Math 231 Calculus 1 Spring 12 Midterm 2a

Name: Solutions

- Do any 8 of the following 10 questions.
- You may use a calculator, but no notes.

1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
	80	

Midterm 2	
Overall	

(1) (10 points) Find the derivative of $f(x) = e^{-3x^2}\sin(x)$.

$f'(x) = e^{-3x^2} \cdot -6x \sin(x) + e^{-3x^2} \cdot \cos(x)$

(2) (10 points) Find the derivative of

$$f(x) = \frac{\cos(x)}{x^2+1}.$$

$$f'(x) = \frac{(x^2+1).(-\sin(x)) - 2x.\cos(x)}{(x^2+1)^2}$$

(3) (10 points) Find the derivative of $f(x) = \tan^{-1}(1/x)$.

$$f'(x) = \frac{1}{1 + (1/x)^2} \cdot -x^{-2} = \frac{-1/x^2}{1 + 1/x^2} = \frac{-1}{x^2 + 1}$$

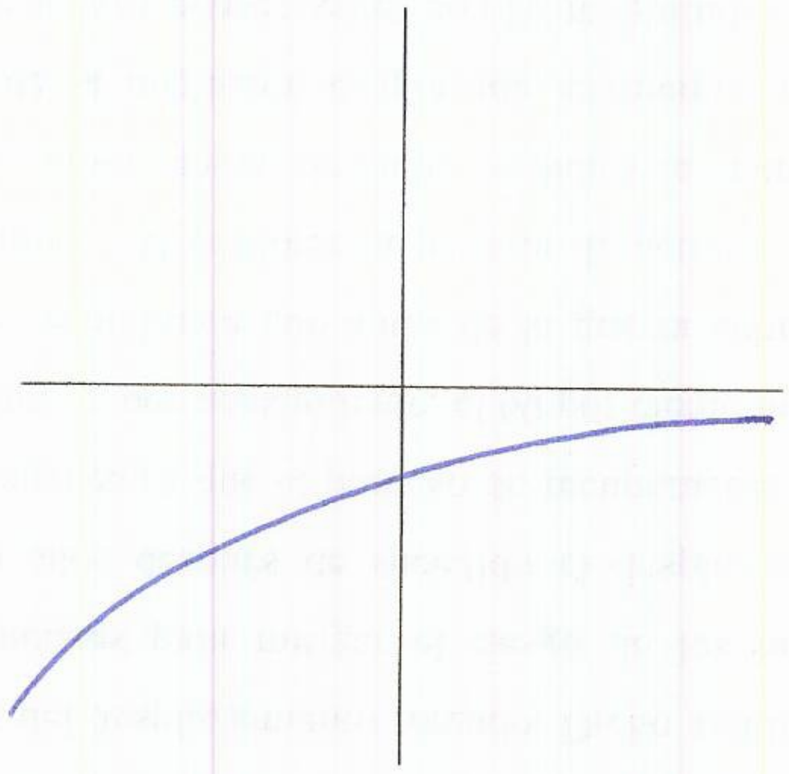
(4) (10 points) Find the second derivative of $f(x) = \sqrt{x^2+1}$ $= (x^2+1)^{1/2}$.

$$f'(x) = \frac{1}{2}(x^2+1)^{-1/2} \cdot 2x = x(x^2+1)^{-1/2}$$

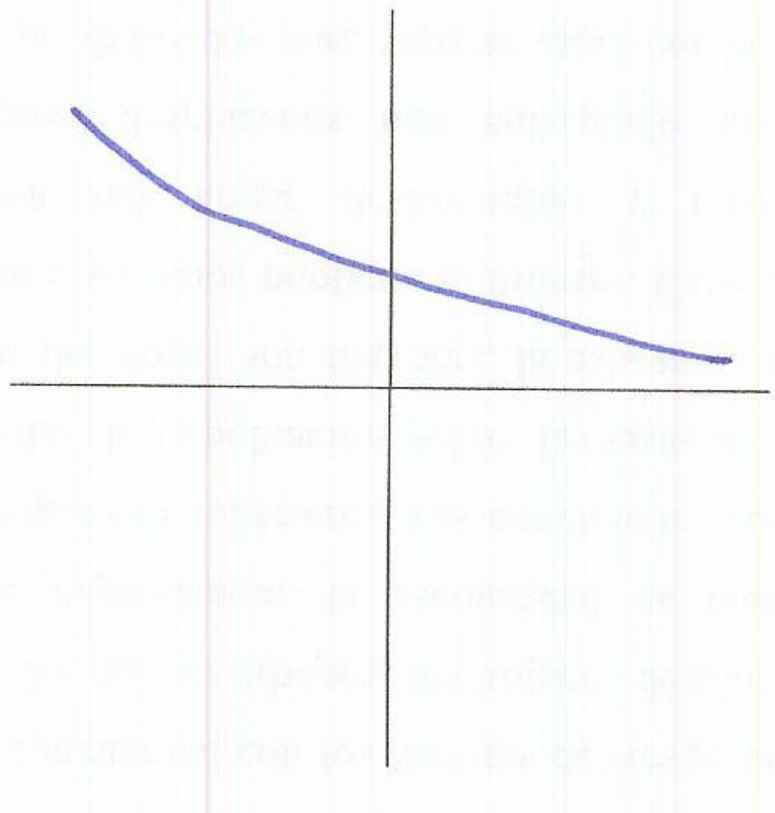
$$f''(x) = (x^2+1)^{-1/2} + x \cdot -\frac{1}{2}(x^2+1)^{-3/2} \cdot 2x$$

$$= (x^2+1)^{-1/2} - x^2(x^2+1)^{-3/2}$$

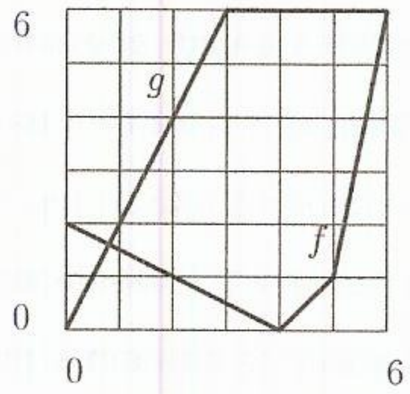
(5) (10 points) Draw the graph of a function $f(x)$ with $f(x) < 0$ and $f'(x) > 0$.



Sketch $f'(x)$ for the graph you drew above.



(6) (10 points) The graphs of the functions $f(x)$ and $g(x)$ are shown below.



(a) If $h(x) = g(f(x))$, what is $h'(1)$?
(b) If $h(x) = f(x)/g(x)$, what is $h'(2)$?

a) $h'(x) = g'(f(x)) \cdot f'(x)$

$h'(1) = g'(f(1)) \cdot f'(1) = g'\left(\frac{3}{2}\right) \cdot -\frac{1}{2} = 2 \cdot -\frac{1}{2} = -1$

b) $h'(x) = \dfrac{g'(x)f(x) - g(x)f'(x)}{g(x)^2}$

$h'(2) = \dfrac{g(2)f'(2) - g'(2)f(2)}{(g(2))^2} = \dfrac{4 \cdot -\frac{1}{2} - 2 \cdot 1}{4^2} = \dfrac{-4}{16} = -\dfrac{1}{4}$

(7) (10 points) Find the tangent line to the curve given by $x^3 + 2xy^2 + xy = 4$ at the point $(1, 1)$.

$$3x^2 + 2y^2 + 2x2y\frac{dy}{dx} + y + x\frac{dy}{dx} = 0$$

at $(1,1)$:

$$6 + 5\frac{dy}{dx} = 0 \qquad \frac{dy}{dx} = -\frac{6}{5}$$

tangent line:

$$y - 1 = -\frac{6}{5}(x - 1)$$

(8) (10 points) Use linear approximation to estimate $\sqrt{83}$. What is the percentage error?

use $f(x) = \sqrt{x} = x^{1/2}$ $\quad 9^2 = 81$ $\quad f'(x) = \frac{1}{2} x^{-1/2}$

$f(x+a) \approx f(a) + f'(a)(x-a)$

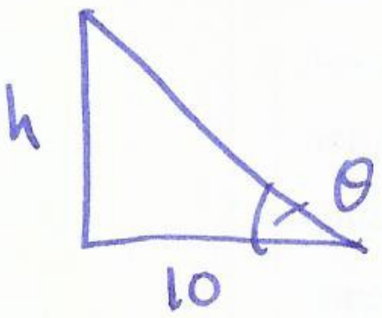
$\sqrt{81+2} \approx 9 + \frac{1}{18} \cdot 2 = 9\frac{1}{9} \approx 9.1111$

actual value $\sqrt{83} \approx 9.1104$

absolute error $= |9.1111 - 9.1104| = 0.0006$

percentage error $= \dfrac{0.0006 \times 100}{9.1104} = 0.007\%$

(9) (10 points) A rocket is launched vertically upwards from a point 10 km away. When you see the rocket at an angle of $\pi/6$ radians, the angle is increasing at a rate of 0.1 radians per second. How fast is the rocket moving?



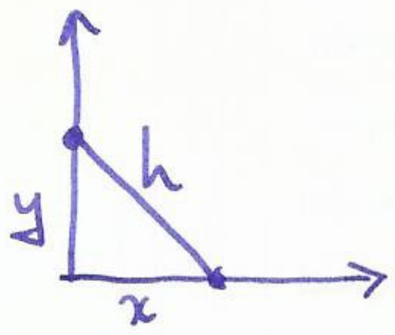
$$\frac{h}{10} = \tan\theta$$

$$\frac{1}{10}\frac{dh}{dt} = \sec^2\theta \frac{d\theta}{dt}$$

$$\frac{dh}{dt} = 10\sec^2\left(\frac{\pi}{6}\right) \cdot 0.1 = \sec^2\left(\frac{\pi}{6}\right) = \frac{4}{3}\ \text{km/s}$$

$$= 4800\ \text{km/h}.$$

(10) We start from the same point, and I drive east at 20 mph, and you drive north at 40 mph. How fast is the distance between us increasing?



$$h^2 = x^2 + y^2$$

$$2h\frac{dh}{dt} = 2x\frac{dx}{dt} + 2y\frac{dy}{dt}$$

after 1 hour.

$x = 20 \quad \frac{dx}{dt} = 20$

$y = 40 \quad \frac{dy}{dt} = 40$

$$h = \sqrt{20^2 + 40^2} = 20\sqrt{5}$$

$$\frac{dh}{dt} = \frac{20 \cdot 20 + 40 \cdot 40}{20\sqrt{5}} = \frac{100}{\sqrt{5}} = 20\sqrt{5} \text{ mph}.$$