

Math 231 Calculus 1 Spring 12 Midterm 1b

Name: Solutions

- Do any 8 of the following 10 questions.
- You may use a calculator, but no notes.

1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
	80	

Midterm 1	
Overall	

- (1) (10 points) Plot the points  $(-3, 4)$  and  $(3, -3)$  on the grid below, and draw the straight line through the two points. Find the equation of the straight line.

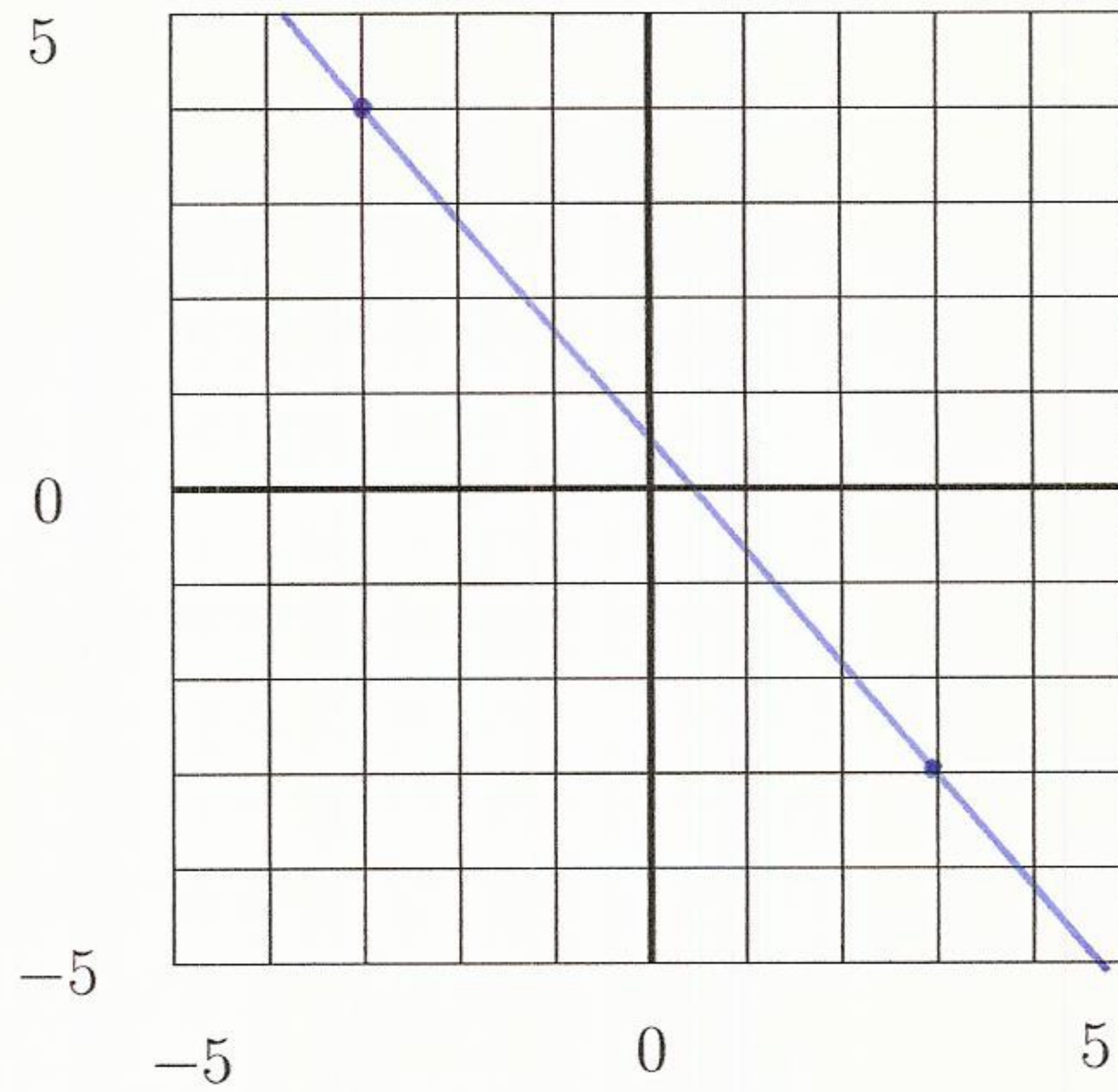


FIGURE 1

$$\text{slope } m = \frac{-3 - 4}{3 - (-3)} = -\frac{7}{6}$$

$$\begin{aligned} \text{equation} \quad y - y_0 &= m(x - x_0) \\ y - 4 &= -\frac{7}{6}(x + 3) \end{aligned}$$

- (2) (10 points) The graph of  $y = f(x)$  is shown below. Evaluate each limit, or write DNE if the limit does not exist. No justifications are necessary.

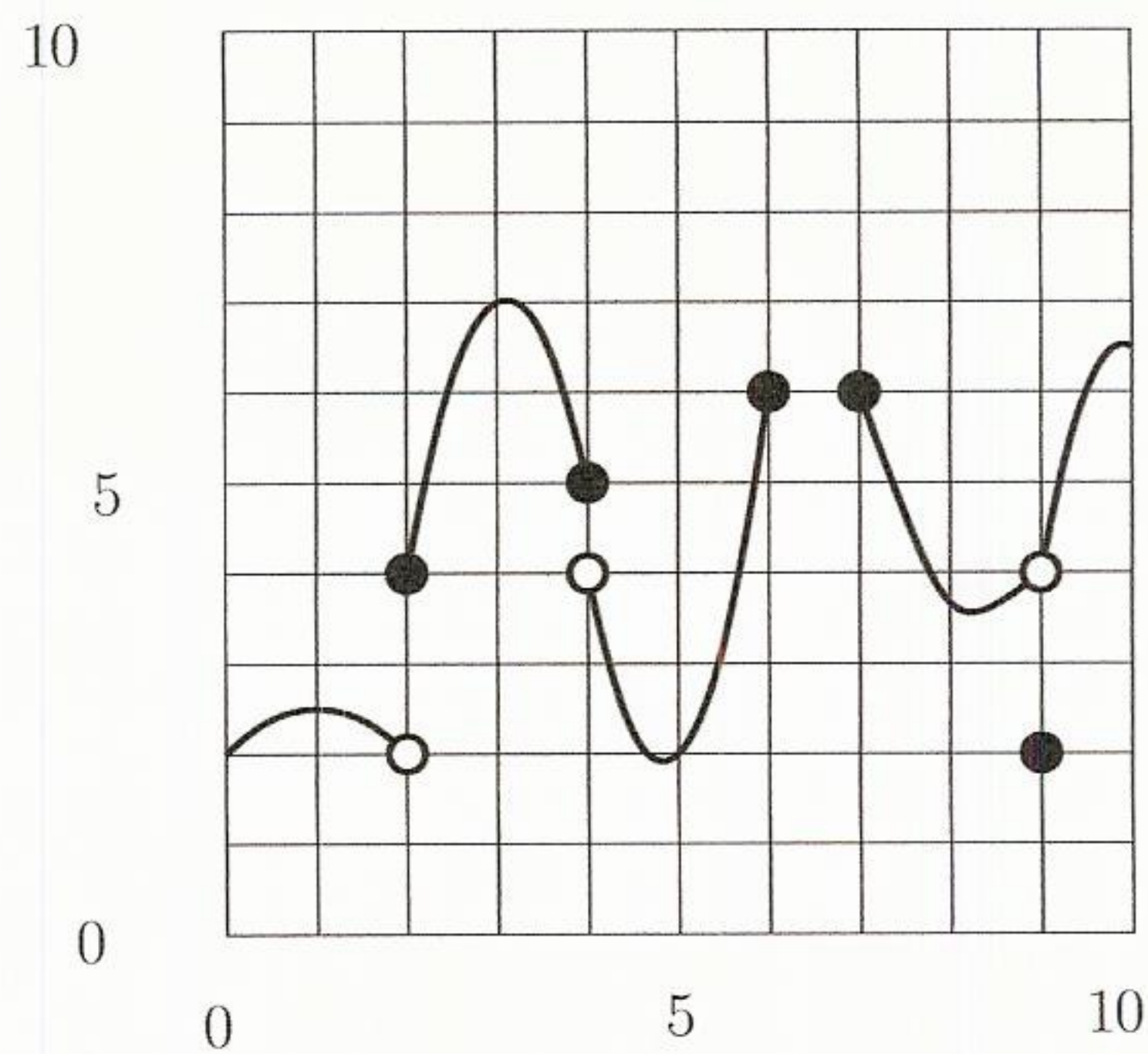


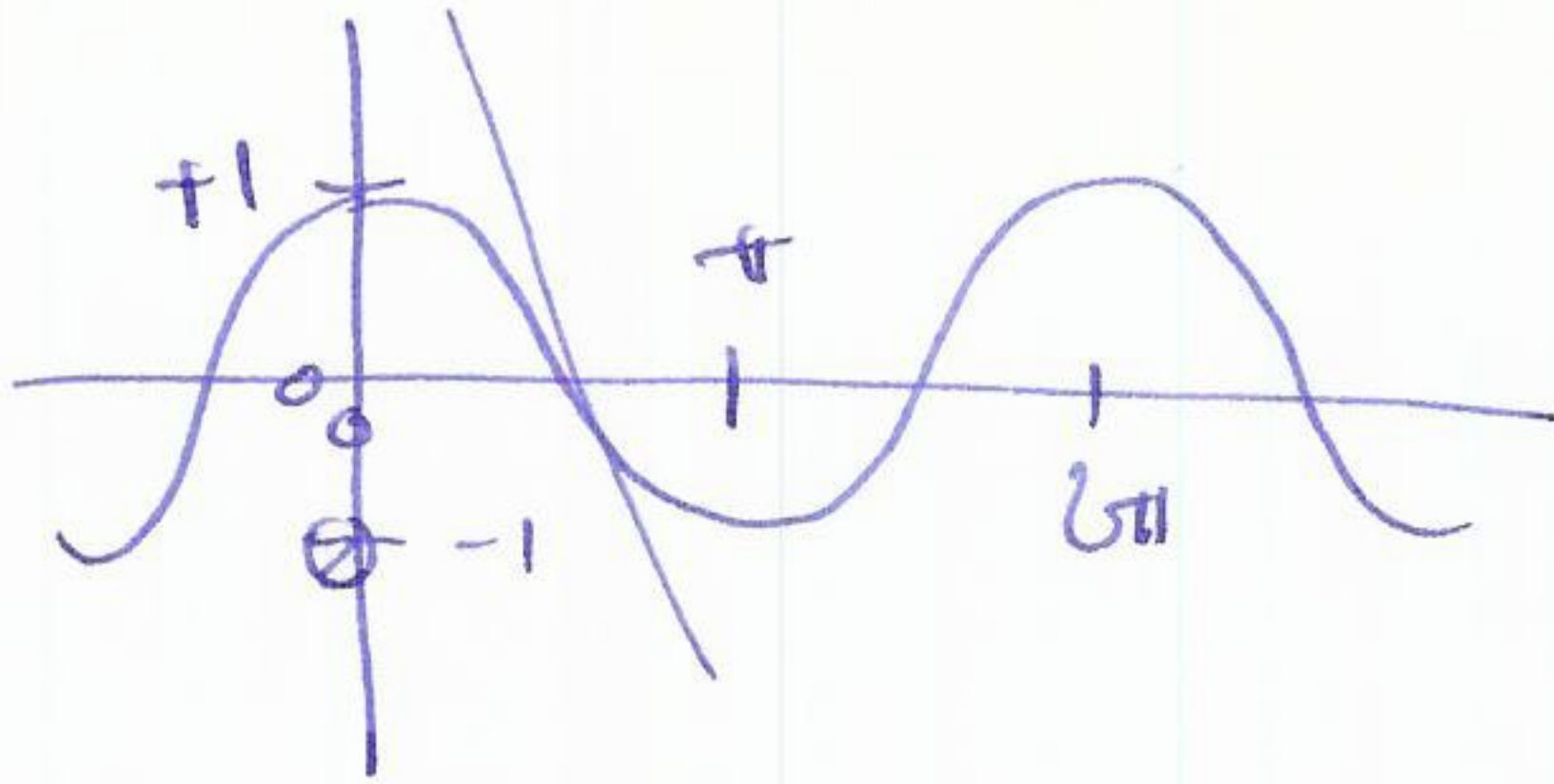
FIGURE 2.  $f(x)$

- (a)  $\lim_{x \rightarrow 2^+} f(x)$  2  
 (b)  $\lim_{x \rightarrow 4} f(x)$  DNE  
 (c)  $\lim_{x \rightarrow 6^+} f(x)$  DNE  
 (d)  $\lim_{x \rightarrow 6^-} f(x)$  6  
 (e)  $\lim_{x \rightarrow 9} f(x)$  4

(3) (10 points) Sketch the graph of  $f(x) = \cos x$ .

(a) What is the average rate of change from  $x = 0$  to  $x = \pi/2$ ?

(b) Looking at the graph, do you expect this to be bigger or smaller than the actual rate of change at  $x = \pi/2$ ?



$$a) \quad \frac{\cos\left(\frac{\pi}{2}\right) - \cos(0)}{\frac{\pi}{2} - 0} = -\frac{2}{\pi}$$

b) bigger

- (4) (10 points) Evaluate the limit algebraically. For an infinite limit, write  $+\infty$  or  $-\infty$ . If a limit does not exist (DNE), you must justify why this is the case.

$$\lim_{x \rightarrow 0} \frac{\sin 3x}{5x}$$

$$\begin{aligned} 3x &= \theta \\ x &= \theta/3 \end{aligned} \quad = \lim_{\theta \rightarrow 0} \frac{\sin \theta}{5\theta/3} = \frac{3}{5} \lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta} = \frac{3}{5}$$

- (5) (10 points) Evaluate the limit algebraically. For an infinite limit, write  $+\infty$  or  $-\infty$ . If a limit does not exist (DNE), you must justify why this is the case.

$$\lim_{x \rightarrow -3} \frac{x^2 + x - 6}{x + 3}$$

$$= \lim_{x \rightarrow -3} \frac{(x+3)(x-2)}{x+3} = \lim_{x \rightarrow -3} x-2 = -5$$

- (6) (10 points) Evaluate the limit algebraically. For an infinite limit, write  $+\infty$  or  $-\infty$ . If a limit does not exist (DNE), you must justify why this is the case.

$$\lim_{x \rightarrow 9} \frac{1}{\sqrt{x}-3} - \frac{6}{x-9}$$

$$= \lim_{x \rightarrow 9} \frac{\sqrt{x}+3-6}{(\sqrt{x}-3)(\sqrt{x}+3)} = \lim_{x \rightarrow 9} \frac{\sqrt{x}-3}{(\sqrt{x}-3)(\sqrt{x}+3)} = \lim_{x \rightarrow 9} \frac{1}{\sqrt{x}+3} = \frac{1}{6}$$

(7) (10 points) Use the limit definition of the derivative to differentiate  $f(x) = x^2 - 3x$ .

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{(x+h)^2 - 3(x+h) - x^2 + 3x}{h} = \lim_{h \rightarrow 0} \frac{x^2 + 2xh + h^2 - 3x - 3h - x^2 + 3x}{h}$$

$$= \lim_{h \rightarrow 0} 2x + h - 3 = 2x - 3.$$



(8) (10 points) Differentiate the function  $f(x) = x^3e^x$ .

$$f'(x) = 3x^2e^x + x^3e^x$$

10

(9) (10 points) Differentiate the function

$$f(x) = \frac{\sqrt{x}}{2x^2 - 3}$$

$$f'(x) = \frac{(2x^2 - 3) \frac{1}{2} x^{-1/2} - 6x\sqrt{x}}{(2x^2 - 3)^2}$$

(10) (10 points) Differentiate the function  $f(x) = (x + 1/x)^2$ .

$$f(x) = x^2 + 2 + \frac{1}{x^2} = x^2 + 2 + x^{-2}$$

$$f'(x) = 2x - 2x^{-3}$$