Math 230 Calculus 1/Precalc Fall 11 Midterm 3b

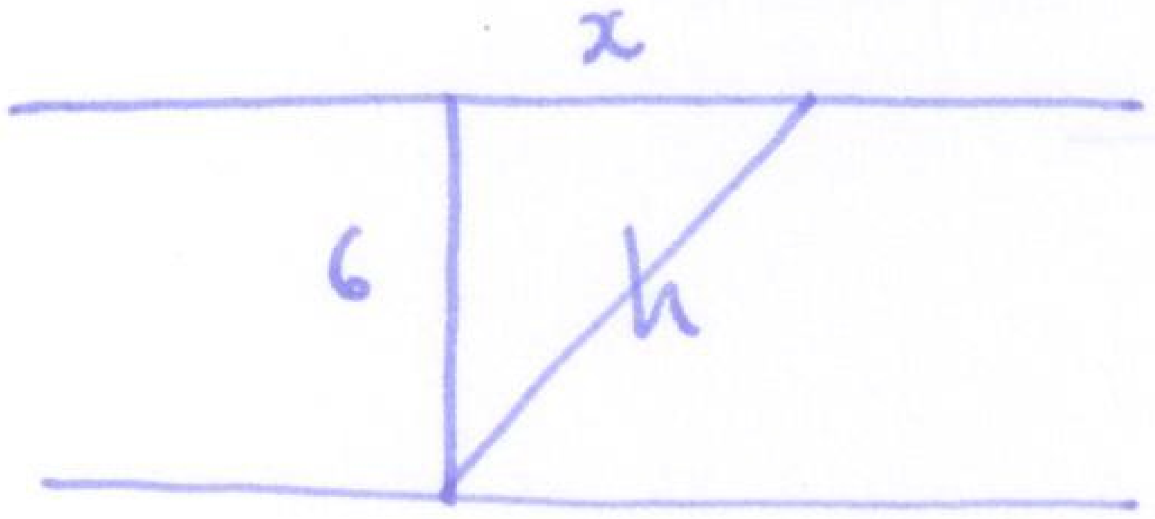
Name: Solutions

- Do any 8 of the following 10 questions.
- You may use a calculator, but no notes.

1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
	80	

Midterm 1	
Overall	

(1) (10 points) A plane flies directly over your head at a height of 6 miles and a speed of 300 mph. How fast is the distance from you to the plane changing 2 minutes later?



$$6^2 + x^2 = h^2$$

$$2x \frac{dx}{dt} = 2h \frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{x}{h} \frac{dx}{dt}$$

$$x = \frac{300}{30} = 10$$

$$h = \sqrt{6^2 + 10^2} \simeq 11.66$$

$$\frac{dx}{dt} = 300$$

$$\frac{dh}{dt} = \frac{10}{11.66} \cdot 300 \approx 257.25$$

(2) (10 points) Use a linear approximation to estimate $\sqrt[3]{65}$, using the fact that $\sqrt[3]{64} = 4$.

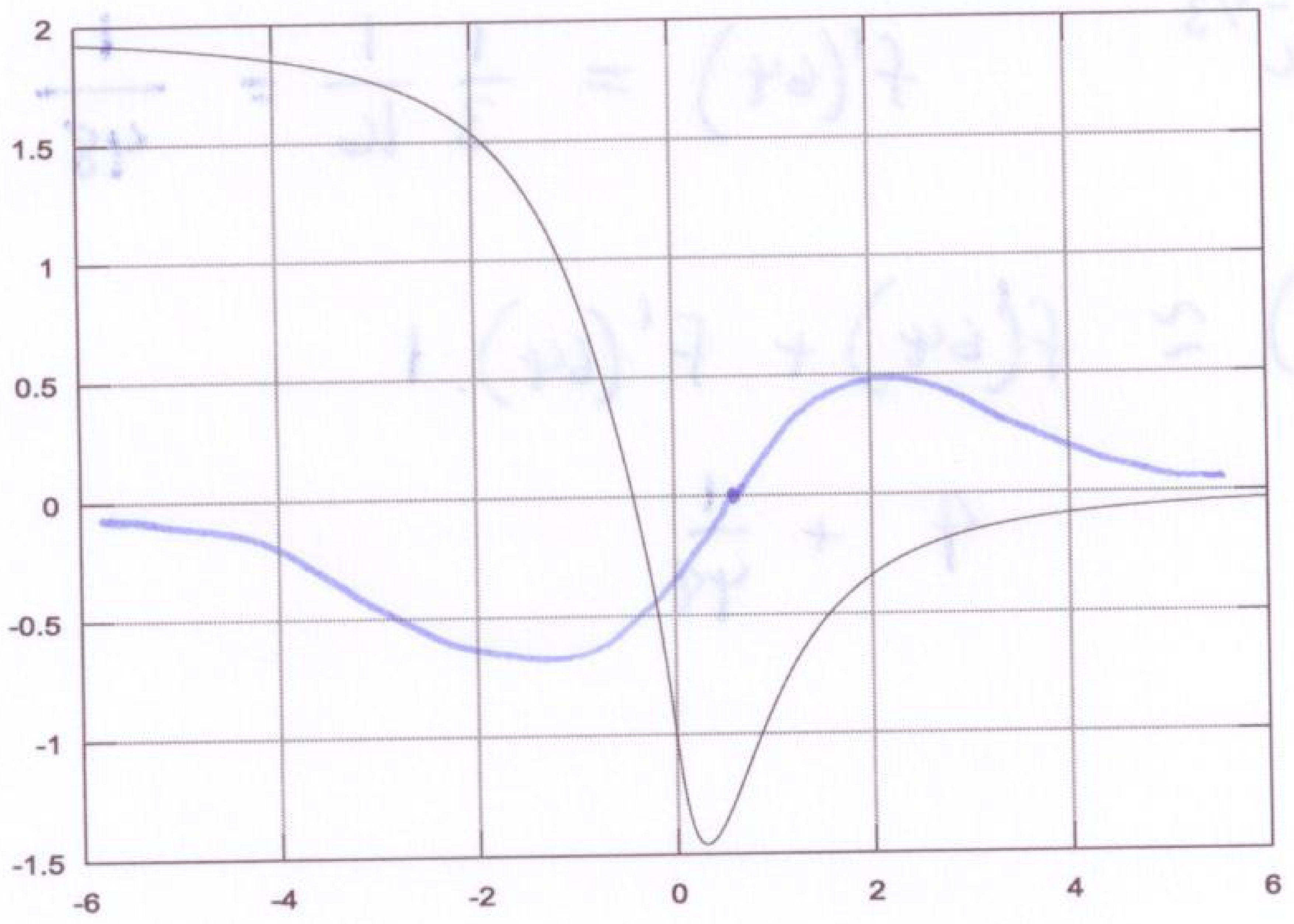
$f(x) = x^{1/3}$ $\qquad f(64) = 4$

$f'(x) = \frac{1}{3}x^{-2/3}$ $\qquad f'(64) = \frac{1}{3}\frac{1}{16} = \frac{1}{48}$

$f(64+1) \simeq f(64) + f'(64).1$

$4 + \frac{1}{48}$

(3) (10 points)



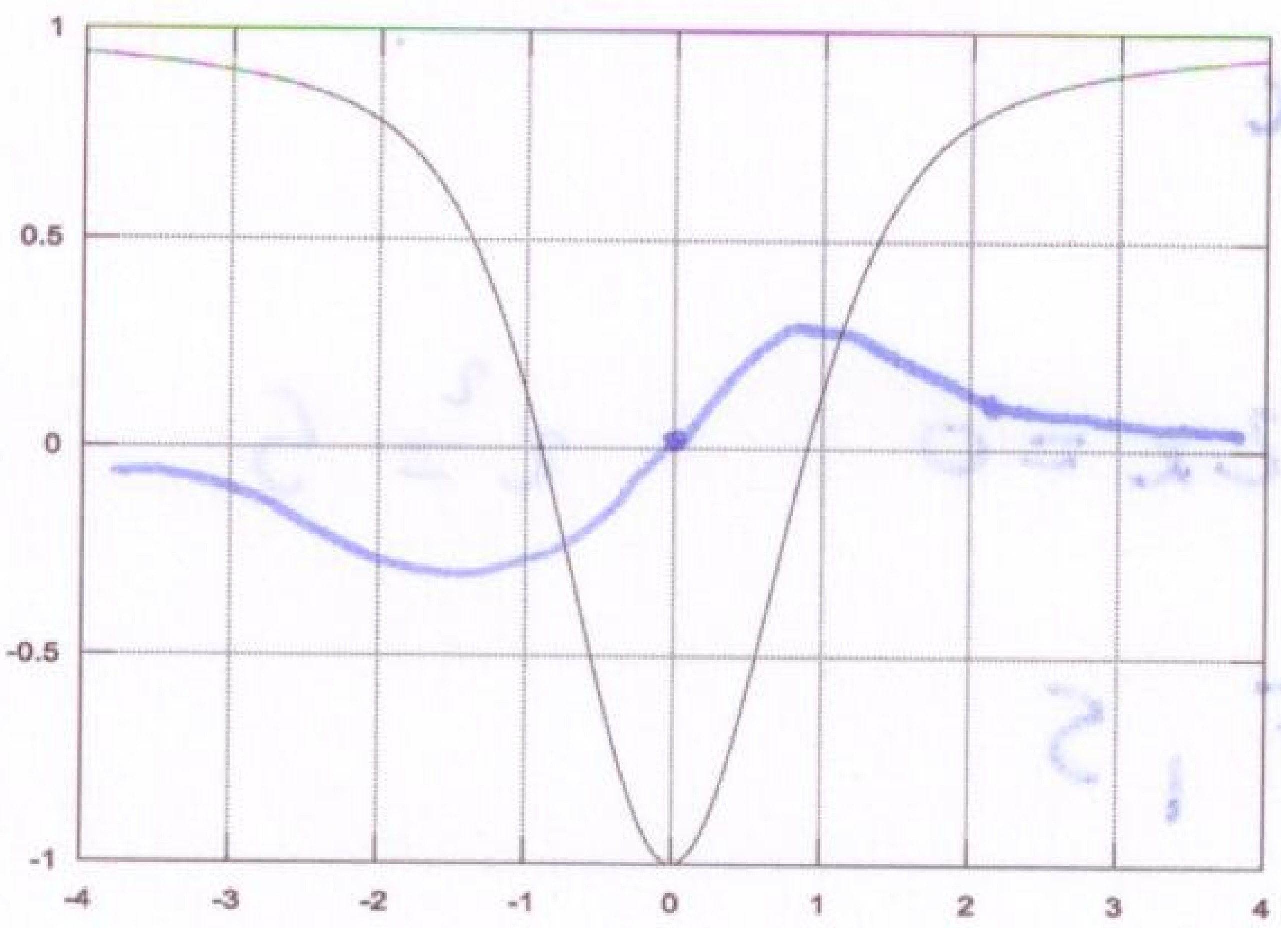
(a) Sketch the graph of $f'(x)$ on the picture above.
(b) Where is the function increasing?
(c) Where is the function decreasing?
(d) What is $\lim_{x\to\infty} f'(x)$?

b) $(\frac{1}{2}, \infty)$

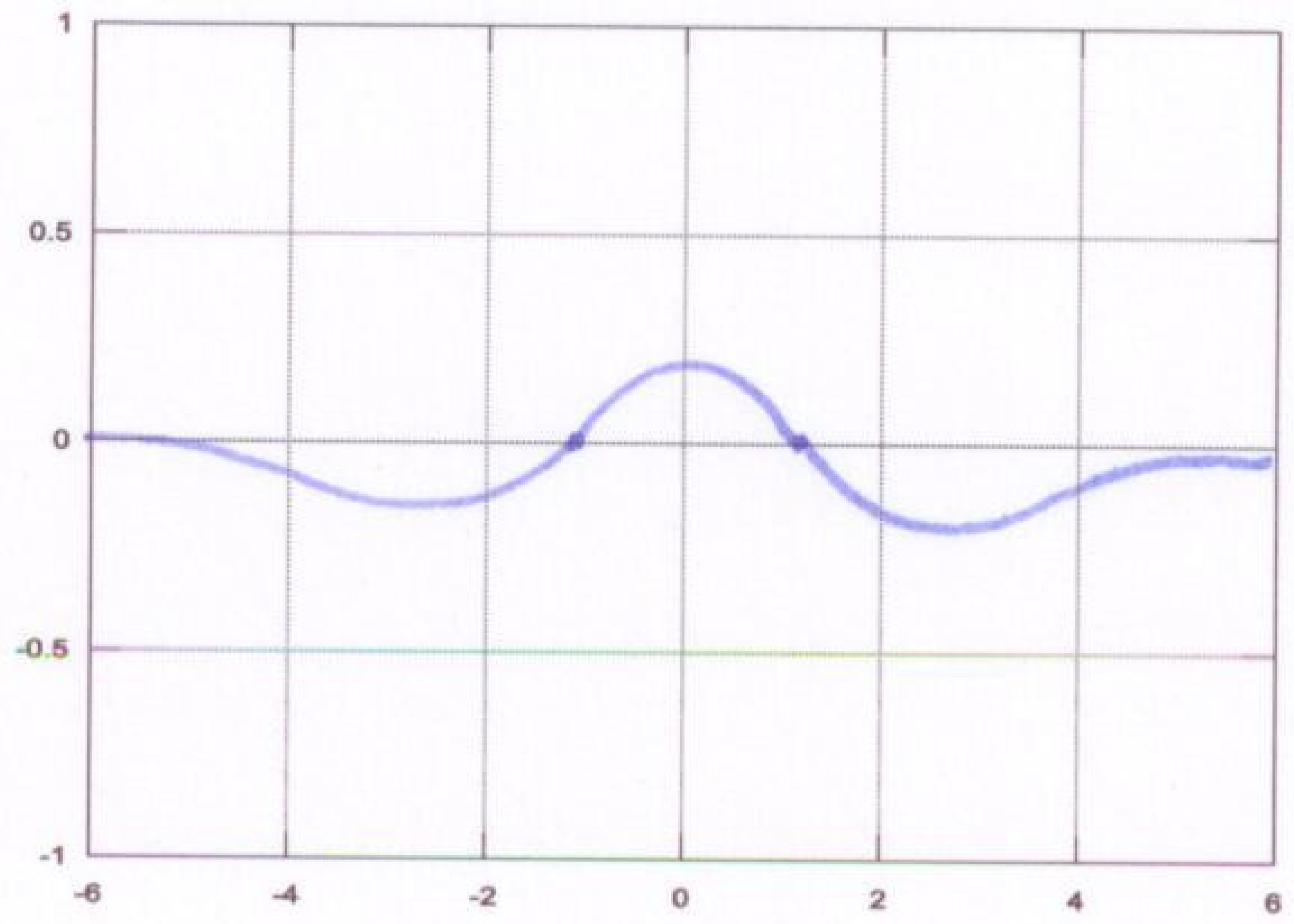
c) $(-\infty, \frac{1}{2})$

d) 0

(4) (10 points)



(a) Sketch the graph of $f'(x)$ on the picture above.
(b) Sketch the graph of $f''(x)$ on the picture below.
(c) Where is the graph concave up and concave down?
(d) Where are the points of inflection?



c) concave up : $(-1,1)$
concave down : $(-\infty,1) \cup (1,\infty)$

d) ± 1

(5) (10 points) Find the maximum and minimum values of the function $f(x) = 10\ln(x) - x^2$ on the interval $[1, 5]$.

$f'(x) = \frac{10}{x} - 2x$

$f'(x) = 0 :$ $\quad \frac{10}{x} - 2x = 0 \quad x^2 = 5 \quad x = \pm\sqrt{5}$

check : $\quad 1, \sqrt{5}, 5$

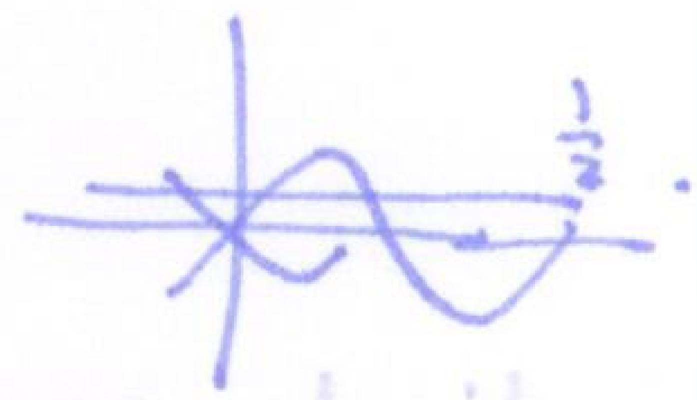
$f(1) = -1$

$f(\sqrt{5}) = 10\ln(\sqrt{5}) - 5 = 5\ln 5 - 5 \simeq 3.05$ maximum

$f(5) = 10\ln 5 - 25 \simeq -8.91$ minimum

(6) (10 points) Find the maximum and minimum values of the function $f(x) = \cos(2x) + x$ on the interval $[0, 1]$.

$$f'(x) = -2\sin(2x)+1 \} = 0 \Rightarrow \sin(2x) = \frac{1}{2}$$



$$2x = \frac{\pi}{6}, \frac{5\pi}{6} \pm 2\pi n$$

only sol$^{\underline{n}}$ in $[0,1]$ is $\frac{\pi}{12}$

check $f(0) = 1$

$$f\left(\frac{\pi}{12}\right) = \cos\left(\frac{\pi}{6}\right) + \frac{\pi}{12} \approx 1.28 \quad \text{maximum}$$

$$f(1) = \cos(2) + \cancel{2}1 \approx 0.58 \quad \text{minimum}$$

(7) (10 points) Consider the function

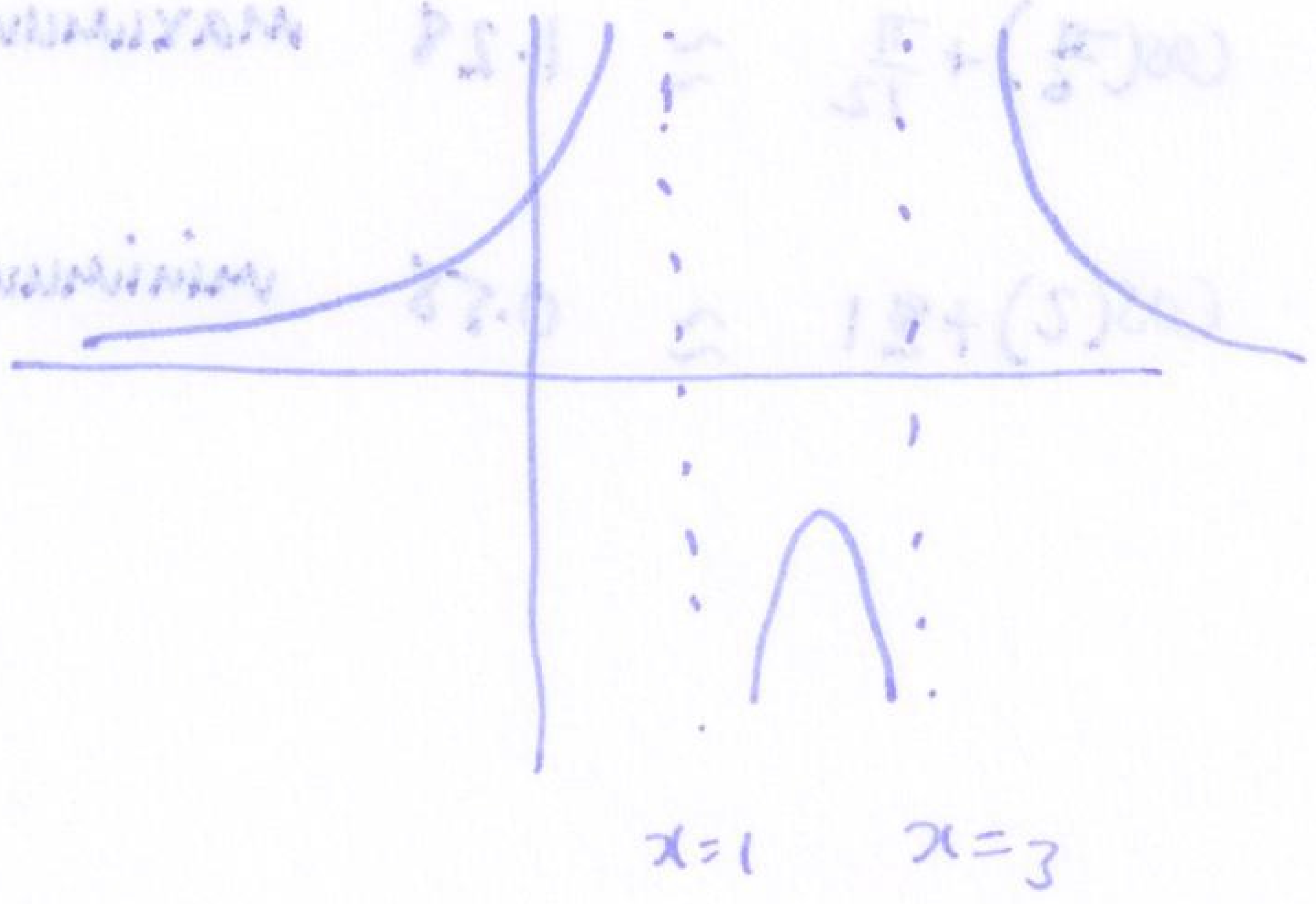
$$f(x) = \frac{1}{x^2 - 4x + 3} = \frac{1}{(x-3)(x-1)}$$

(a) Find all the vertical and horizontal asymptotes.
(b) Find all the critical points.
(c) Sketch the graph of the function.

vertical asymptotes : $x=3$, $x=1$ horizontal asymptotes $y=0$

critical points : $f'(x) = -(x^2-4x+3)^{-2} \cdot (2x-4)$

critical points : $x=2$



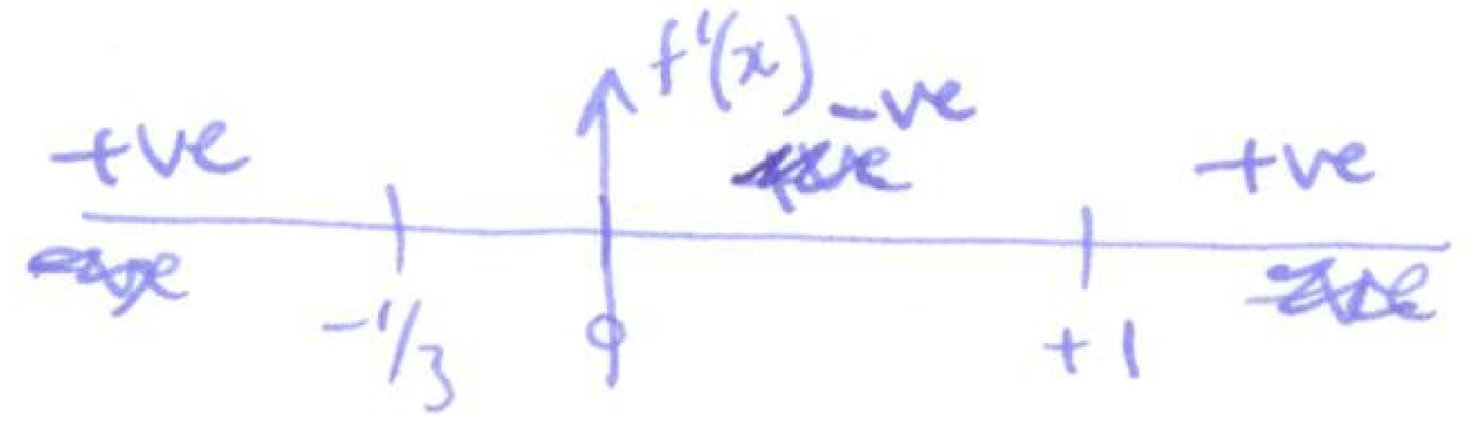
(8) (10 points) Consider the function

$$f(x) = x^3 - x^2 - x - 1$$

Find all the critical points and use the first derivative test to classify them.

$f'(x) = 3x^2 - 2x - 1 = (3x+1)(x-1)$

critical points : solve $f'(x) = 0$: $x = -1/3$, $x = 1$



$-1/3$: ~~minimum~~ max

$+1$: ~~maximum~~ min

(9) (10 points) Consider the function

$$f(x) = (x+1)e^{3x}$$

Find all the critical points and use the second derivative test to try and identify them.

$f'(x) = (x+1)3e^{3x} + e^{3x} = (3x+4)e^{3x}$

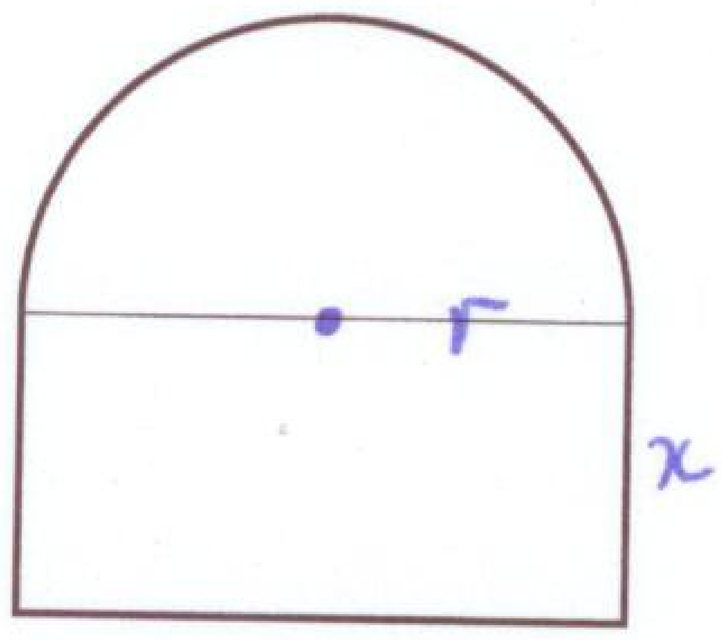
$x = -4/3$.

$f''(x) = (3x+4)3e^{3x} + 3e^{3x} = (9x+15)e^{3x}$.

$f''(-4/3) = (-12+15)e^{3x} > 0$ minimum.

(10) (10 points) I wish to make a window in the shape of a rectangle with a semi-circle attached to one side. If I want the frame of the window to be 3m long, what are the dimensions of the largest area window I can make?

$A = 2rx + \frac{1}{2}\pi r^2$



$P = \pi r + 2r + 2x = 3$

$x = \dfrac{3 - \pi r - 2r}{2}$

$A = \dfrac{\cancel{2}r\left(3 - r(\pi+2)\right)}{\cancel{2}} + \frac{1}{2}\pi r^2 = 3r + r^2\left(\frac{1}{2}\pi - \pi - 2\right)$

$\dfrac{dA}{dr} = 3 + 2r\left(-2 - \frac{\pi}{2}\right) = 0 \Rightarrow r = \dfrac{3}{2\left(2 + \frac{\pi}{2}\right)} = \dfrac{3}{4+\pi}$

$\simeq 0.42$

$x = \dfrac{3 - \left(\frac{3}{4+\pi}\right)(\pi+2)}{2} \simeq 0.42$