Sample midterm 2 Solutions

Q1 $y = \frac{1}{\sqrt{3-2x}} = (3-2x)^{-1/2}$ $\frac{dy}{dx} = -\frac{1}{2}(3-2x)^{-3/2} \cdot -2 = (3-2x)^{-3/2}$

$\frac{d^2y}{dx^2} = -\frac{3}{2}(3-2x)^{-5/2} \cdot -2 = 3(3-2x)^{-5/2}$

Q2 a) $\frac{dy}{dx} = -\sin(2x^{1/2}) \cdot x^{-1/2}$

b) $\frac{dy}{dx} = \frac{1}{1+(e^{-2x})^2} \cdot -2e^{-2x}$

c) $y = \sec(2x)\tan(2x) \cdot 2\ln(x^2+1) + \sec(2x) \cdot \frac{1}{x^2+1} \cdot 2x$

Q3 a) $h'(x) = f'(x)g(x) + f(x)g(x)$

$h'(4) = f'(4)g(4) + f(4)g'(4) = 1.5 + 2\frac{1}{2} = 6$

b) $h'(x) = f'(g(x)) \cdot g'(x)$

$h'(3) = f'(g(3)) \cdot g'(3) = f'(4\frac{1}{2}) \cdot \frac{1}{2} = 1 \cdot \frac{1}{2} = \frac{1}{2}$

Q4 $4x^2 + y^2 = 8$ $8x + 2y\frac{dy}{dx} = 0$ $(1,2)$ $\frac{dy}{dx} = -\frac{8x}{2y} = -2$

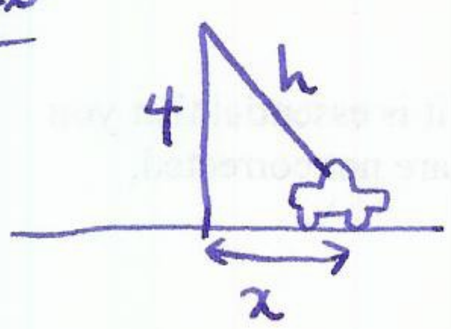
Q5 $x^2y^3 + 2xy^2 = x+y$

$2xy^3 + x^2 3y^2\frac{dy}{dx} + 2y^2 + 4xy\frac{dy}{dx} = 1 + \frac{dy}{dx}$

$\frac{dy}{dx}(x^2 3y^2 + 4xy - 1) = 1 - 2xy^3 - 2y^2$

$\frac{dy}{dx} = \frac{1-2xy^3-2y^2}{x^2 3y^2 + 4xy - 1}$

Q6



$4^2 + x^2 = h^2$

$2x\frac{dx}{dt} = 2h\frac{dh}{dt}$

$x=1, \frac{dx}{dt} = 60, h = \sqrt{17}$ $\frac{dh}{dt} = \frac{120}{2\cdot\sqrt{17}}$ mph $\simeq 14.55$

Q7 $f(x) = x^{1/3}$ $\quad f'(x) = \frac{1}{3}x^{-2/3}$

$f(x+\Delta x) \approx f(x) + f'(x)\Delta x$

$f(64+1) \approx f(64) + \frac{1}{3}(64)^{-2/3} \cdot 1 \approx 4 + \frac{1}{3.16} \cdot 1 \approx 4.02083$

absolute error : $\sqrt[3]{65} - 4.02083 \approx 0.04156$

percentage error : $\frac{100 \times 0.04156}{4.02607257} \approx 1.0\%$.

Q8 $f(x) = x^2 - 4x + 2$ $\quad f'(x) = 2x - 4$

find critical points solve $f'(x) = 0 \Rightarrow 2x - 4 = 0 \quad x = 2$

evaluate f at endpoints and critical points

$f(-1) = 7$ abs max

$f(2) = -2$ abs min

$f(5) = 7$