Math 329 Geometry Spring 11 Final

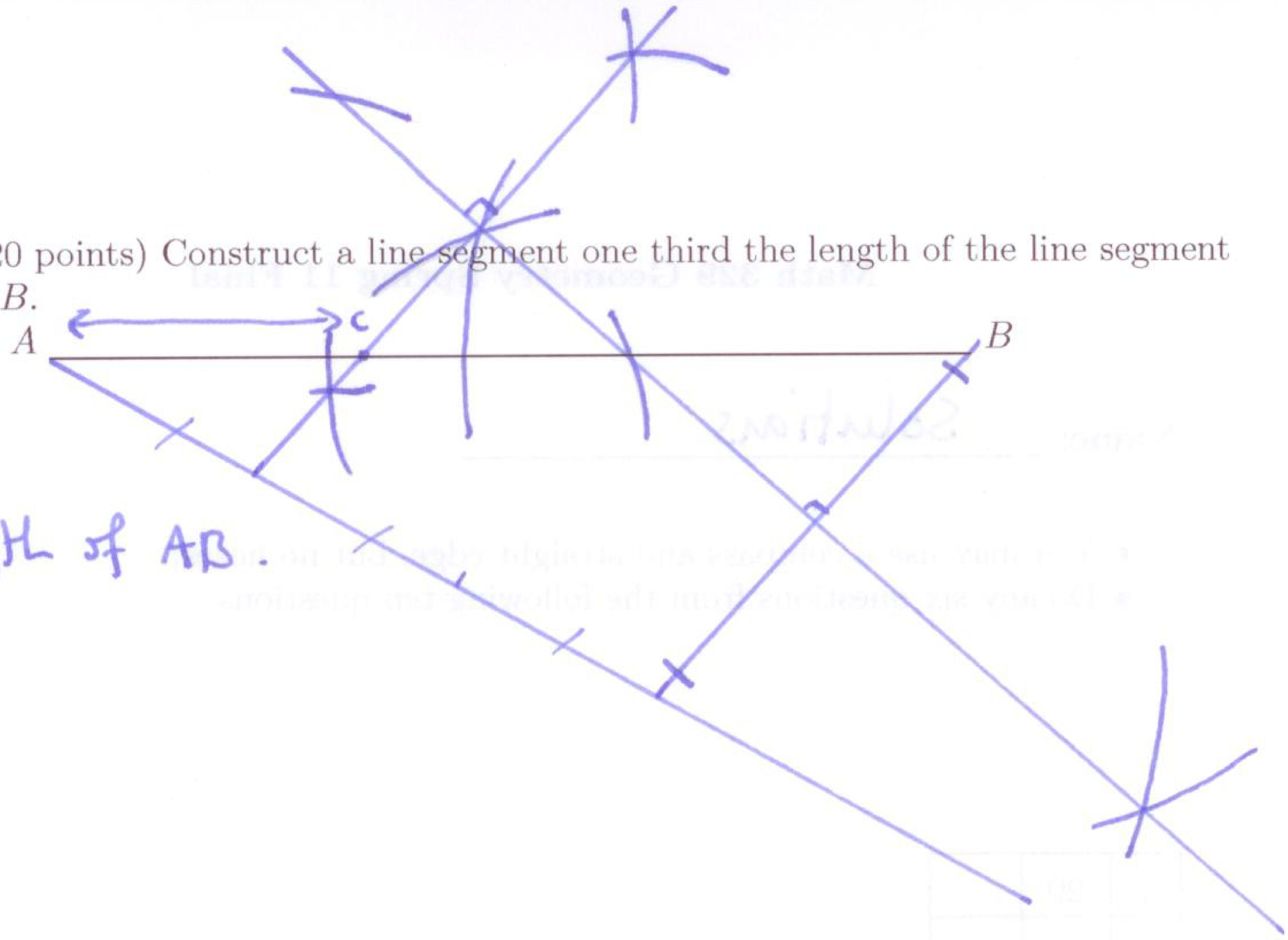
Name: Solutions

- You may use a compass and straight edge, but no notes.
- Do any six questions from the following ten questions.

1	20	
2	20	
3	20	
4	20	
5	20	
6	20	
7	20	
8	20	
9	20	
10	20	
	120	

Final	
Overall	

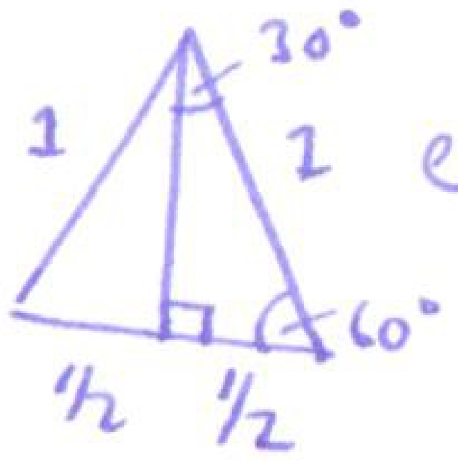
(1) (20 points) Construct a line segment one third the length of the line segment AB.



AC $\frac{1}{3}$ length of AB.

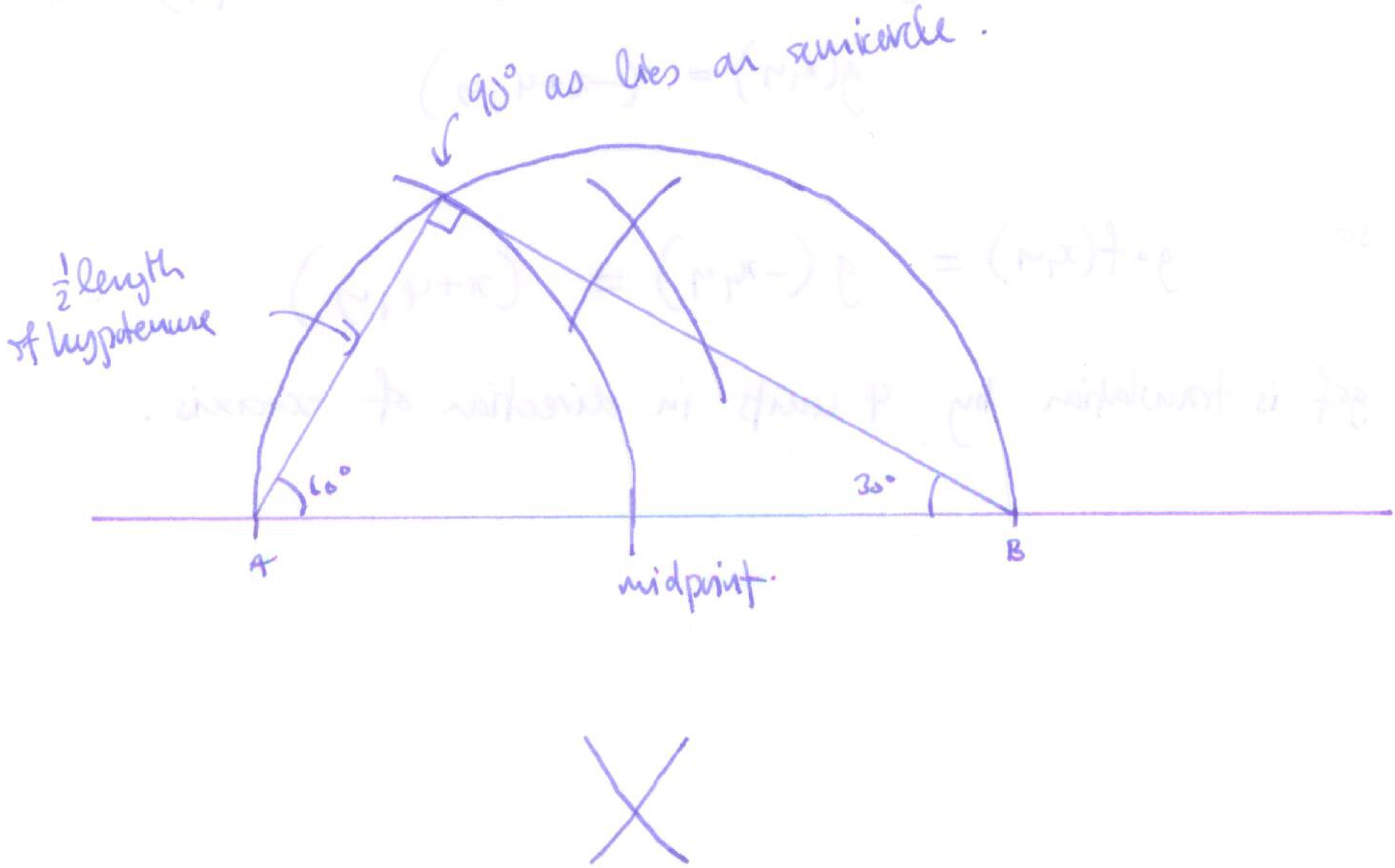
(2) (20 points) Given the line segment AB, construct a triangle with angles $30°, 60°$ and $90°$, with hypotenuse the length of AB.

A ____ B

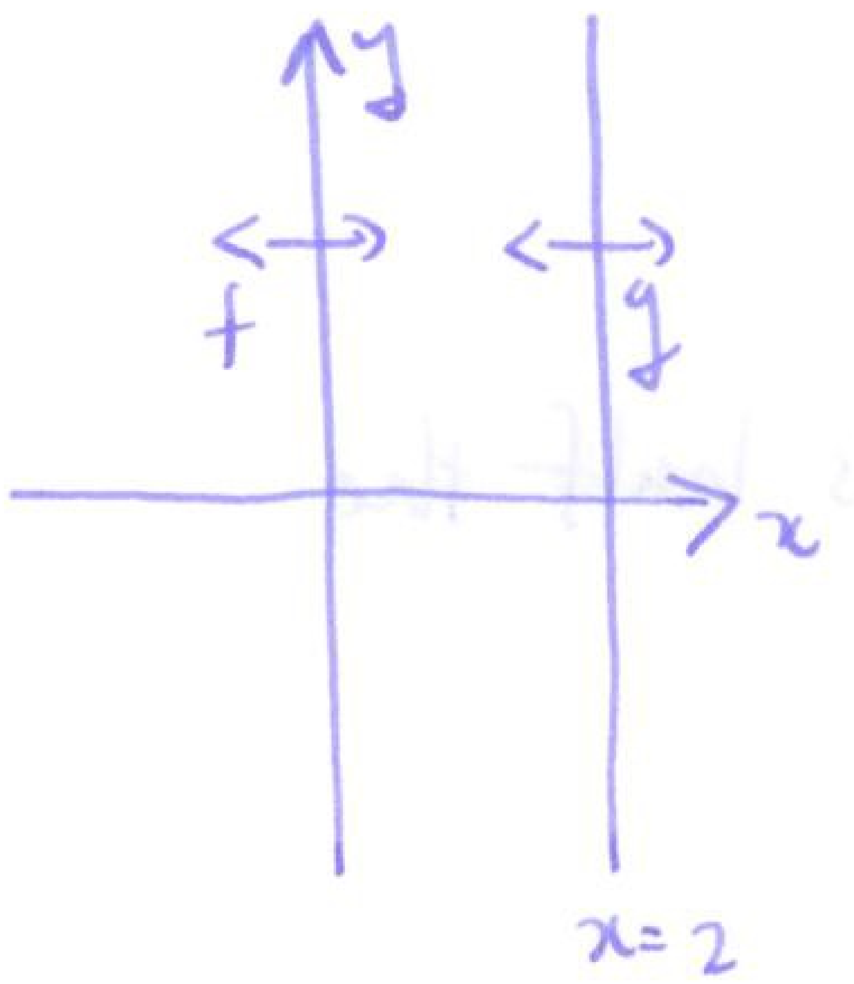


equilateral triangle has equal sides.

so for a 30°, 60°, 90° triangle, one side is half the length of the hypotenuse.



(3) (20 points) Show that the composition of a reflection in the y-axis, followed by a reflection in $x = 2$ is a translation. Find an explicit description of the translation.



$$f(x,y) = (-x,y)$$

$$g: \quad (x,y) \mapsto (x-2,y)$$

$$(x,y) \mapsto (-x,y)$$

$$(x,y) \mapsto (x+2,y)$$

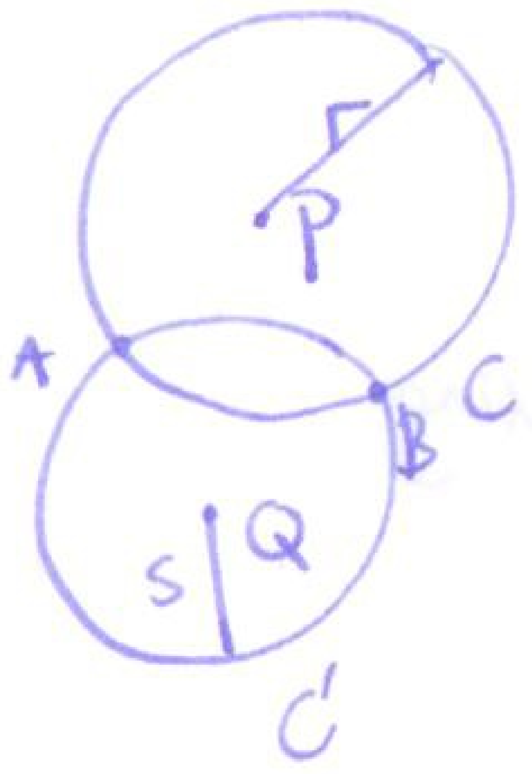
so $g(x,y) \mapsto (x-2,y) \mapsto (-x+2,y) \mapsto (-x+4,y)$

$$g(x,y) = (-x+4,y)$$

so $\quad g\circ f(x,y) = g(-x,y) = (x+4,y)$

$g\circ f$ is translation by 4 units in direction of x-axis.

(4) (20 points) Show that a Euclidean isometry takes circles to circles. Show that it takes a pair of circles which intersect exactly twice to a pair of circles which intersect exactly twice.



a circle C with center P of radius r consists of all points distance r from P. So $f(C)$ consists of all points distance r from $f(P)$, so is the circle of radius r about $f(P)$.

Let C' be all points distance s from Q.

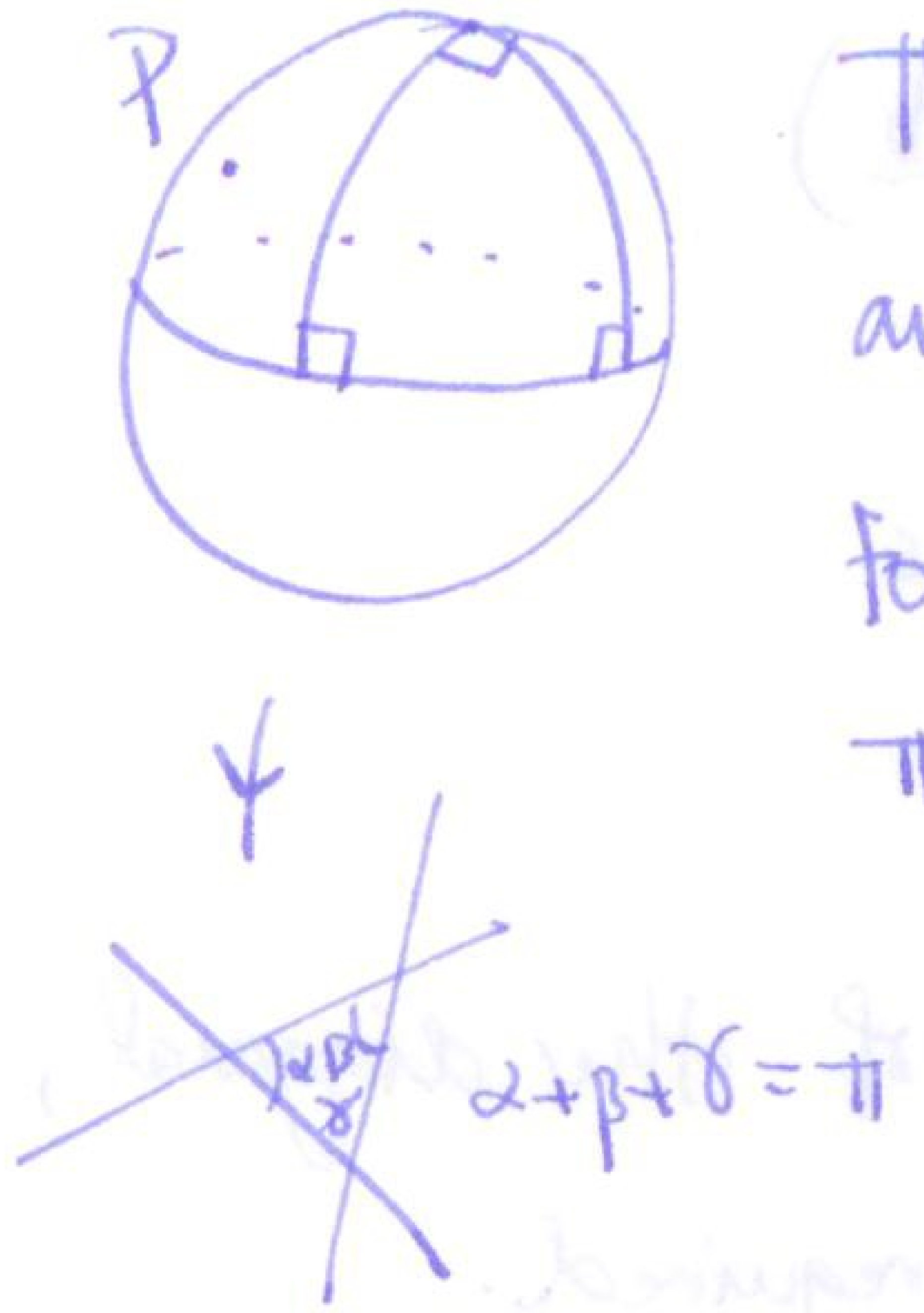
If there are exactly two points, A, B distance both r from P and s from Q, then there are exactly two points $f(A), f(B)$ distance both r from $f(P)$ and s from $f(Q)$, so $f(C)$ and $f(C')$ intersect exactly twice.

(5) (20 points) Describe an isometry of S^2 which swaps the points $(1, 0, 0)$ and $(-1, 0, 0)$. (There's more than one, just describe a particular one.)

$(x, y, z) \mapsto (-x, y, z)$ works

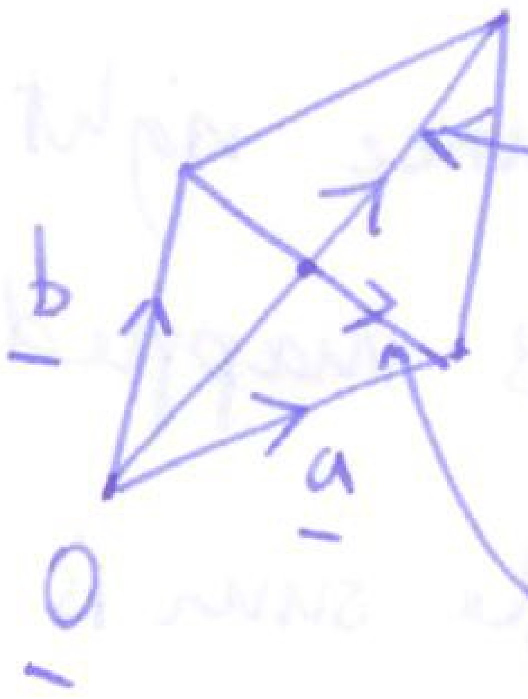
(this is reflection in the great circle corresponding to the y-z-plane).

(6) (20 points) Show that any map from the sphere minus a point to the plane, which takes great circles to straight lines, does not preserve angles.



There is a spherical triangle with three right angles not containing P, so this gets mapped to a Euclidean triangle, whose angle sum is π, so angles are not preserved.

(7) (20 points) Use vectors to show that the diagonals of a parallelogram bisect each other.



$\underline{a}+\underline{b}$ halfway point is $\frac{1}{2}(\underline{a}+\underline{b})$

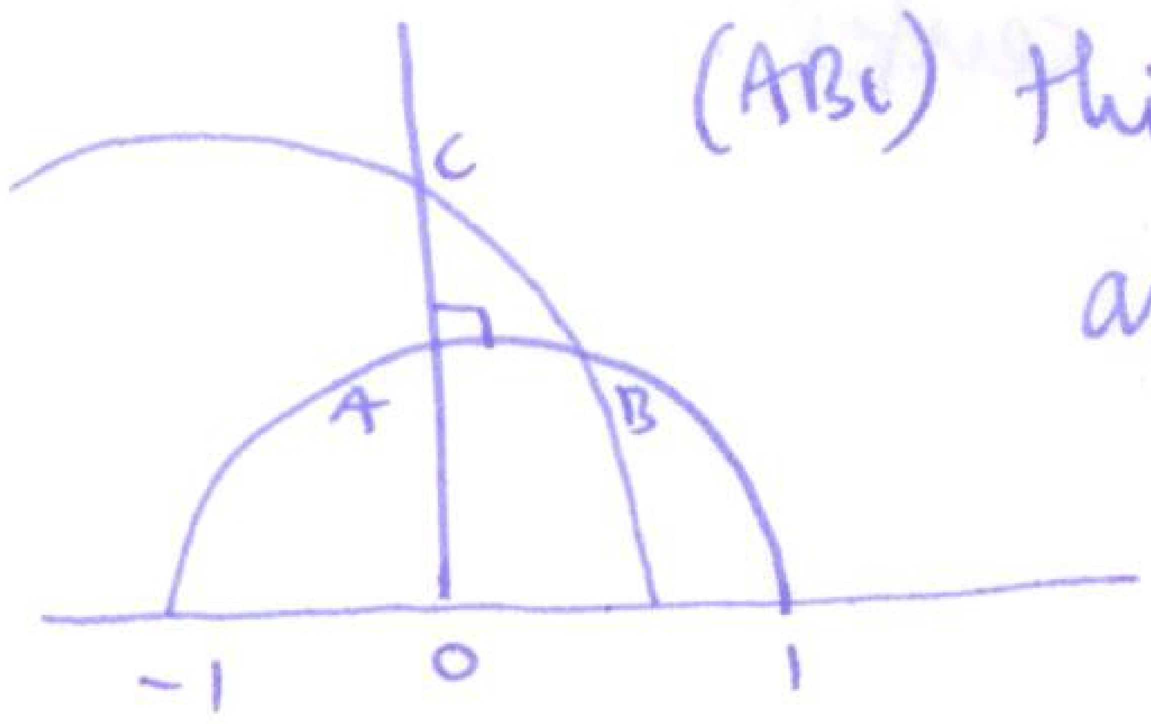
this vector is $-\underline{b}+\underline{a}$, so midpoint is

$$\underline{b}+\frac{1}{2}(-\underline{b}+\underline{a}) = \frac{1}{2}\underline{b}+\underline{a}$$

$=$ midpoint of other diagonal, as required.

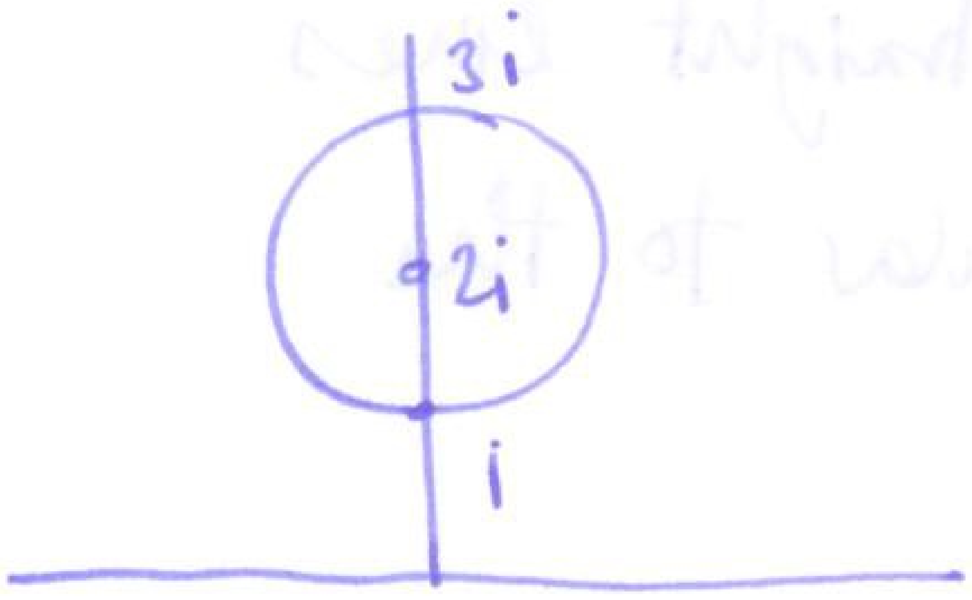
(8) (20 points) Describe the hyperbolic lines in the upper half space model for the hyperbolic plane. Draw a hyperbolic right angled triangle, and explain briefly why you know that it has a right angle.

Hyperbolic lines are straight lines and circles perpendicular to the boundary.



(ABC) this is a right angled triangle, as hyperbolic angles are the same as Euclidean angles.

(9) (20 points) In the upper half space model for hyperbolic space, it turns out that hyperbolic circles are also Euclidean circles. Explain why the point $2i$ cannot be the center in the hyperbolic metric for the Euclidean circle of radius 1 about $2i$.



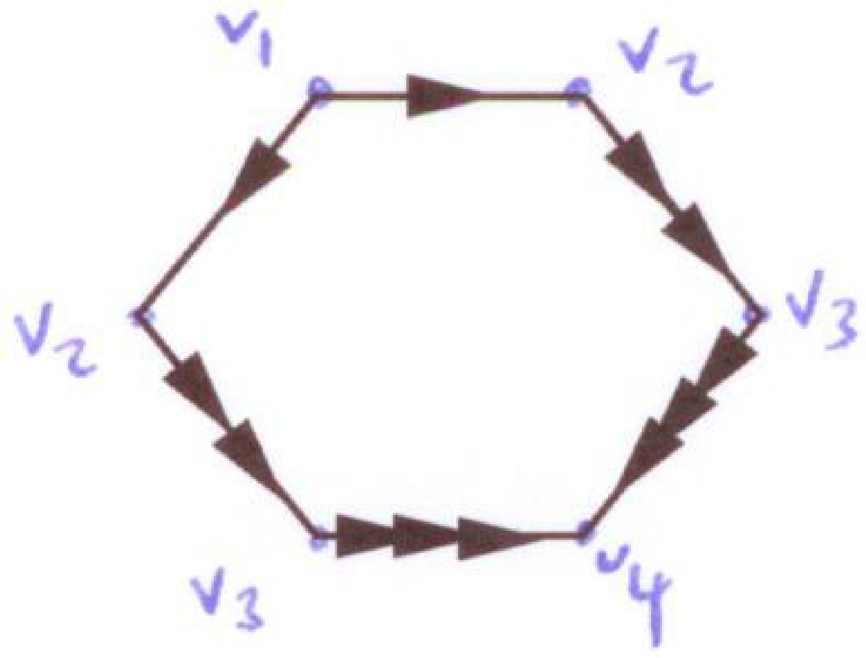
$$d_{\mathbb{H}}(2i, 3i) = \ln(3/2)$$

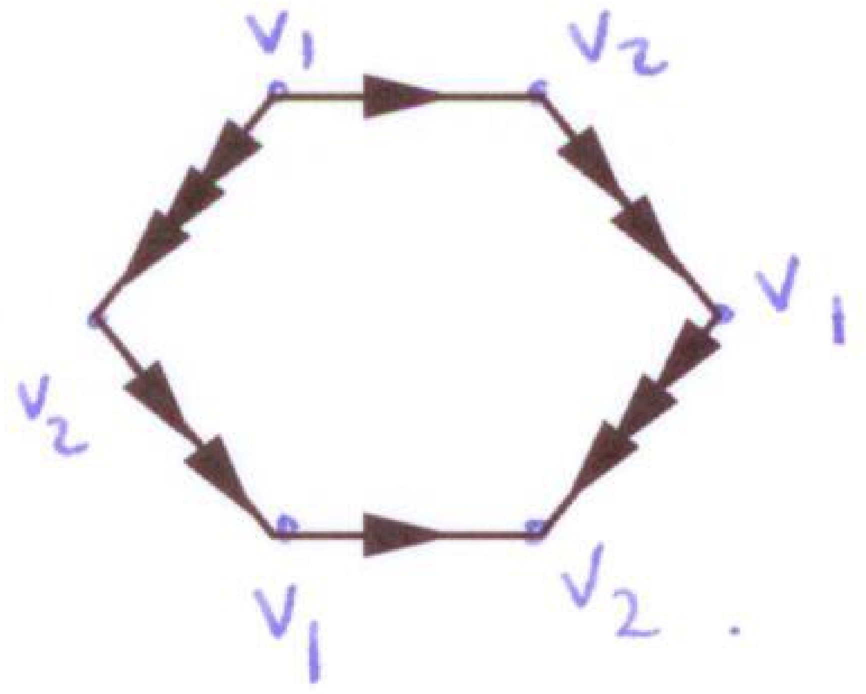
$$d_{\mathbb{H}}(2i, i) = \ln(2)$$

not equal

so $2i$ cannot be the center.

(10) (20 points) Two surfaces are constructed by identifying sides of polygons, as illustrated below. Find the Euler characteristic of the surfaces. Identify the surfaces.





$\chi = V - E + F$

$4 - 3 + 1$

$= 2$

sphere

$\chi = V - E + F$

$2 - 3 + 1 = 0$

torus