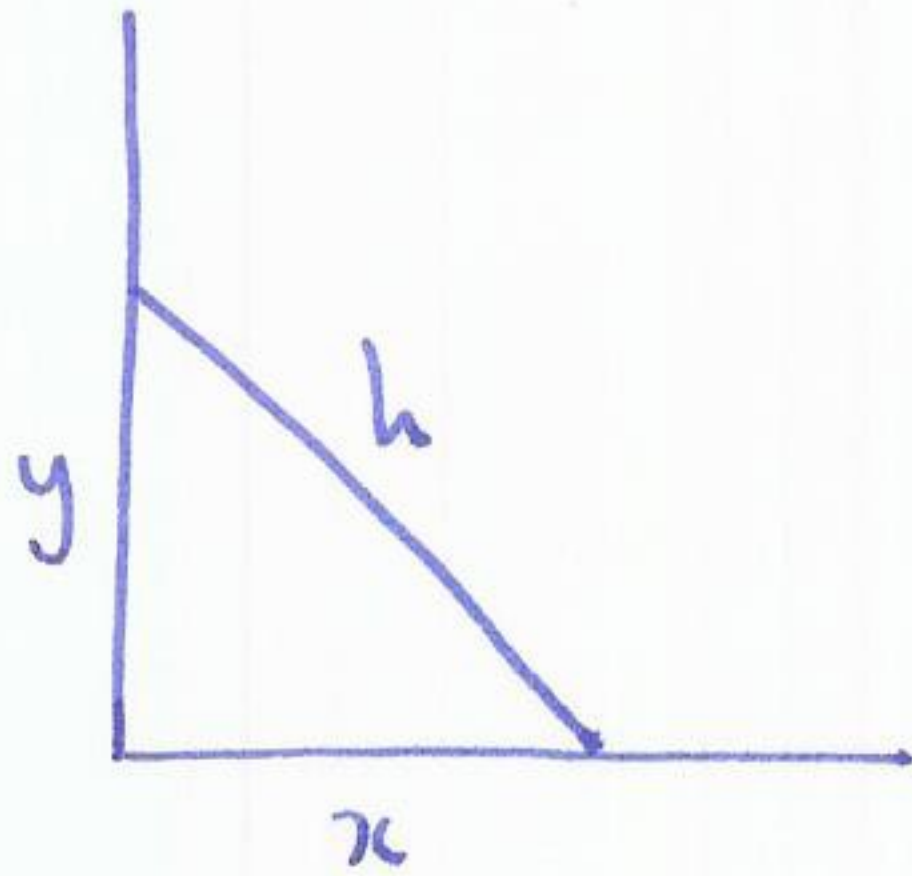


$$\frac{h}{20} = \tan \theta$$

$$\frac{dh}{dt} = 20 \sec^2 \theta \frac{d\theta}{dt}$$

$$\frac{dh}{dt} = 20 \sec^2\left(\frac{\pi}{6}\right) \cdot 0.2 = 4 \cdot \frac{4}{3} = \frac{16}{3} \text{ km/s}$$

- (10) We start from the same point, and I drive east at 30 mph, and you drive north at 20 mph. How fast is the distance between us increasing after two hours?



$$h^2 = x^2 + y^2$$

$$2h \frac{dh}{dt} = 2x \frac{dx}{dt} + 2y \frac{dy}{dt}$$

$$\frac{dx}{dt} = 30 \quad \frac{dy}{dt} = 20$$

after two hours

$$x = 60$$

$$y = 40$$

$$h = \sqrt{40^2 + 60^2}$$

$$= 20\sqrt{13}$$

$$\frac{dh}{dt} = \frac{30 \cdot 60 + 40 \cdot 20}{20\sqrt{13}} = \frac{130}{\sqrt{13}} = 10\sqrt{13}$$