Math 230 Calculus 1/Precalc Fall 11 Midterm 3a

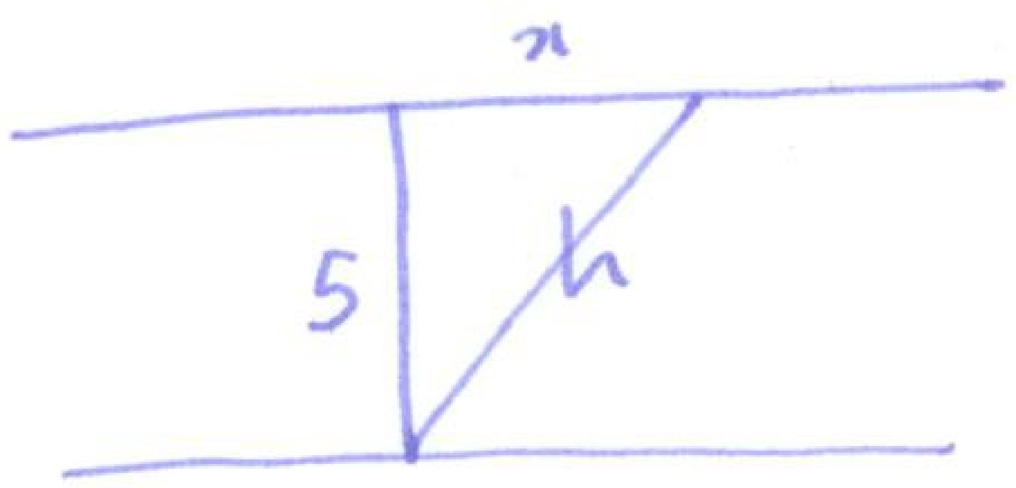
Name: Solutions

- Do any 8 of the following 10 questions.
- You may use a calculator, but no notes.

1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
	80	

Midterm 1	
Overall	

(1) (10 points) A plane flies directly over your head at a height of 5 miles and a speed of 600 mph. How fast is the distance from you to the plane changing 1 minute later?



$$5^2 + x^2 = h^2$$

$$2x\frac{dx}{dt} = 2h\frac{dh}{dt}$$

$$x = \frac{600}{60} = 10$$

$$h = \sqrt{5^2+10^2} \simeq 11.18$$

$$\frac{dx}{dt} = 600$$

$$\frac{dh}{dt} = \frac{x}{h}\frac{dx}{dt} = \frac{10}{11.18}600 \approx 536.66 \text{ mph}$$

(2) (10 points) Use a linear approximation to estimate $\sqrt[3]{30}$, using the fact that $\sqrt[3]{27} = 3$.

$f(x) = \sqrt[3]{x} = x^{1/3}.$

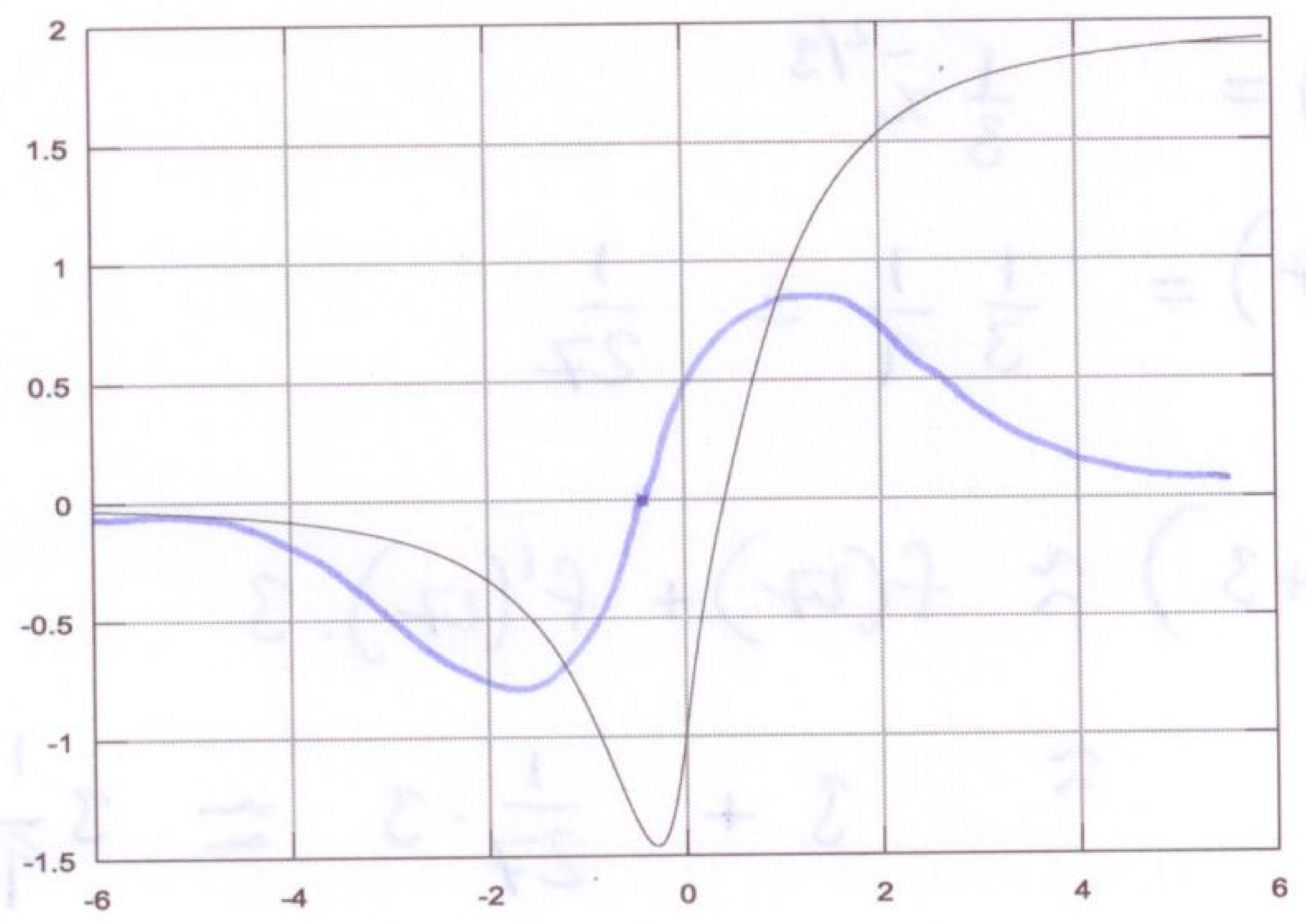
$f'(x) = \frac{1}{3} x^{-2/3}$

$f'(27) = \frac{1}{3} \frac{1}{9} = \frac{1}{27}$

$f(27+3) \approx f(27) + f'(27) \cdot 3$

$\approx 3 + \frac{1}{27} \cdot 3 \approx 3\frac{1}{9}.$

(3) (10 points)



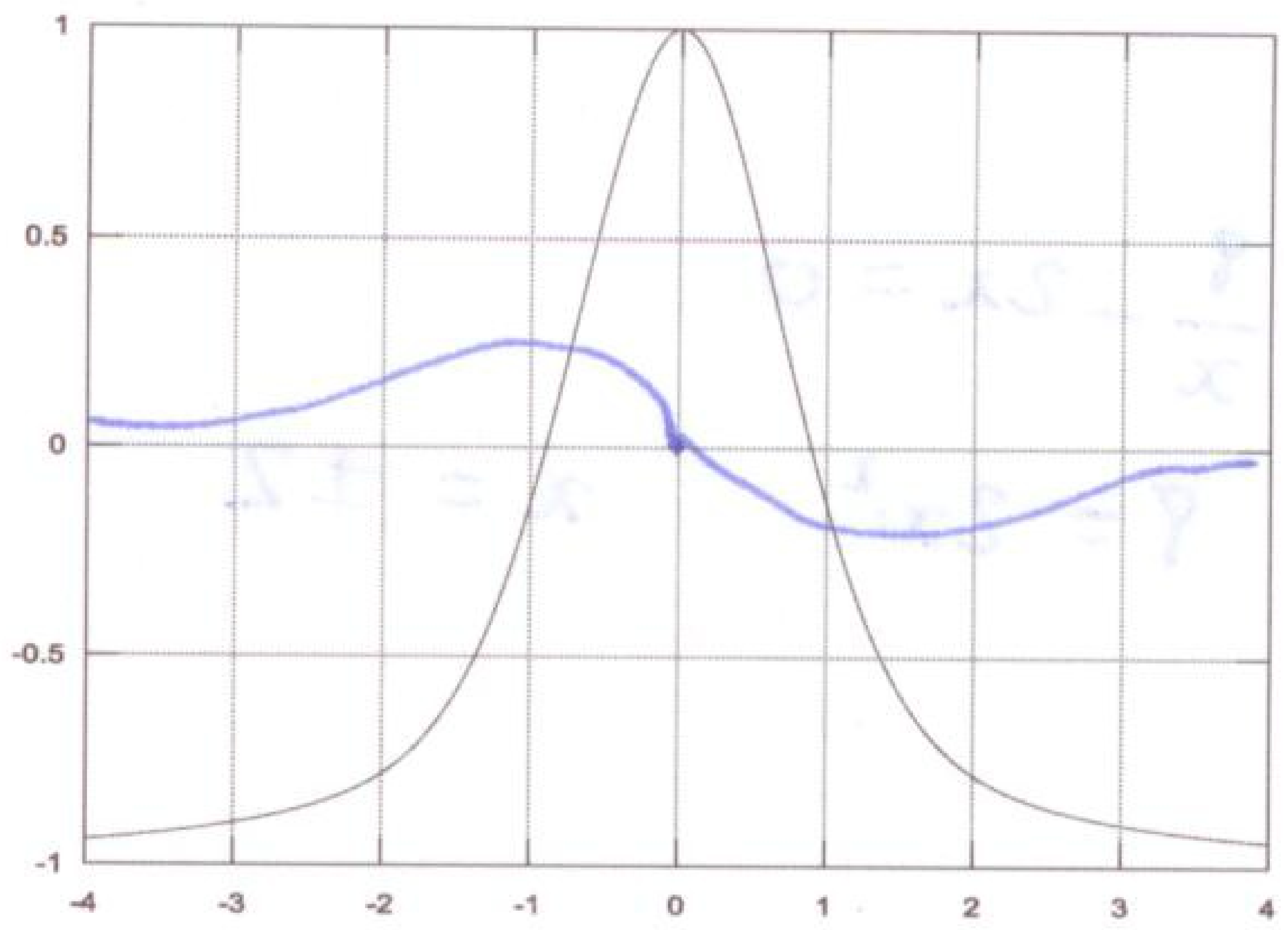
(a) Sketch the graph of $f'(x)$ on the picture above.
(b) Where is the function increasing?
(c) Where is the function decreasing?
(d) What is $\lim_{x\to\infty} f'(x)$?

b) ~~(-∞,~~ $(-\frac{1}{2}, \infty)$

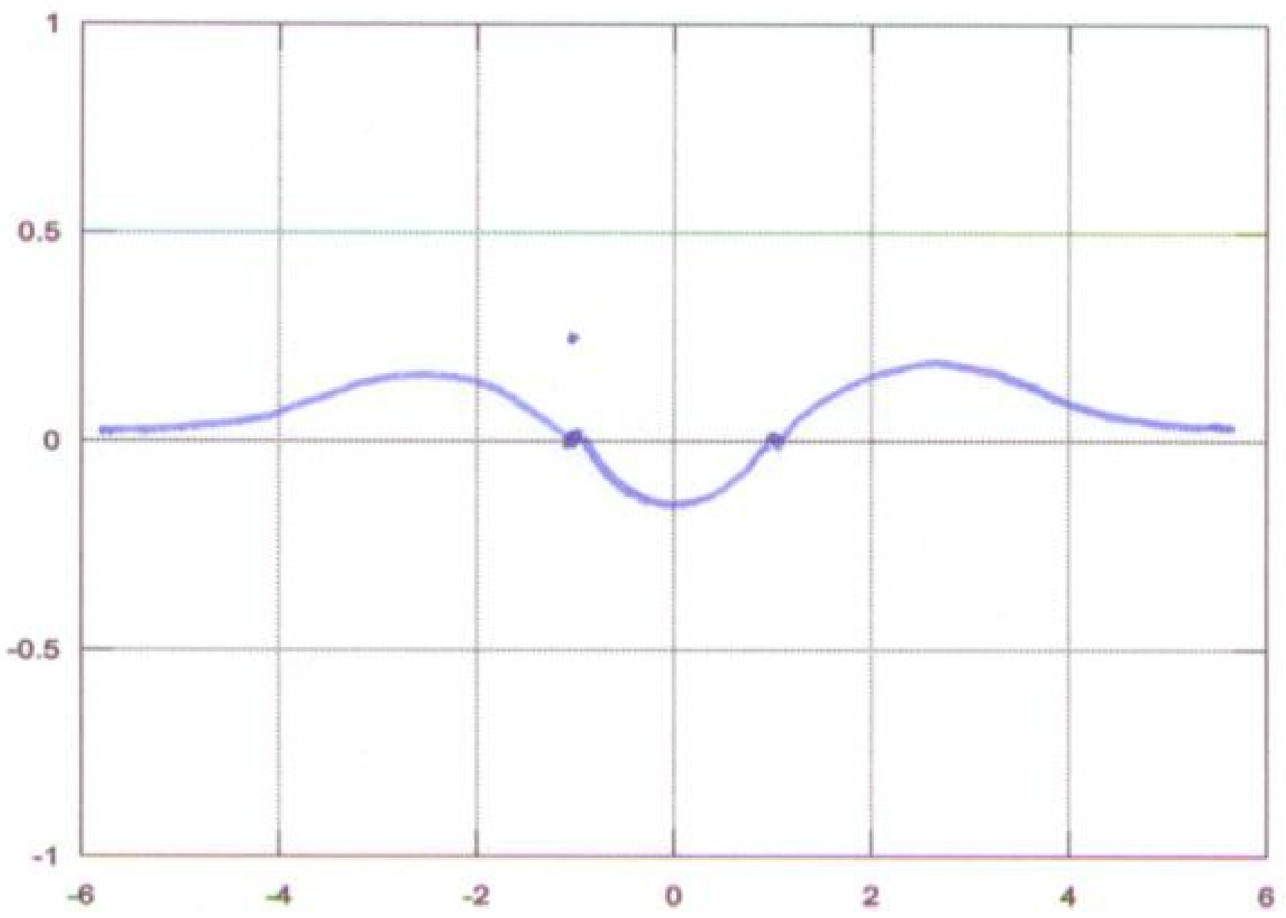
c) $(-\infty, -\frac{1}{2})$

d) 0

(4) (10 points)



(a) Sketch the graph of $f'(x)$ on the picture above.
(b) Sketch the graph of $f''(x)$ on the picture below.
(c) Where is the graph concave up and concave down?
(d) Where are the points of inflection?



c) concave up : $(-\infty,1) \cup (1,\infty)$
concave down: $(-1,1)$

(5) (10 points) Find the maximum and minimum values of the function $f(x) = 8\ln(x) - x^2$ on the interval $[1, 5]$.

$f'(x) = \frac{8}{x} - 2x$

solve $f'(x) = 0$: $\frac{8}{x} - 2x = 0$

$8 = 2x^2$ $\quad x = \pm 2$

check 1, 2, 5

$f(1) = -1$

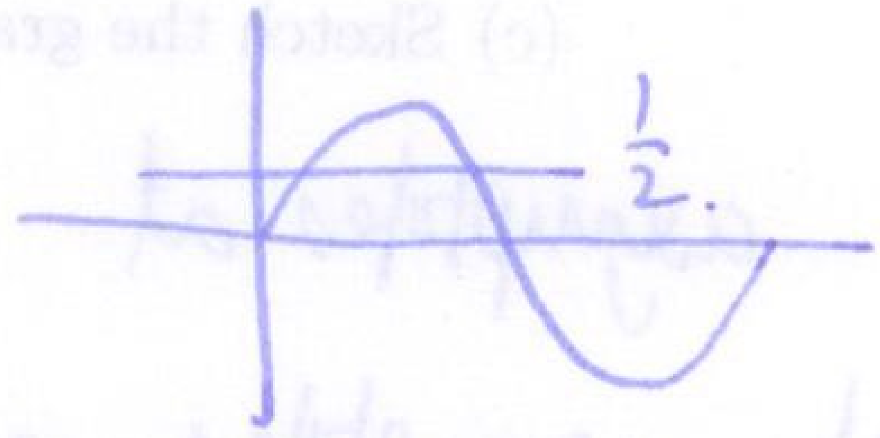
$f(2) = 8\ln(2) - 4 \approx 1.55$ maximum

$f(5) = 8\ln(5) - 25 \approx -12.12$ minimum

(6) (10 points) Find the maximum and minimum values of the function $f(x) = \cos(2x) + x$ on the interval $[0, 1]$.

$$f'(x) = -2\sin(2x) + 1 = 0$$

$$\sin(2x) = \frac{1}{2}.$$



$$2x = \frac{\pi}{6}, \frac{5\pi}{6} \pm 2\pi.$$

$$x = \frac{\pi}{12} \text{ (only soln in } [0,1]).$$

check $0, \frac{\pi}{12}, 1$

$$f(0) = 1$$

$$f\left(\frac{\pi}{12}\right) = \cos\left(\frac{\pi}{6}\right) + \frac{\pi}{12} \approx 1.28 \quad \text{maximum}$$

$$f(1) = \cos(2) + 1 \approx 0.58 \quad \text{minimum}$$

(7) (10 points) Consider the function

$$f(x) = \frac{1}{x^2 - 3x + 2} = \frac{1}{(x-2)(x-1)}$$

(a) Find all the vertical and horizontal asymptotes.
(b) Find all the critical points.
(c) Sketch the graph of the function.

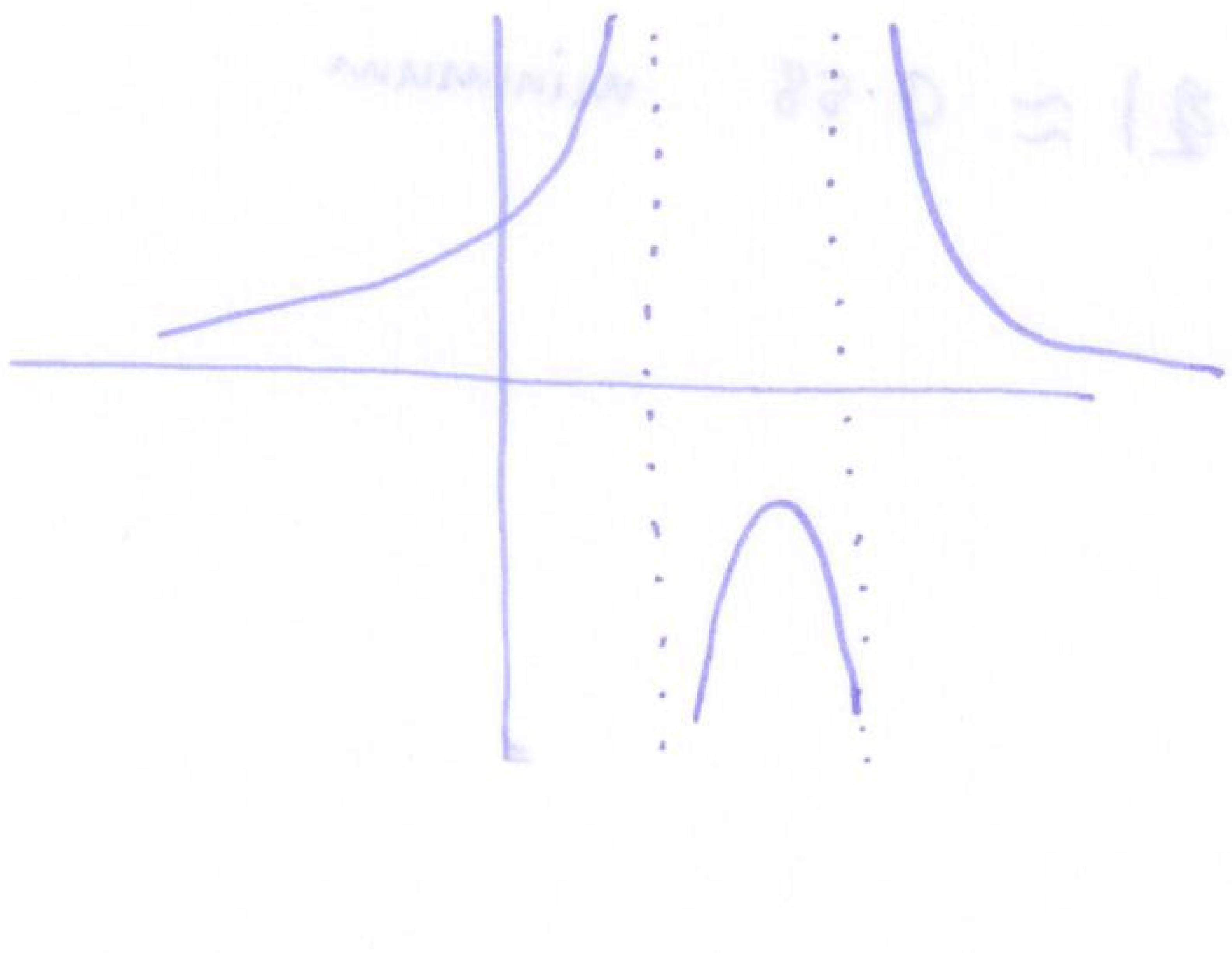
vertical asymptotes at $x = 2$, $x = 1$

horizontal asymptotes at : $\lim_{x \to \pm\infty} f(x) = 0$.

critical points : $f'(x) = -(x^2 - 3x + 2)^{-2} \cdot (2x - 3)$

$$= -\frac{2x-3}{(x^2-3x+2)^2}$$

critical point at $2x - 3 = 0$ $x = 1\frac{1}{2}$.



(8) (10 points) Consider the function

$$f(x) = x^3 - 2x^2 + x$$

Find all the critical points and use the first derivative test to classify them.

$$f'(x) = 3x^2 - 4x + 1 = (3x-1)(x-1)$$

critical points: $x = 1/3$ $\quad x = 1$

$f'(x)$

$+$ $\quad -$ $\quad +$

$0 \quad 1/3 \quad 1$

$1/3$: local max

1 : local min

(9) (10 points) Consider the function

$$f(x) = (x+1)e^{2x}$$

Find all the critical points and use the second derivative test to try and identify them.

$f'(x) = (x+1)2e^{2x} + e^{2x} = (2x+3)e^{2x}$

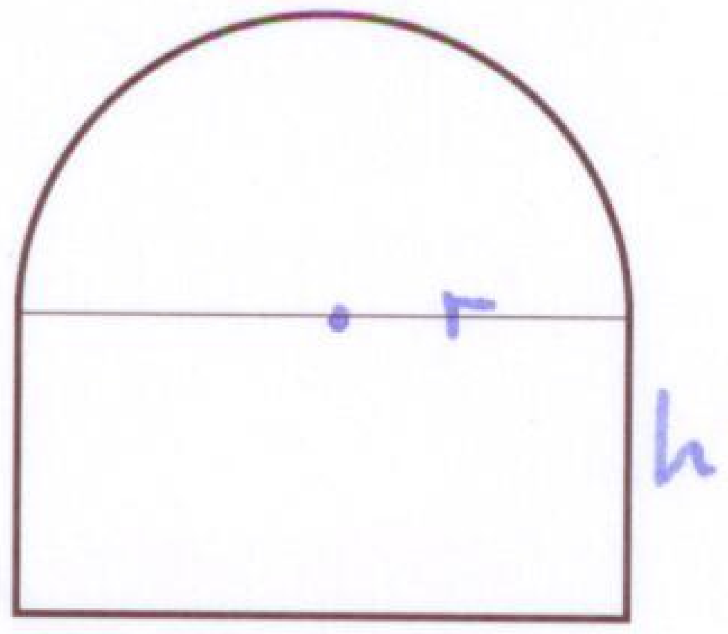
$f'(x) = 0 \Rightarrow x = -3/2$

$f''(x) = (2x+3)2e^{2x} + 2e^{2x} = (4x+8)e^{2x}$

$f''(-3/2) = (-6+8)e^{-3} > 0$

$\Rightarrow$ local min

(10) (10 points) I wish to make a window in the shape of a rectangle with a semicircle attached to one side. If I want the frame of the window to be 2m long, what are the dimensions of the largest area window I can make?



$A = 2rh + \frac{1}{2}\pi r^2$

$P = \pi r + 2h + 2r = 2 \Rightarrow h = 1 - r\left(\frac{\pi}{2} + 1\right)$

$A = 2r\left(1 - r\left(\frac{\pi}{2}+1\right)\right) + \frac{1}{2}\pi r^2 = 2r + r^2\left(\frac{1}{2}\pi - \pi - 2\right)$

$\frac{dA}{dr} = 2 + 2r\left(-\frac{\pi}{2} - 2\right) = 0$

$2r = \frac{2}{\frac{\pi}{2}+2} \Rightarrow r = \frac{1}{\frac{\pi}{2}+2} = \frac{2}{\pi+4} \approx 0.28$

$h = 1 - \frac{2}{\pi+4}\left(\frac{\pi}{2}+1\right) \simeq 0.28$