Calc I Sample midterm solutions

Q1 a) 3 b) 4 c) 7 d) 7 e) 3 f) DNE

Q2 a) -2 should be $\lim_{x\to -3} \frac{x^2-x-12}{|x+3|}$.

$$\lim_{x\to 3+} \frac{x^2-x-12}{x+3} = \lim_{x\to -3+} \frac{(x+3)(x-4)}{(x+3)} = \lim_{x\to -3+} x-4 = -7$$

$$\lim_{x\to -3-} \frac{x^2-x-12}{-(x+3)} = \lim_{x\to -3-} \frac{(x+3)(x-4)}{-(x+3)} = \lim_{x\to -3-} -x+4 = +7$$

not equal so $\lim_{x\to -3} \frac{x^2-x-12}{|x-3|}$ DNE.

b) $$\lim_{x\to 6} \frac{\sqrt{x+3}-3}{x-6} = \lim_{x\to 6} \frac{\sqrt{x+3}-3}{x+3-9} = \lim_{x\to 6} \frac{\sqrt{x+3}-3}{(\sqrt{x+3}-3)(\sqrt{x+3}+3)} = \lim_{x\to 6} \frac{1}{\sqrt{x+3}+3} = \frac{1}{6}$$

c) $$\lim_{x\to 0} \frac{\tan 2x}{3x} = \lim_{x\to 0} \frac{\sin 2x}{3x}\cdot\frac{1}{\cos 2x} = \lim_{x\to 0} \frac{\sin 2x}{3x}\cdot \underbrace{\lim_{x\to 0} \frac{1}{\cos 2x}}_{=1}$$ set $\theta = 2x$

$$= \lim_{x\to 0} \frac{\sin\theta}{3\theta/2} = \frac{2}{3}\lim_{x\to 0} \frac{\sin\theta}{\theta} = \frac{2}{3}.$$

d) $$\lim_{x\to 0+} \left(\frac{1}{\sqrt{x}} - \frac{1}{\sqrt{x^2+2x}}\right) = \lim_{x\to 0+} \frac{\sqrt{x^2+2x}-\sqrt{x}}{\sqrt{x}\sqrt{x^2-2x}}\cdot\frac{\sqrt{x^2+2x}+\sqrt{x}}{\sqrt{x^2+2x}+\sqrt{x}}$$

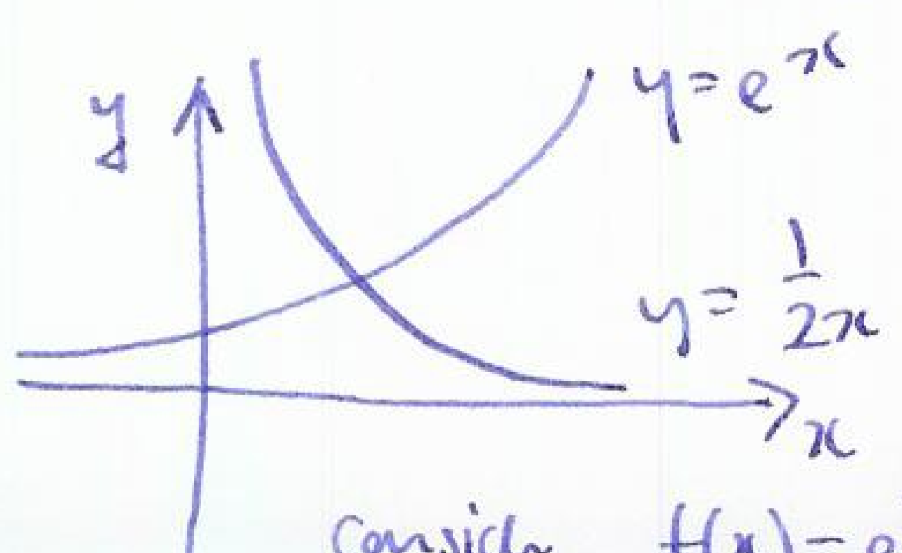
$$= \lim_{x\to 0+} \frac{x^2+2x-x}{\sqrt{x^3-2x^2}\,(\sqrt{x^2+2x}+\sqrt{x})} = \lim_{x\to 0+} \frac{1+x}{\sqrt{x+2}\,(\sqrt{x^2+2x}+\sqrt{x})} \to \infty$$ as $x \to 0_+$.

Q3) $$\lim_{x\to 1-} x + \frac{4}{x-3} = 1 + \frac{4}{-2} = -1$$

$$\lim_{x\to 1+} \frac{\cos(\pi x)}{x} = \lim \frac{\cos(\pi)}{1} = -1$$

choose $c = -1$ then cts.

4) $S = 4\pi r^2$ average rate of change from $r=3$ to $r=4$ is $\frac{4\pi(4)^2 - 4\pi(3^2)}{4-3} = 28\pi$.

5) 

Consider $f(x) = e^x - \frac{1}{2x}$

$x = 1:\ e^1 - \frac{1}{2} \simeq 1.5 > 0$

$x = 0.1:\ e^{0.1} - 5 \simeq -4 < 0$

so there is a soln in $[0.1, 1]$ by I.V.T.

②

6) a) $\left(\frac{4x}{x+2}\right)' = \frac{(x+2)(4)-(4x).1}{(x+2)^2} = \frac{4x+8-4x}{(x+2)^2} = \frac{8}{(x+2)^2}$

b) $(-2x^3e^x)' = (-2x^3)'e^x + (-2x^3)(e^x)' = -6x^2e^x - 2x^3e^x$

c) $\frac{x^2-1}{x^2+1} = \frac{(x^2+1)(x^2-1)' - (x^2-1)(x^2+1)'}{(x^2+1)^2} = \frac{2x(x^2+1)-2x(x^2-1)}{(x^2+1)^2} = \frac{4x}{(x^2+1)^2}$

7) a) $\left(\frac{8}{(x+2)^2}\right)' = \frac{(x+2)^2.0-(x^2+4x+4)'.8}{(x+2)^4}$

$= \frac{8(2x+4)}{(x+2)^4} = \frac{16x+32}{(x+2)^4} = \frac{16}{(x+2)^3}$

b) $(-6x^2e^x-2x^3e^x)' = (-6x^2)'e^x + (-6x^2)(e^x)' + (-2x^3)'e^x + (-2x^3)(e^x)'$

$= -12xe^x - 6x^2e^x - 6x^2e^x - 2x^3e^x.$

$= (-12x-12x^2-2x^3)e^x.$

c) $\left(\frac{16}{(x+2)^3}\right)' = \frac{((x+2)^3).(16)' - (x^3+6x^2+6x+8)'16}{(x+2)^6} = \frac{16(3x^2+12x+6)}{(x+2)^6}$