Math 230 Calculus 1/Precalc Fall 11 Midterm 2a / b

Name: Solutions

- Do any 8 of the following 10 questions.
- You may use a calculator, but no notes.

1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
	80	

Midterm 1	
Overall	

(1) (10 points) Find the derivative of the function

$$f(x) = e^{-5x}$$

$-5e^{-5x}$

(2) (10 points) Find the derivative of the function

$$f(t) = \frac{\cos(t)}{1+3t^2}$$

$$\frac{(1+3t^2).(-\sin(t)) - \cos(t)(6t)}{(1+3t^2)^2}$$

(3) (10 points) Find the derivative of the function

$$f(x) = (\sqrt{x})\tan(x)$$

$\frac{1}{2}x^{-1/2}\tan(x) + x^{1/2}\sec^2(x)$

(4) (10 points) Find the derivative of the function

$$f(x) = \ln(x^{5/3} - 3x)$$

$$\frac{1}{x^{5/3} - 3x} \cdot \left(\frac{5}{3}x^{2/3} - 3\right)$$

(5) (10 points) Find the second derivative of the function

$$f(x) = \tan^{-1}(3x)$$

$$f'(x) = \frac{1}{1+(3x)^2} = (1+9x^2)^{-1}$$

$$f''(x) = -(1+9x^2)^{-2} \cdot 18x$$

(6) (10 points) Use the limit definition of the derivative to find the derivative of the following function

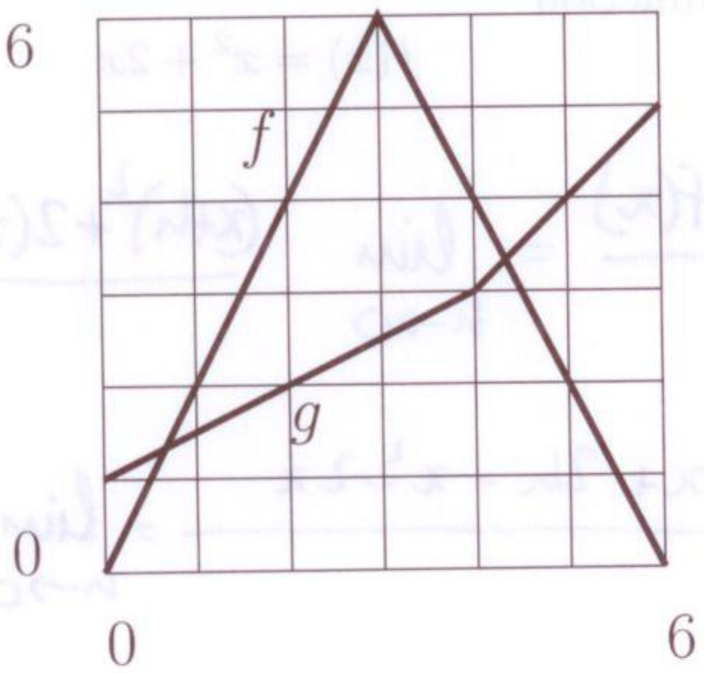
$$f(x) = x^2 + 2x$$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{(x+h)^2+2(x+h) - x^2-2x}{h}$$

$$= \lim_{h\to 0} \frac{x^2+2xh+h^2+2x+2h-x^2-2x}{h} = \lim_{h\to 0} \frac{2xh+h^2+2h}{h}$$

$$= \lim_{h\to 0} 2x+h+2 = 2x+2$$

(7) (10 points) Graphs of two functions $f(x)$ and $g(x)$ are shown below.



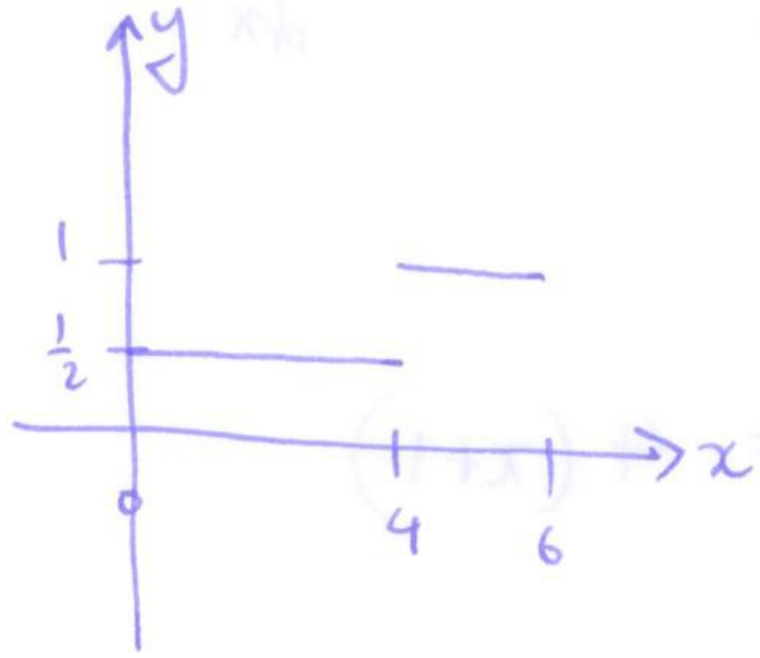
(a) Let $A(x) = f(x)g(x)$. Find $A'(2)$.
(b) Let $B(x) = f(g(x))$. Find $B'(5)$.

a) $A'(2) = f'(2)g(2) + f(2)g'(2) = 2 \cdot 2 + 4 \cdot \frac{1}{2} = 6$

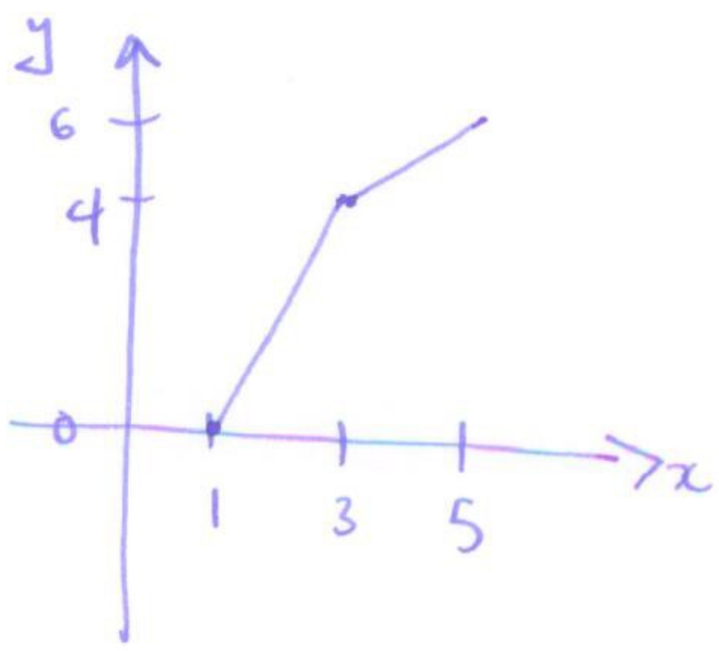
b) $B'(5) = f'(g(5)) \cdot g'(5) = f'(4) \cdot 1 = -2$

(8) (10 points) Use the function g from the previous question.

(a) Sketch $g'(x)$. (Label the scale on your axes.)



(b) Sketch $g^{-1}(x)$. (Label the scale on your axes.)



(9) (10 points) Find an equation for the tangent line to $4x^2 + y^2 = 5$ at the point $(-1, 1)$.

$8x + 2y\frac{dy}{dx} = 0$ $\qquad \frac{dy}{dx} = -\frac{4x}{y}$

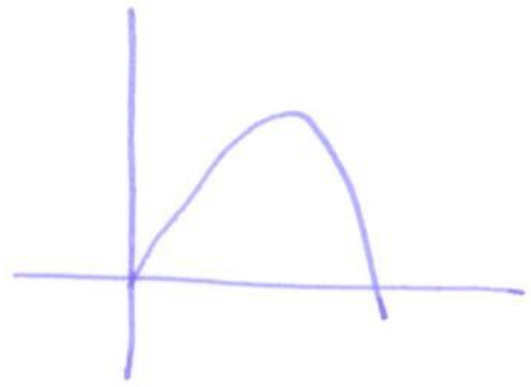
at $(-1,1)$ $\frac{dy}{dx} = 4$

tangent line : $y - 1 = 4(x+1)$

(10) (10 points) How fast do you need to throw a ball from the ground to get it to the top of a 20m building? (Feel free to use g = 10m/s.)

$h(t) = -\frac{1}{2}gt^2 + v_0 t + h_0$

$h'(t) = -gt + v_0$



max height when $h'(t)=0$: $-gt+v_0 = 0$

$t = \frac{v_0}{g}$

height at $t = \frac{v_0}{g}$: $-\frac{1}{2}g\left(\frac{v_0}{g}\right)^2 + v_0\frac{v_0}{g} = \frac{1}{2}\frac{v_0^2}{g}$

set max height = 20

$\frac{1}{2}\frac{v_0^2}{g} = 20$

$v_0^2 = 40g$

$v_0 = \sqrt{40g} = 20\,m/s$