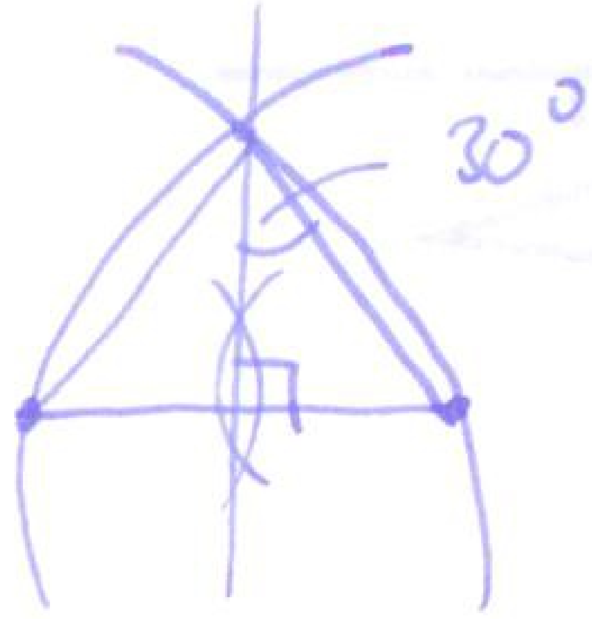
Sample final solutions

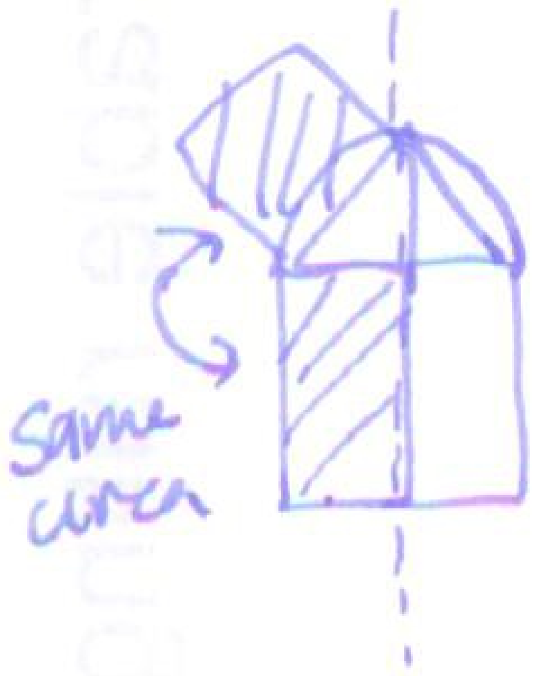
Q1 construct an equilateral triangle (60° angles) and then bisect it.



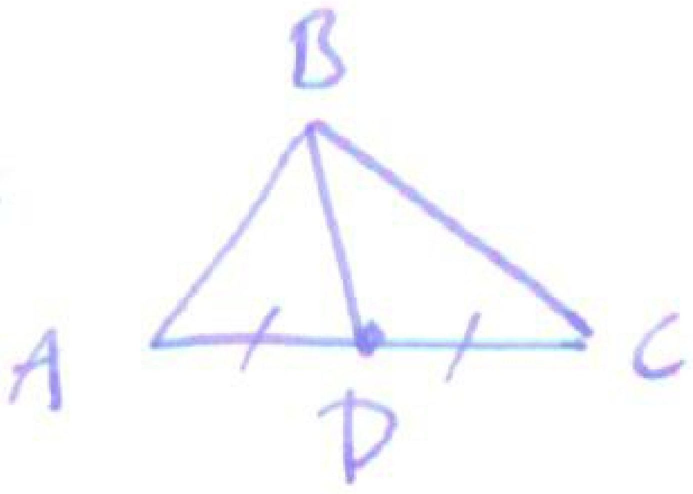
Q2 construct the altitude, and then bisect the resulting ~~triangle~~ rectangle.

This gives a rectangle with the same area.

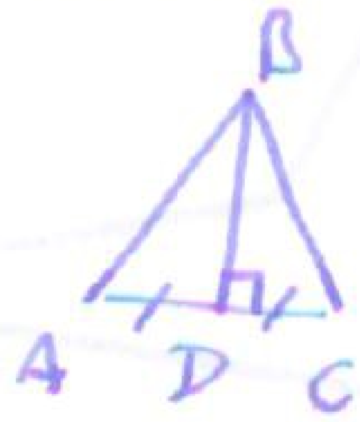
now apply standard rectangle ⇝ square construction, i.e. embed rectangle in square of side length longest side, and construct semicircle on extended edge, then extend inner side of rectangle to intersect semi-circle, and construct right angle triangle. Square on corresponding side has same area as rectangle



Q3



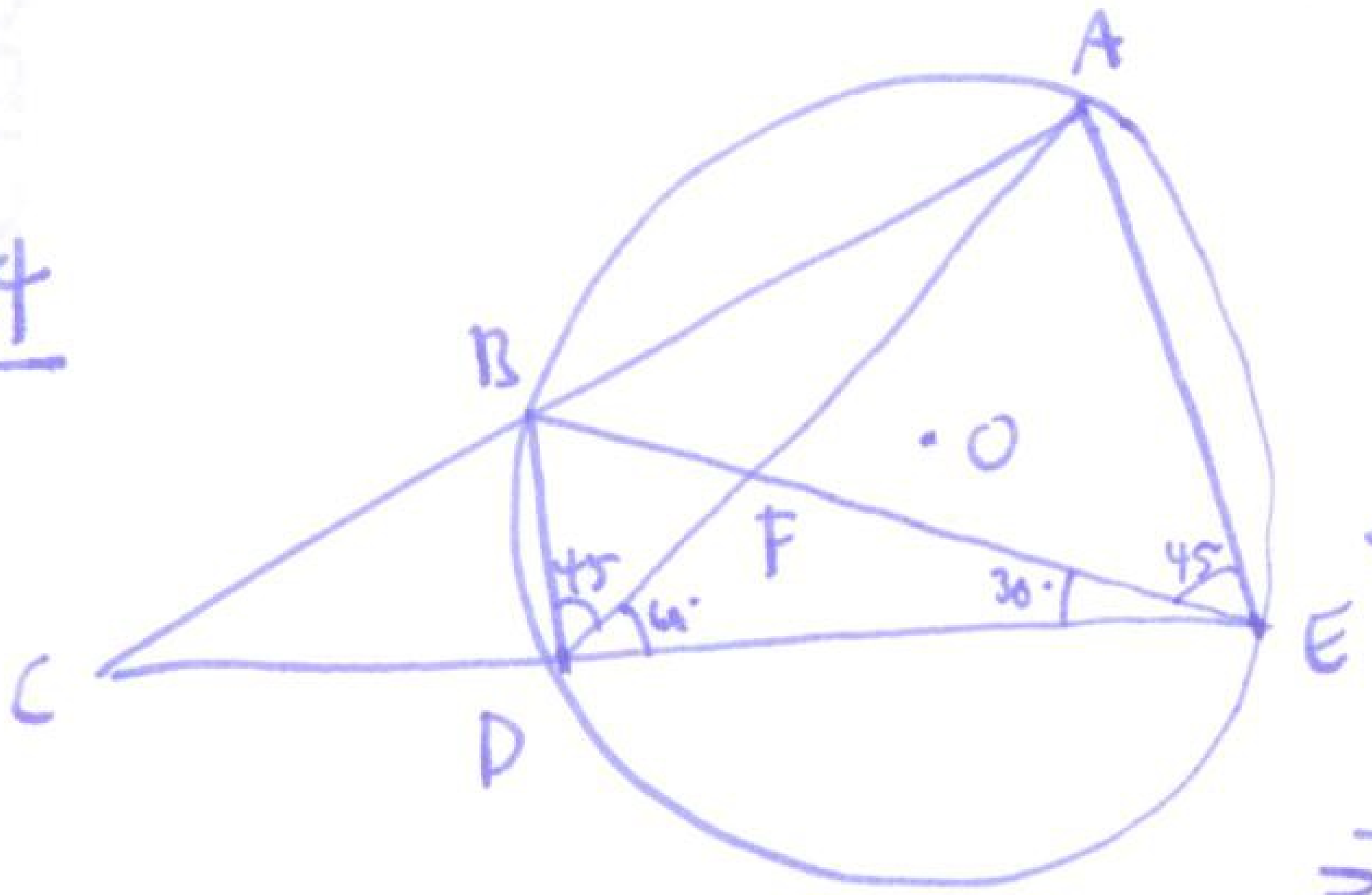
spose $BD \perp AC$



then ADB, CDB congruent by SAS

$\Rightarrow \overline{AB} = \overline{BC}$. ↯.

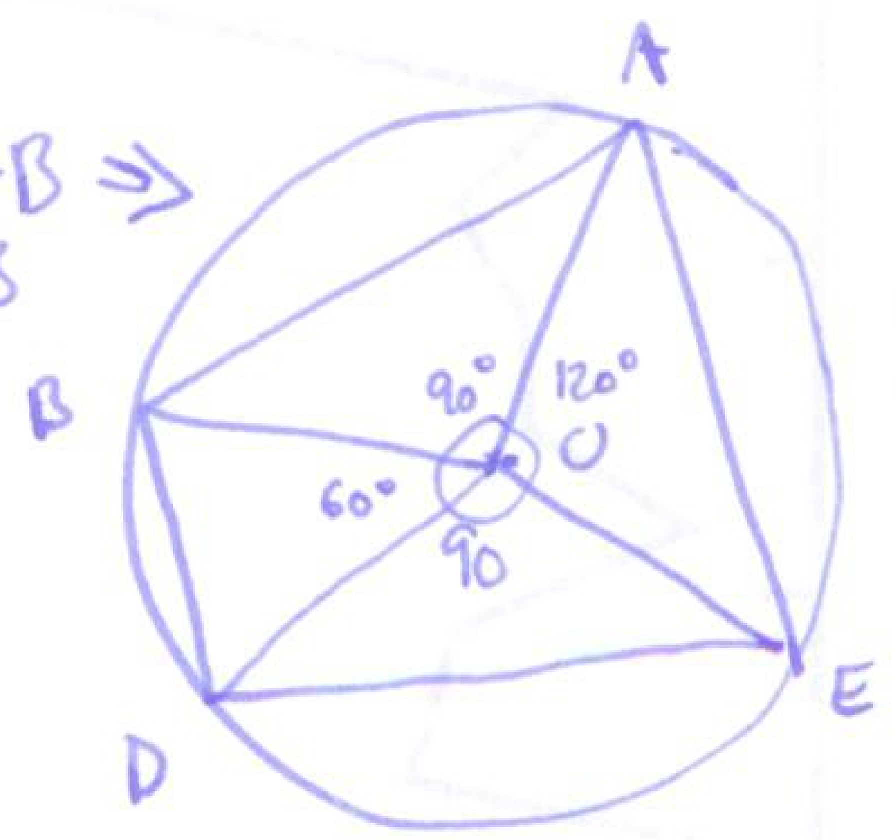
Q4



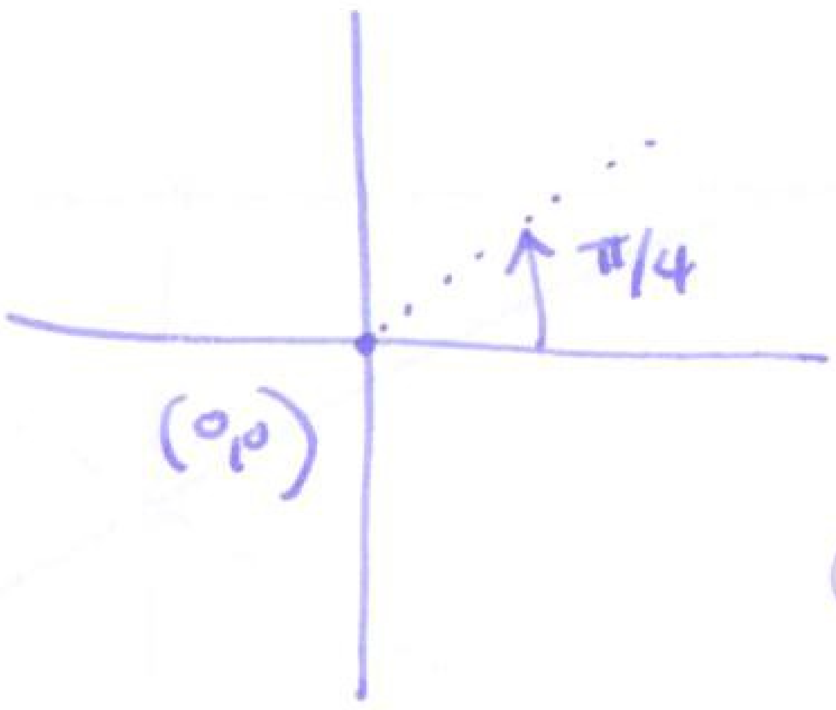
⇐ angle at center twice angle at circumference

$\Rightarrow DFE = 90°$

$BD : DE : EA : AB \Rightarrow$
$2 : 3 : 4 : 3$



Q5



rotⁿ of $\frac{\pi}{4}$ about $(0,0)$ $\quad z \mapsto e^{\pi i/4} z$

translation by 4 in +ve x-axis: $z \mapsto z+4$.

composition: $z \mapsto e^{\pi i/4} z + 4$

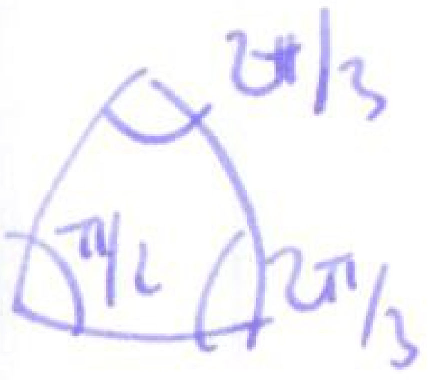
find fixed point: $z = e^{\pi i/4} z + 4$

$z\left(1 - e^{\pi i/4}\right) = 4 \qquad z = \dfrac{4}{1-e^{\pi i/4}}$ $\quad$ unique fixed pt $\Rightarrow$ rotation.

$$z = \frac{4}{1 - \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i} = \frac{4}{\frac{2-\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i} \Rightarrow \frac{2}{(2-\sqrt{2}+\sqrt{2}i)} \frac{(2-\sqrt{2}-\sqrt{2}i)}{(2-\sqrt{2}-\sqrt{2}i)} = \frac{4-2\sqrt{2}-2\sqrt{2}i}{4-2\sqrt{2}+2+2}$$

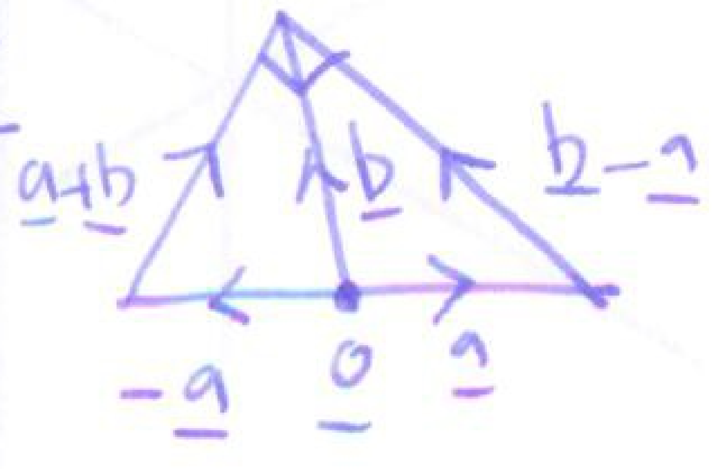
$$z = \frac{4-2\sqrt{2}-2\sqrt{2}i}{8-2\sqrt{2}}.$$ $\leftarrow$ rotation of $\pi/4$ about this point.

Q6 area spherical triangle $= \alpha + \beta + \gamma - \pi$.



area $\frac{2\pi}{3} + \frac{2\pi}{3} + \frac{\pi}{2} - \pi = \frac{4+4+3-6}{6}\pi = \frac{5}{6}\pi$ does not divide 4π, so can't tile S^2.

Q7



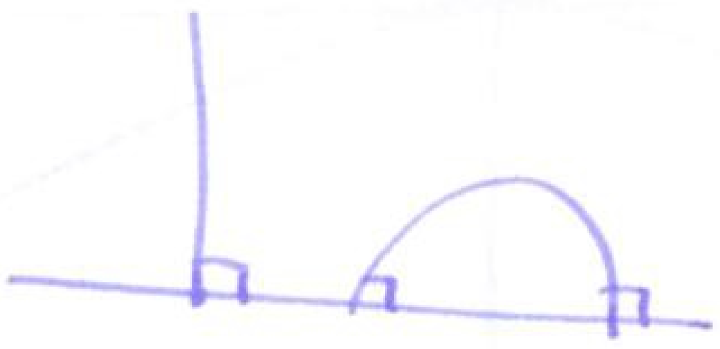
$\underline{a}+\underline{b}$, $\underline{b}-\underline{a}$ perpendicular so

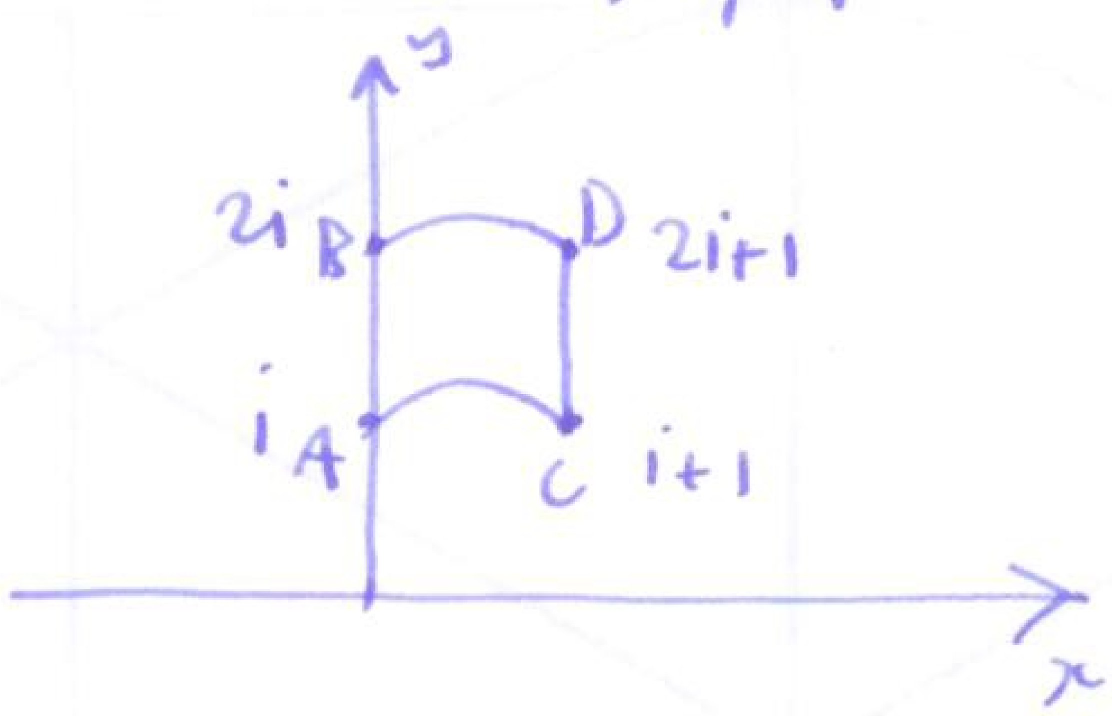
$(\underline{b}+\underline{a})\cdot(\underline{b}-\underline{a}) = 0$

$\Rightarrow \quad |\underline{b}|^2 - |\underline{a}|^2 = 0$

$\Rightarrow \quad |\underline{a}|^2 = |\underline{b}|^2 \Rightarrow$ length of median $|\underline{b}|$ is equal to $\frac{1}{2}$ length of hyp ($2|\underline{a}| = 2|\underline{b}|$).

Q8 hyperbolic lines: straight lines and circles, both perpendicular to x-axis:

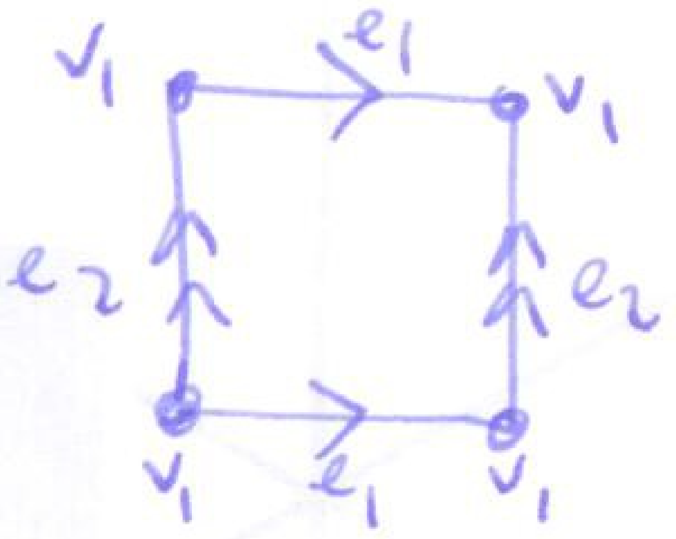




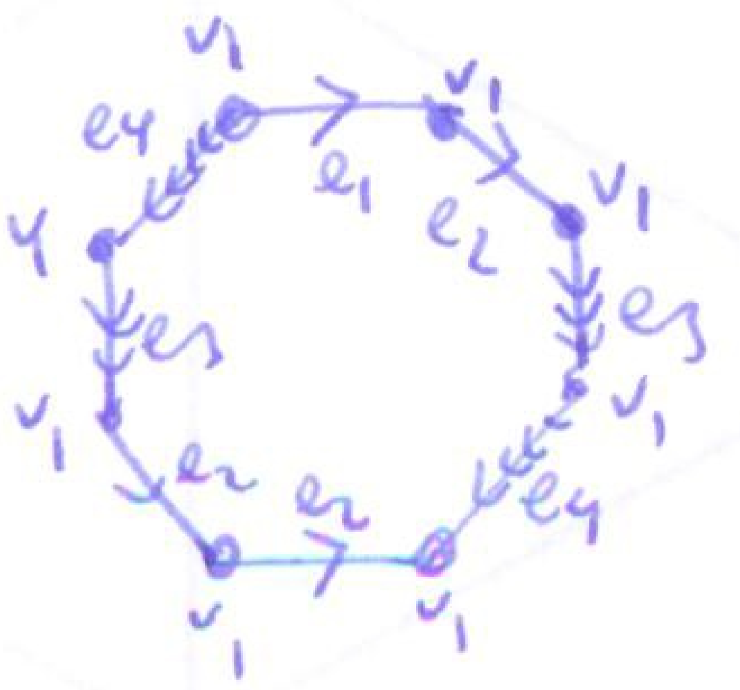
$z \mapsto z+1$ is an isometry of the upper half space, so $\overline{AB} = \overline{CD}$.

horizontal distances increase as $y \to 0$ so $\overline{AC}$ is greater than $\overline{BD}$.

Q9 count vertices, edges, faces.



$V - E + F = 1 - 2 + 1 = 0$ (torus).



$V - E + F = 1 - 4 + 1 = -2$ (genus 2).