**Math 329 Geometry Spring 11 Sample Midterm 2**

(1) (10 points) Let $f$ be any isometry of $\mathbb{R}^2$. Show that $f$ preserves angles. (Do *not* use the classification of planar isometries.)

(2) (10 points) Let $P$ be a point on $S^2$, and let $f$ be a map from $S^2 \setminus P$ to $\mathbb{R}^2$ which takes great circles to lines. Show that $f$ cannot preserve angles.

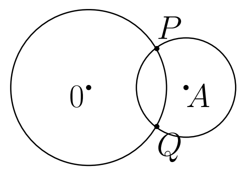
(3) (20 points) Let $f$ be an isometry of $\mathbb{R}^2$ given by three reflections,

$$f = r_c r_b r_a.$$

(a) Suppose the three lines intersect at a single point. Describe the isometry $f$.

(b) Suppose that $a$ is parallel to $b$, and $c$ is perpendicular to $a$. Describe the isometry $f$.

(4) (20 points) Use vectors to show that $OA$ is perpendicular to $PQ$.




Hint: assume that 0 is the origin.

(5) (20 points) Let $A(1,0,0)$, $B(0,1,0)$ and $C(0,0,1)$ be three points on the unit sphere $S^2$ in $\mathbb{R}^3$.

(a) What is the center of $\triangle ABC$ in $\mathbb{R}^3$?

(b) What is the center of $\triangle ABC$ in $S^2$?

(6) (20 points) Suppose a rotation of $\mathbb{R}^2$ takes the point $(1,0)$ to the point $(3,0)$.

(a) Where can the fixed point of the rotation be?

(b) If the rotation is an anticlockwise rotation by $\pi/2$, find the fixed point of the rotation.

(7) (20 points)

(a) Describe all the isometries of $\mathbb{R}^2$ which fix the origin $(0,0)$.

(b) Use your answer to (a) to describe all isometries of $\mathbb{R}^2$ which take $(0,0)$ to $(1,1)$ as a product of a translation and one other isometry.

(8) (20 points) Let $f$ be reflection in the line $x = 1$, and let $g$ be an anti-clockwise rotation of $\pi/4$ about $(0,0)$. Describe $f \circ g$.

(9) (30 points)

(a) Find an explicit hyperbolic isometry, described as a Möbius transformation, which takes the line with endpoints $a, b \in \mathbb{R}$ in the upper half space model to the $y$-axis.

(b) Describe the set of hyperbolic isometries which preserve the $y$-axis in the upper half space model.

(c) Use your answers to (a) and (b) to show that the isometries of hyperbolic space act transitively on pairs of points a fixed distance apart.