Math 231 Calculus 1 Spring 12 Midterm 1a

Name: ___Solutions___

- Do any 8 of the following 10 questions.
- You may use a calculator, but no notes.

1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
	80	

Midterm 1	
Overall	

(1) (10 points) Plot the points $(-4, 3)$ and $(2, -4)$ on the grid below, and draw the straight line through the two points. Find the equation of the straight line.
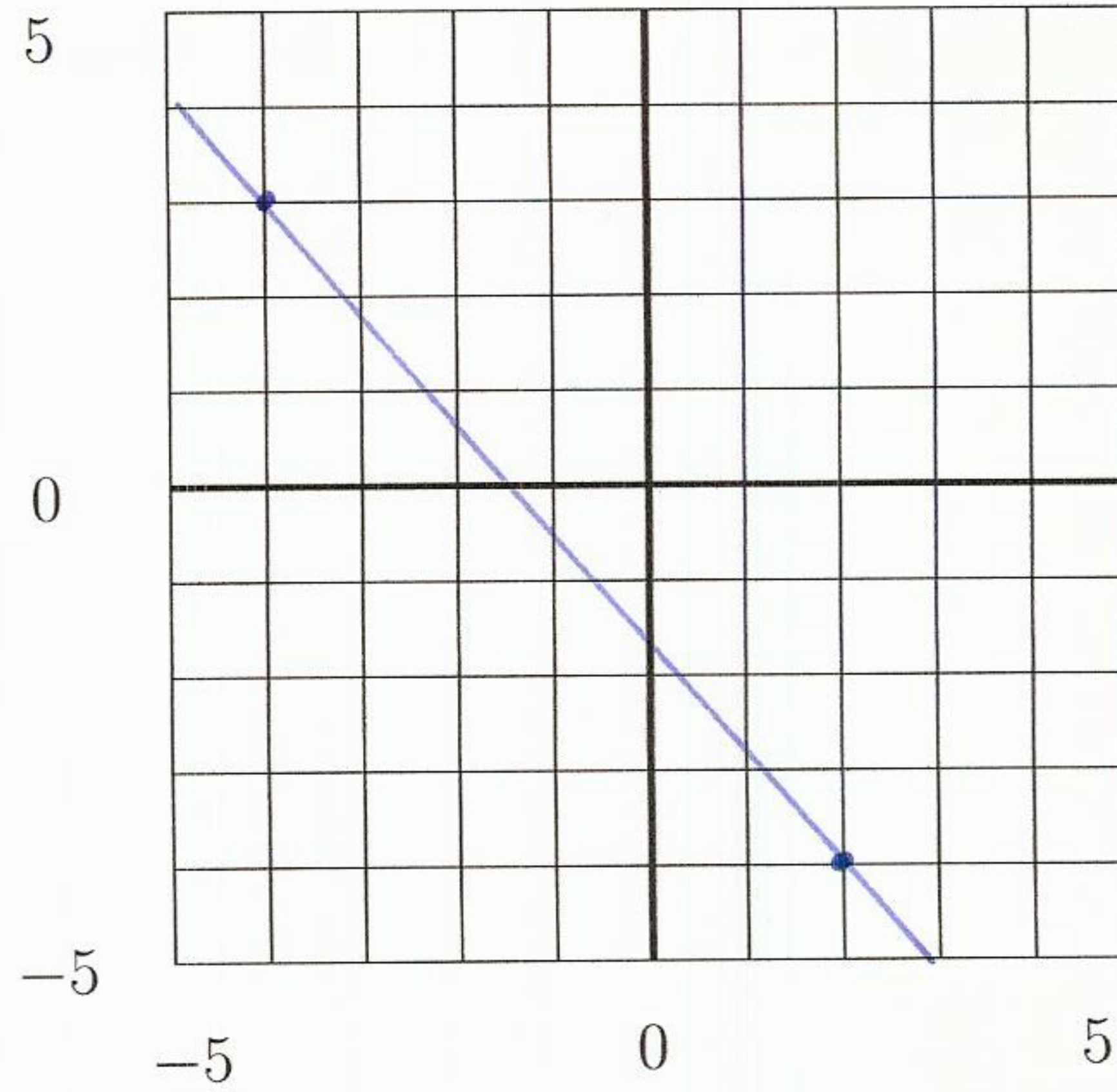

FIGURE 1

slope $m = \dfrac{-4-3}{2+4} = -\dfrac{7}{6}$

equation $\quad y - y_0 = m(x - x_0)$

$\qquad\qquad y - 3 = -\dfrac{7}{6}(x+4)$

(2) (10 points) The graph of $y = f(x)$ is shown below. Evaluate each limit, or write DNE if the limit does not exist. No justifications are necessary.
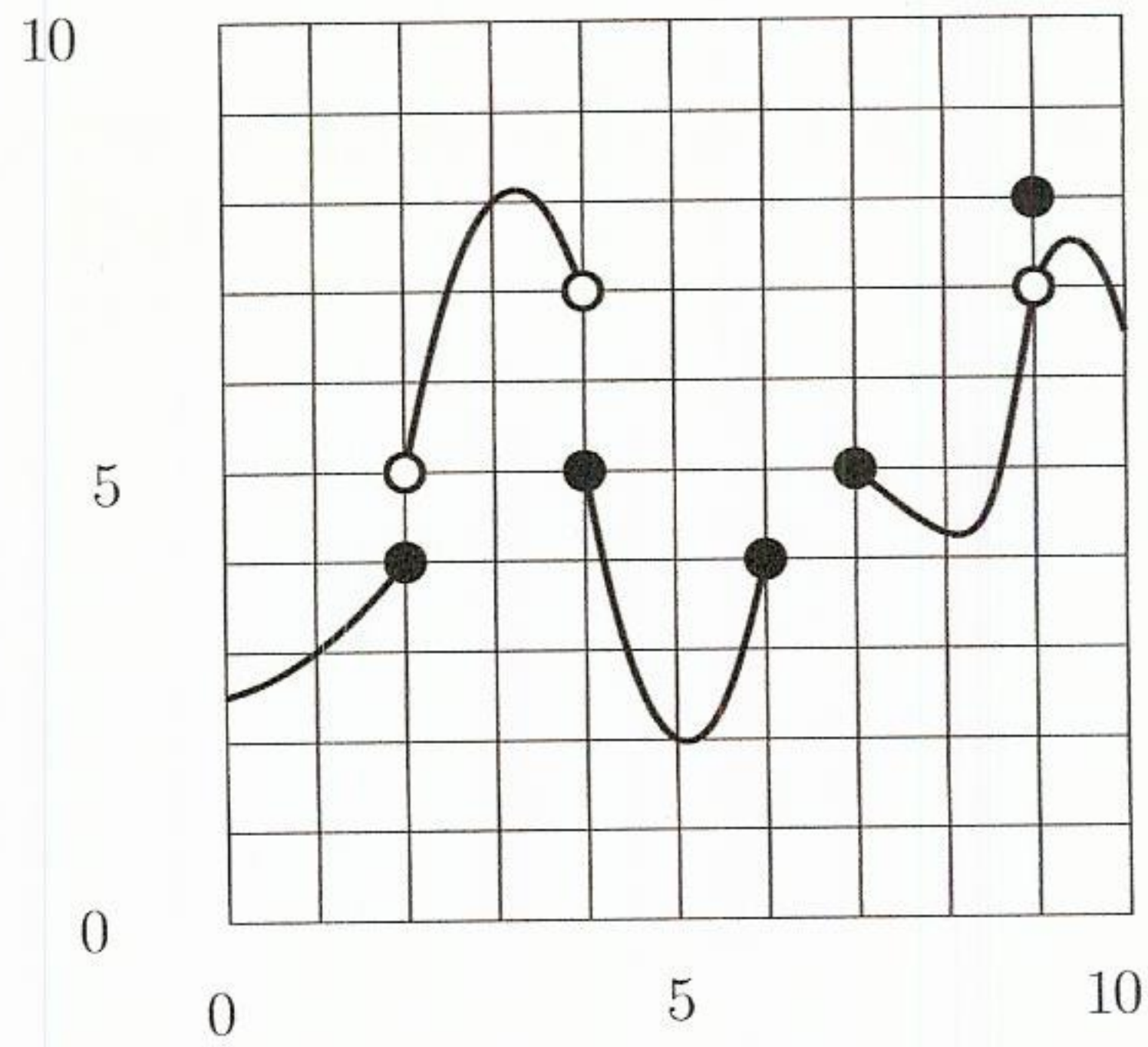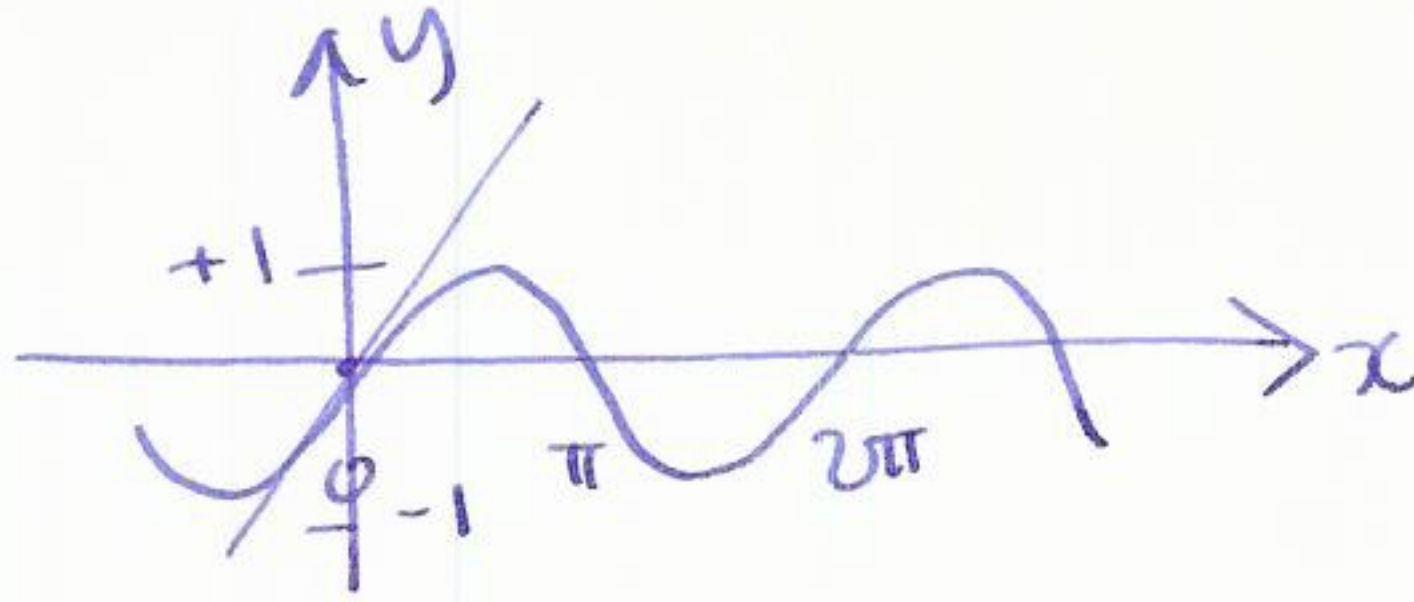

FIGURE 2. $f(x)$

(a) $\lim_{x \to 2-} f(x)$ 4

(b) $\lim_{x \to 4} f(x)$ DNE

(c) $\lim_{x \to 6-} f(x)$ 4

(d) $\lim_{x \to 6+} f(x)$ DNE

(e) $\lim_{x \to 9} f(x)$ 7

(3) (10 points) Sketch the graph of $f(x) = \sin x$.
    (a) What is the average rate of change from $x = 0$ to $x = \pi/2$?
    (b) Looking at the graph, do you expect this to be bigger or smaller than the actual rate of change at $x = 0$?

a) $\dfrac{\sin\left(\pi/2\right) - \sin(0)}{\pi/2 - 0} = \dfrac{2}{\pi}$

b) smaller.

(4) (10 points) Evaluate the limit algebraically. For an infinite limit, write $+\infty$ or $-\infty$. If a limit does not exist (DNE), you must justify why this is the case.

$$\lim_{x \to 0} \frac{\sin 4x}{2x}$$

$4x = \theta$

$x = \theta/4$

$\lim_{\theta \to 0} \frac{\sin \theta}{2 \theta/4} = 2 \lim_{\theta \to 0} \frac{\sin \theta}{\theta} = 2$

(5) (10 points) Evaluate the limit algebraically. For an infinite limit, write $+\infty$ or $-\infty$. If a limit does not exist (DNE), you must justify why this is the case.

$$\lim_{x \to 2} \frac{x^2 + x - 6}{x - 2}$$

$$= \lim_{x \to 2} \frac{(x-2)(x+3)}{x-2} = \lim_{x \to 2} x+3 = 5$$

(6) (10 points) Evaluate the limit algebraically. For an infinite limit, write $+\infty$ or $-\infty$. If a limit does not exist (DNE), you must justify why this is the case.

$$\lim_{x \to 4} \frac{1}{\sqrt{x} - 2} - \frac{4}{x - 4}$$

$$= \lim_{x \to 4} \frac{\sqrt{x} + 2 - 4}{x - 4} = \lim_{x \to 4} \frac{\sqrt{x} + 4 - 2}{(\sqrt{x} + 2)(\sqrt{x} - 2)} = \lim_{x \to 4} \frac{1}{\sqrt{x} + 2} = \frac{1}{4}$$

(7) (10 points) Use the limit definition of the derivative to differentiate $f(x) = x^2 - 2x$.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{(x+h)^2 - 2(x+h) - x^2 + 2x}{h}$$

$$= \lim_{h \to 0} \frac{x^2 + 2xh + h^2 - 2x - 2h - x^2 + 2x}{h} = \lim_{h \to 0} 2x + h - 2 = 2x - 2$$

(8) (10 points) Differentiate the function $f(x) = x^2 e^x$.

$$f'(x) = 2xe^x + x^2 e^x$$

(9) (10 points) Differentiate the function

$$f(x) = \frac{\sqrt{x}}{3x^2 + 2}$$

$$f'(x) = \frac{(3x^2+2)\frac{1}{2}x^{-1/2} - (6x)\sqrt{x}}{(3x^2+2)^2}$$

(10) (10 points) Differentiate the function $f(x) = (x + 1/x)^2$.

$$f(x) = \quad x^2 + 2 + \frac{1}{x^2} \quad = \quad x^2 + 2 + x^{-2}$$

$$f'(x) = \quad 2x - 2x^{-3}$$