Math 329 Geometry Spring 11 Midterm 1

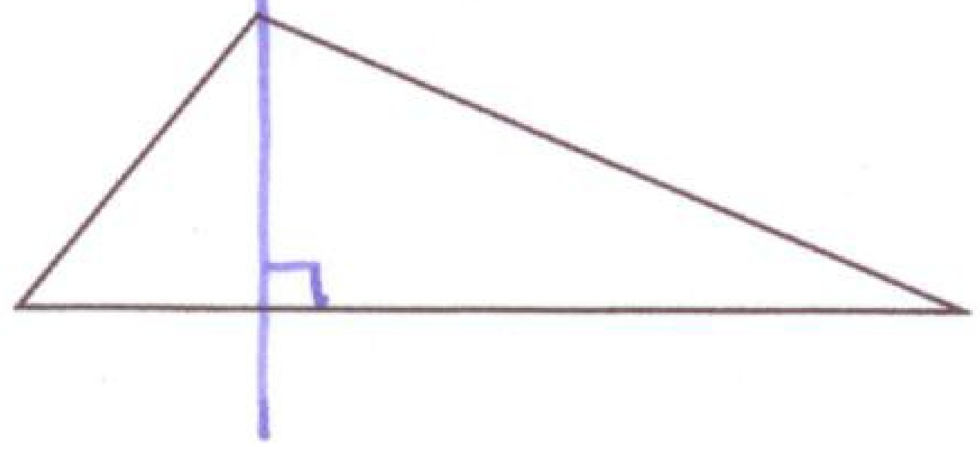
Name: Solutions

- You may use a compass and straight edge, but no notes.

1	20	
2	20	
3	20	
4	20	
5	20	
6	35	
	135	

Midterm 1	
Overall	

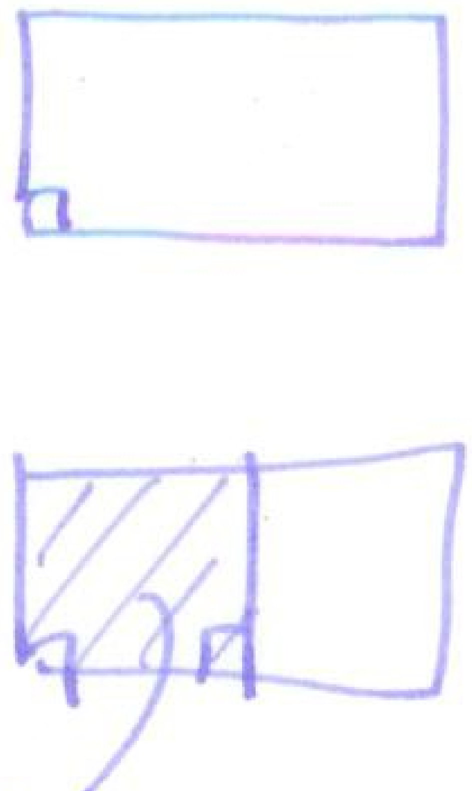
(1) (20 points) Construct a rectangle with the same area as the triangle below.



find perpendicular through vertex.

construct rectangle with height and base:

bisect one edge to get rectangle with half the area

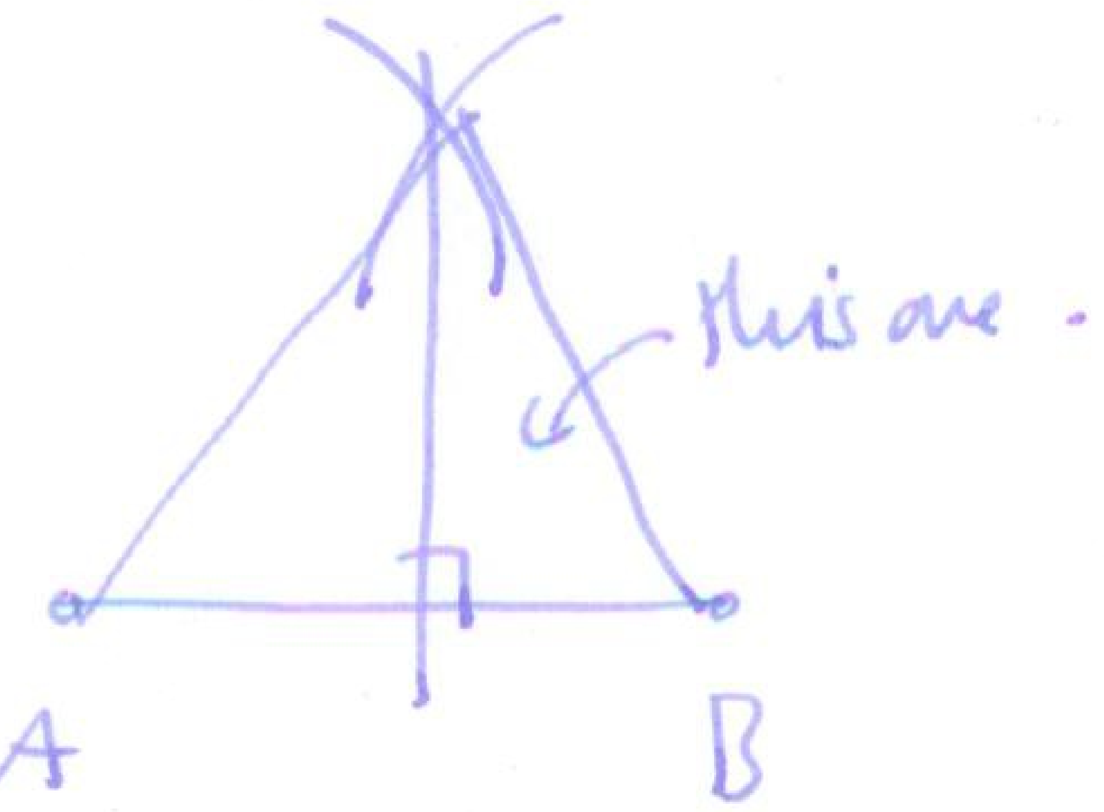


this one.

(2) (20 points) Given the segment AB below, construct a right angled triangle with hypotenuse of length equal to $|AB|$, and with angles $\pi/6, \pi/3$ and $\pi/2$
Hint: construct an equilateral triangle first.

A __________________________ B

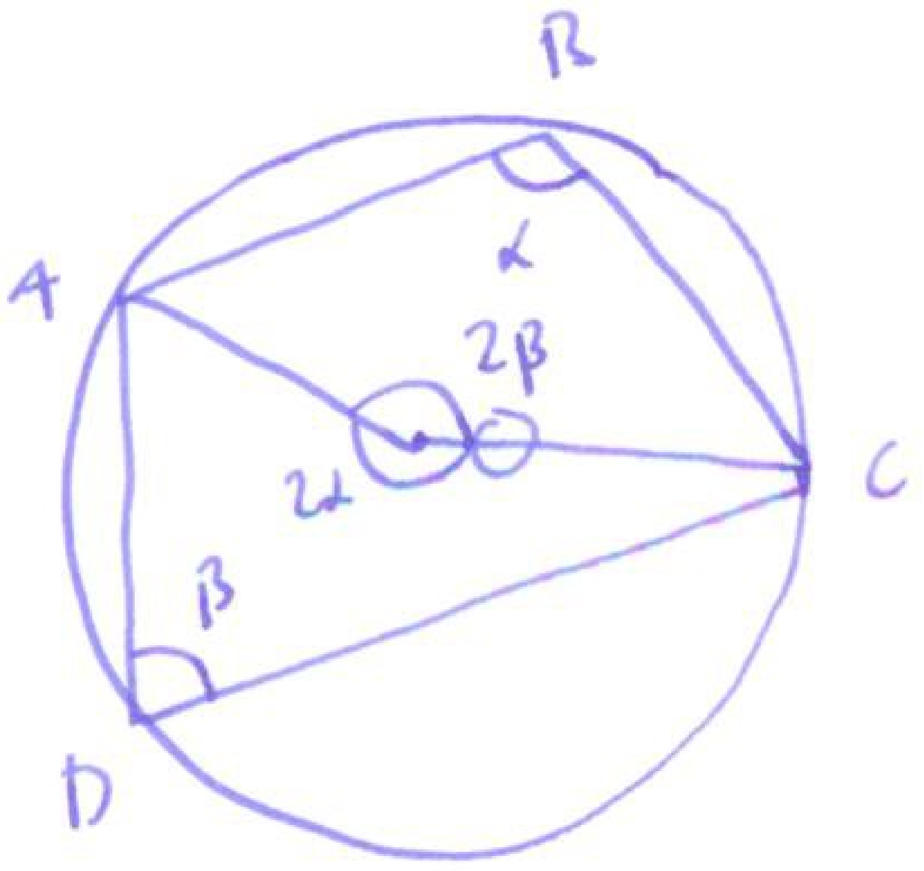
construct an equilateral triangle



then construct perpendicular through vertex

(3) (20 points) If a quadrilateral has all four vertices on a circle, show that any pair of opposite angles sum to π.

Hint: use the relation between the angle at the center and the angle at the circumference..

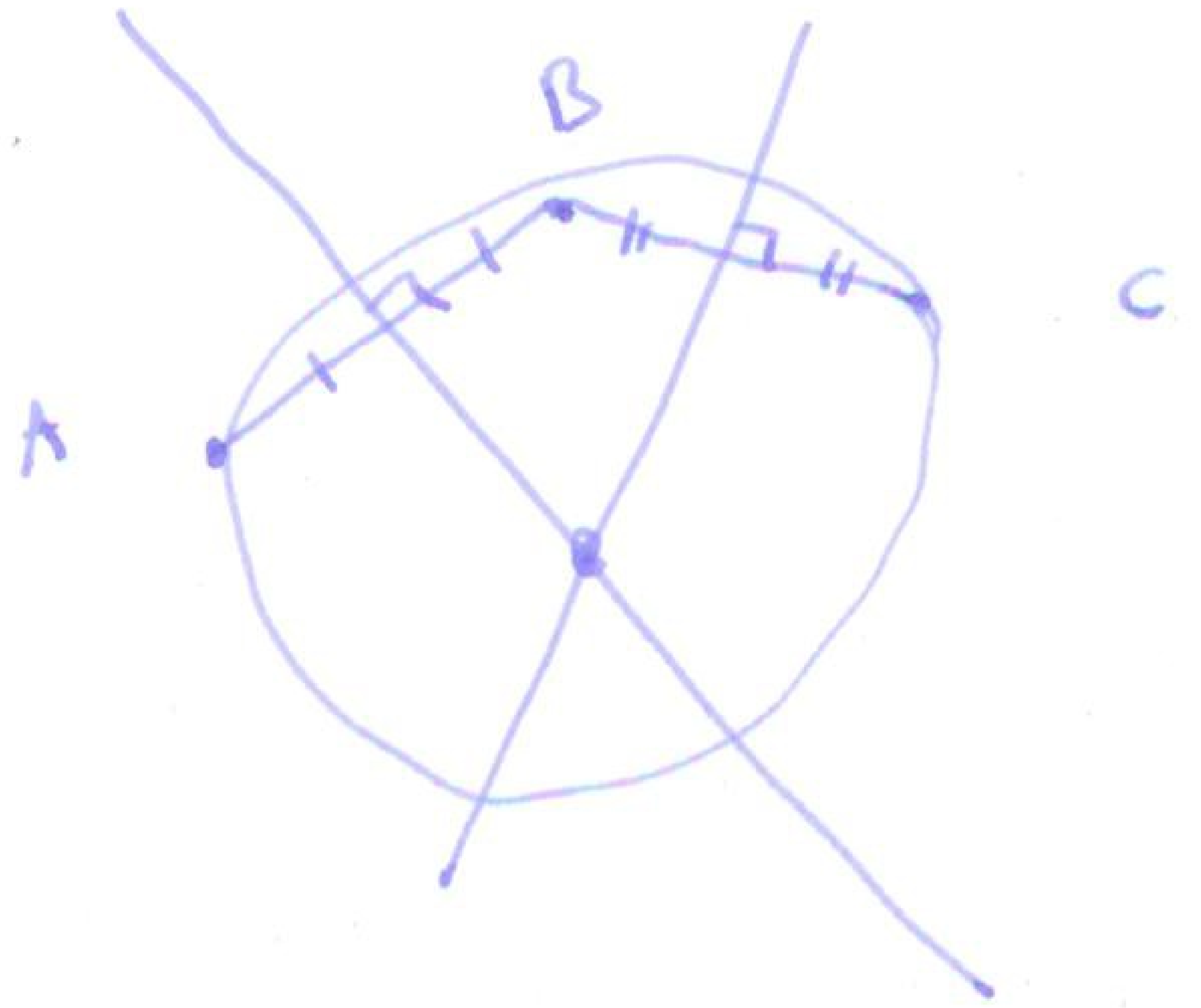


$$2\alpha + 2\beta = 2\pi$$

$$\Rightarrow \alpha + \beta = \pi$$

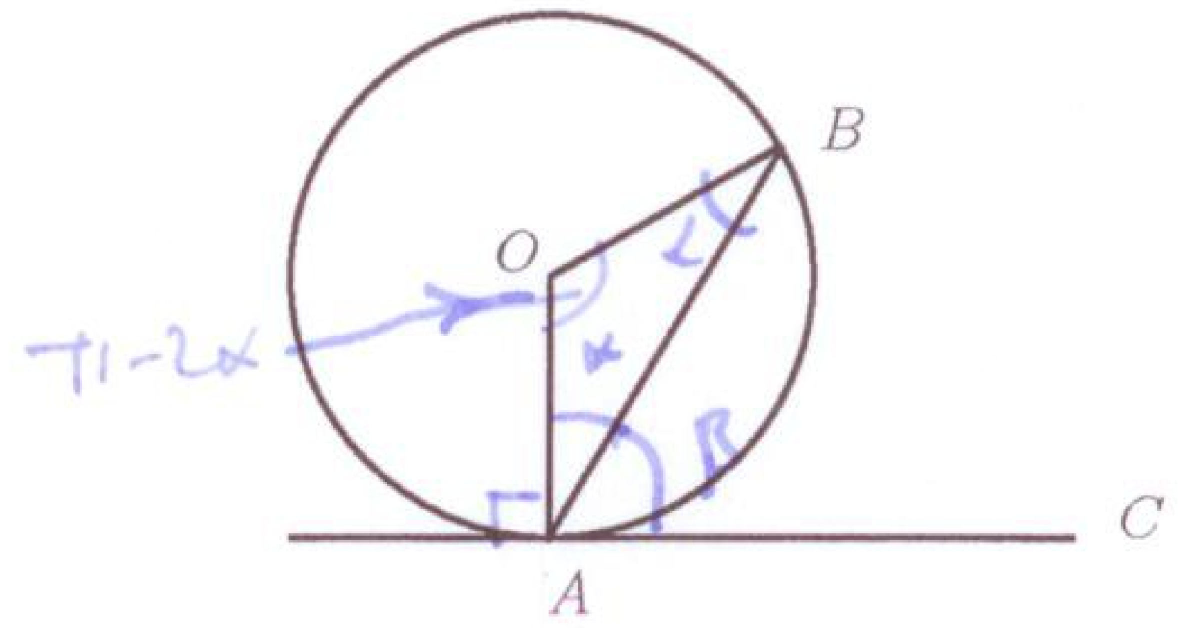
(4) (20 points) Draw three points A, B and C which are *not* colinear. Construct a circle which contains all three points.

Hint: the perpendicular bisector of a chord runs through the center of the circle.



construct perpendicular bisector to AB = set of points equidistant to A, B
construct perpendicular bisector to BC = set of points equidistant to B, C
intersection is equidistant to all 3 of A, B, C so center of circle through A, B, C

(5) (20 points) Show that angle $\angle BAC$ is half the size of angle $\angle AOB$.



Δ AOB iscoceles so $\angle OAB = \angle ABO = \alpha$

sum of angles in a triangle is π so $\angle AOB = \pi - 2\alpha$

tangent meets radius at right angles so $\alpha + \beta = \frac{\pi}{2}$.

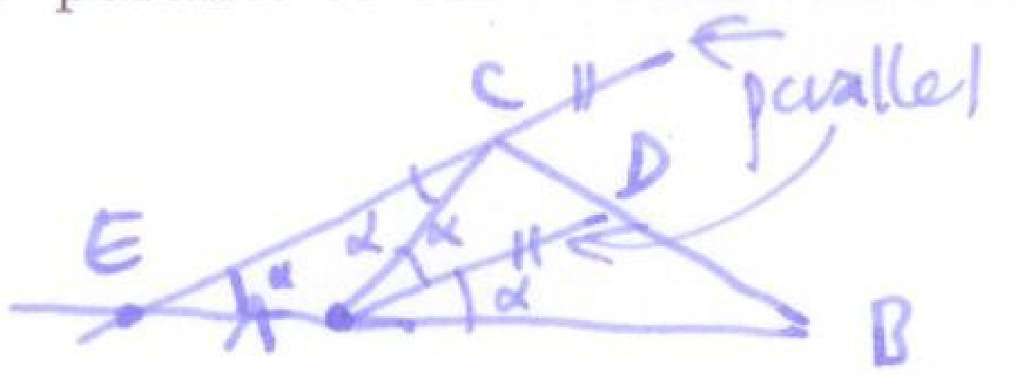
so $\beta = \frac{\pi}{2} - \alpha = \cancel{\frac{\pi}{2}(1-(\pi-2\alpha))} = \frac{\pi - 2\alpha}{2} = \frac{1}{2}\angle AOB$

as required.

(6) (35 points) For any triangle, an angle bisector divides the opposite side in the ratio of the adjacent sides. Below, complete the proof that if AD bisects angle $\angle CAB$ then

$$\frac{|AB|}{|AC|} = \frac{|DB|}{|DC|}.$$

(a) There is a line through C parallel to AD which meets an extension of AB at a point E.



(b) $\angle ACE = \angle CAD$

opposite alternate angles are equal.

(c) $= \angle BAD$

AD bisects the angle $\angle CAB$.

(d) $= \angle AEC$

parallel lines make equal angles.

(e) $AC = AE$ equal angles $\Rightarrow$
iscocales triangle $\Rightarrow$ equal sides.

(f)

$$\frac{|AB|}{|AE|} = \frac{|DB|}{|DC|}$$

Thales' theorem applied to ΔBCE with AD parallel to CE.

(g)

$$\frac{|AB|}{|AC|} = \frac{|DB|}{|DC|}$$

by part e) $|AE| = |AC|$.