

Math 230 Calculus 1/Precalc Fall 11 Midterm 2a

Name: Solutions

- Do any 8 of the following 10 questions.
- You may use a calculator, but no notes.

1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
	80	

Midterm 1	
Overall	

(1) (10 points) Find the derivative of the function

$$f(x) = e^{-3x}$$

$$-3e^{-3x}$$

1	1
2	2
3	3
4	4
5	5
6	6
7	7
8	8
9	9
10	10
11	11
12	12
13	13
14	14
15	15
16	16
17	17
18	18
19	19
20	20

	Midterm 1
	Overall

(2) (10 points) Find the derivative of the function

$$f(t) = \frac{\sin(t)}{1+5t^2}$$

$$\frac{(1+5t^2)\cos(t) - \sin(t)(10t)}{(1+5t^2)^2}$$

4

(3) (10 points) Find the derivative of the function

$$f(x) = (\sqrt{x}) \tan(2x)$$

$$\frac{1}{2} x^{-1/2} \tan(2x) + x^{1/2} \sec^2(2x) \cdot 2(2x) (1+x^2)$$

$$x^{1/2} (1+x^2)$$

(4) (10 points) Find the derivative of the function

$$f(x) = \ln(x^{4/3} - 4x)$$

$$\frac{1}{x^{4/3} - 4x} \cdot \left(\frac{4}{3}x^{1/3} - 4 \right) = \frac{1}{(x^{4/3} - 4x)} \cdot \left(\frac{4}{3}x^{1/3} - 4 \right)$$

$$= \frac{4x^{1/3} - 12}{3(x^{4/3} - 4x)}$$

6

(5) (10 points) Find the second derivative of the function

$$f(x) = \tan^{-1}(4x)$$

$$f'(x) = \frac{1}{1 + (4x)^2} = (1 + 16x^2)^{-1} \cdot 4$$

$$f''(x) = - (1 + 16x^2)^{-2} \cdot 32x$$

- (6) (10 points) Use the limit definition of the derivative to find the derivative of the following function

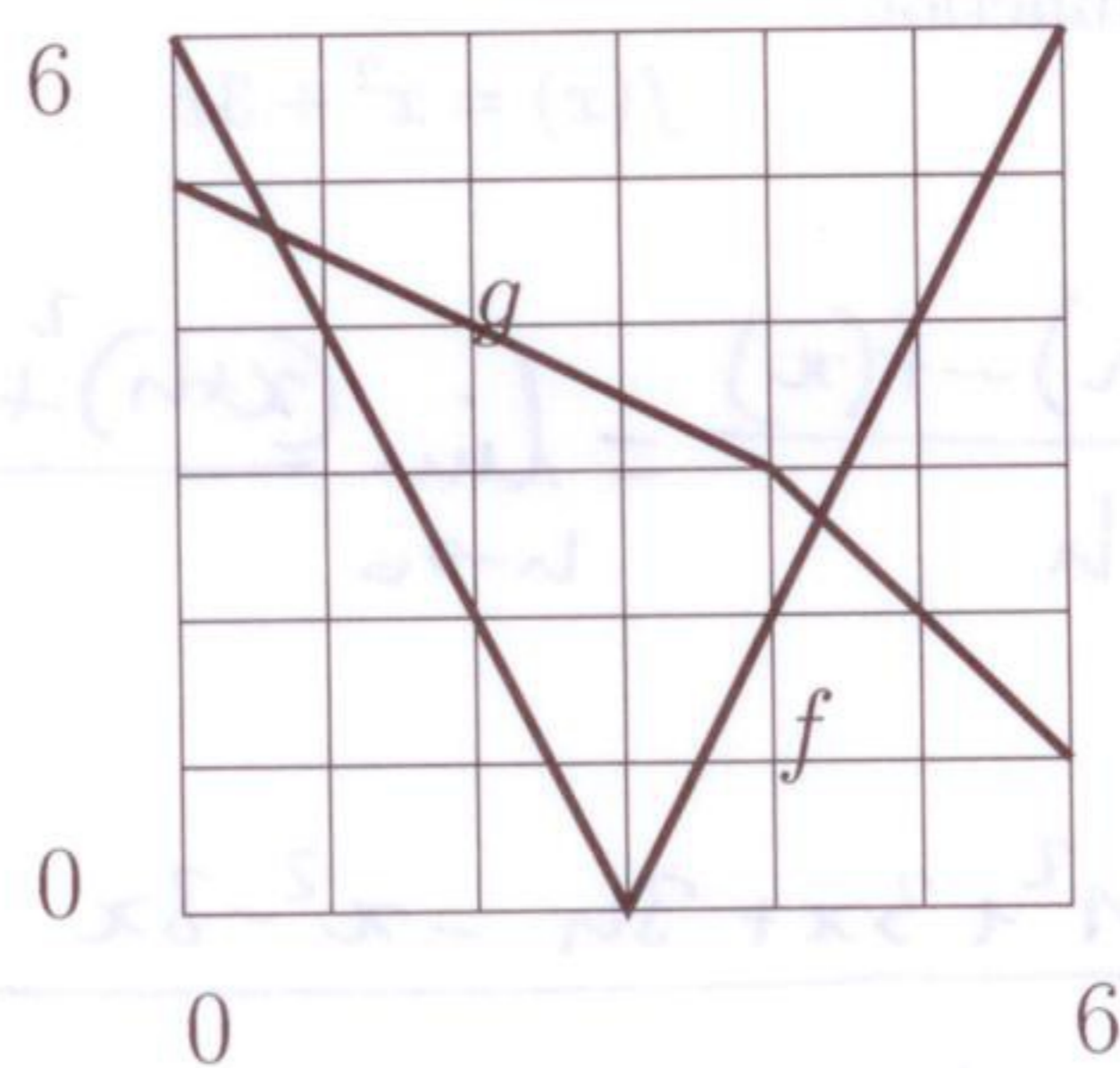
$$f(x) = x^2 + 3x$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{(x+h)^2 + 3(x+h) - x^2 - 3x}{h}$$

$$= \lim_{h \rightarrow 0} \frac{x^2 + 2xh + h^2 + 3x + 3h - x^2 - 3x}{h} = \lim_{h \rightarrow 0} \frac{2xh + h^2 + 3h}{h}$$

$$= \lim_{h \rightarrow 0} 2x + h + 3 = 2x + 3$$

(7) (10 points) Graphs of two functions $f(x)$ and $g(x)$ are shown below.



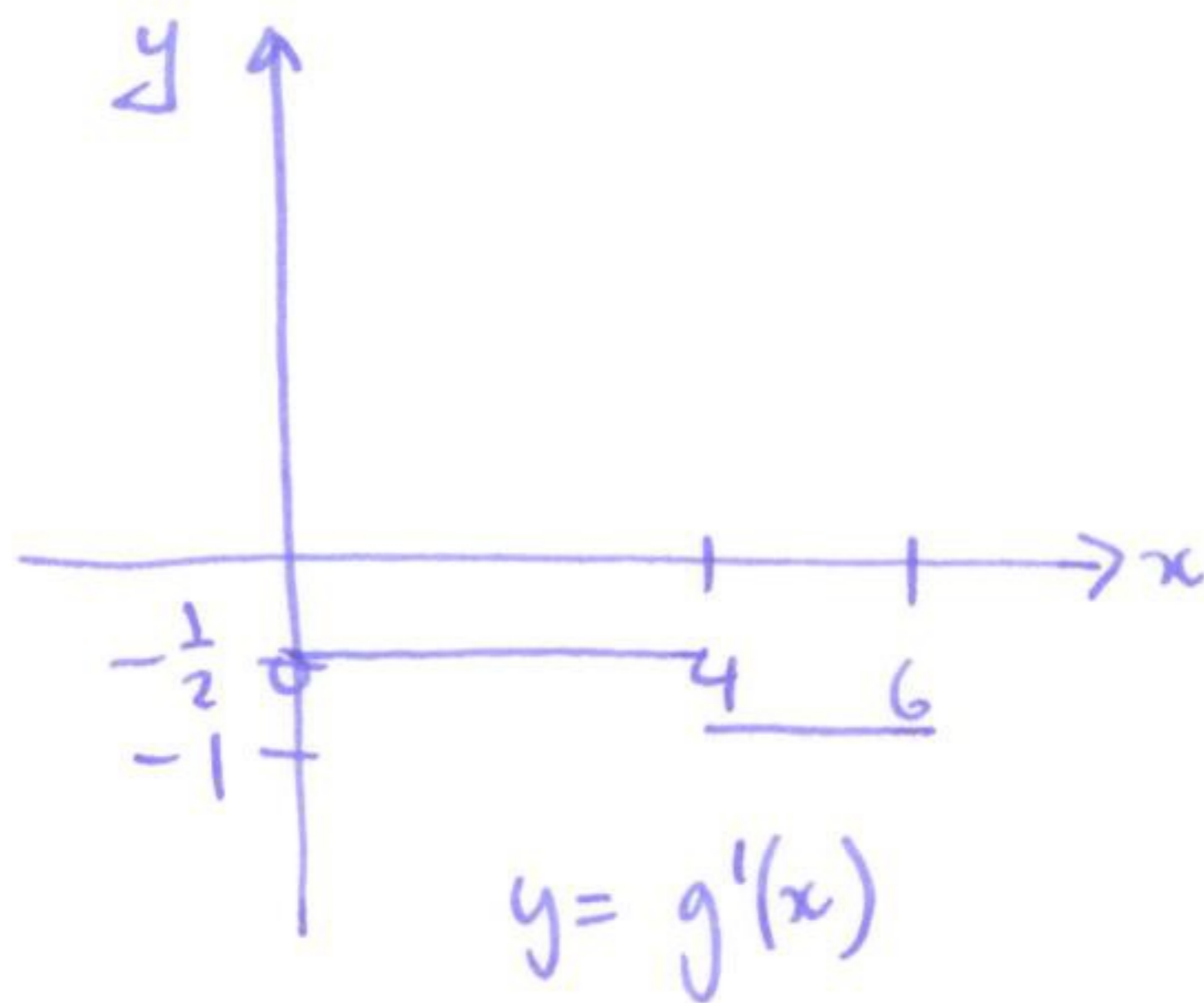
- (a) Let $A(x) = f(x)g(x)$. Find $A'(5)$.
 (b) Let $B(x) = f(g(x))$. Find $B'(2)$.

$$a) A'(5) = f'(5)g(5) + f(5)g'(5) = 2 \cdot 2 + 4 \cdot (-1) = 0$$

$$b) B'(2) = f'(g(2)) \cdot g'(2) = f'(4)$$

$$B'(2) = f'(g(2)) \cdot g'(2) = f'(4) \cdot (-\frac{1}{2}) = 2 \cdot (-\frac{1}{2}) = -1$$

- (8) (10 points) Use the function g from the previous question.
 (a) Sketch $g'(x)$. (Label your axes.)



- (b) Sketch $g^{-1}(x)$. (Label your axes.)



- (9) (10 points) Find an equation for the tangent line to $x^2 + 4y^2 = 5$ at the point $(1, -1)$.

$$2x + 8y \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = -\frac{x}{4y}$$

at $(1, -1)$ $\frac{dy}{dx} = +\frac{1}{4}$

tangent line:

$$y + 1 = \frac{1}{4}(x - 1)$$

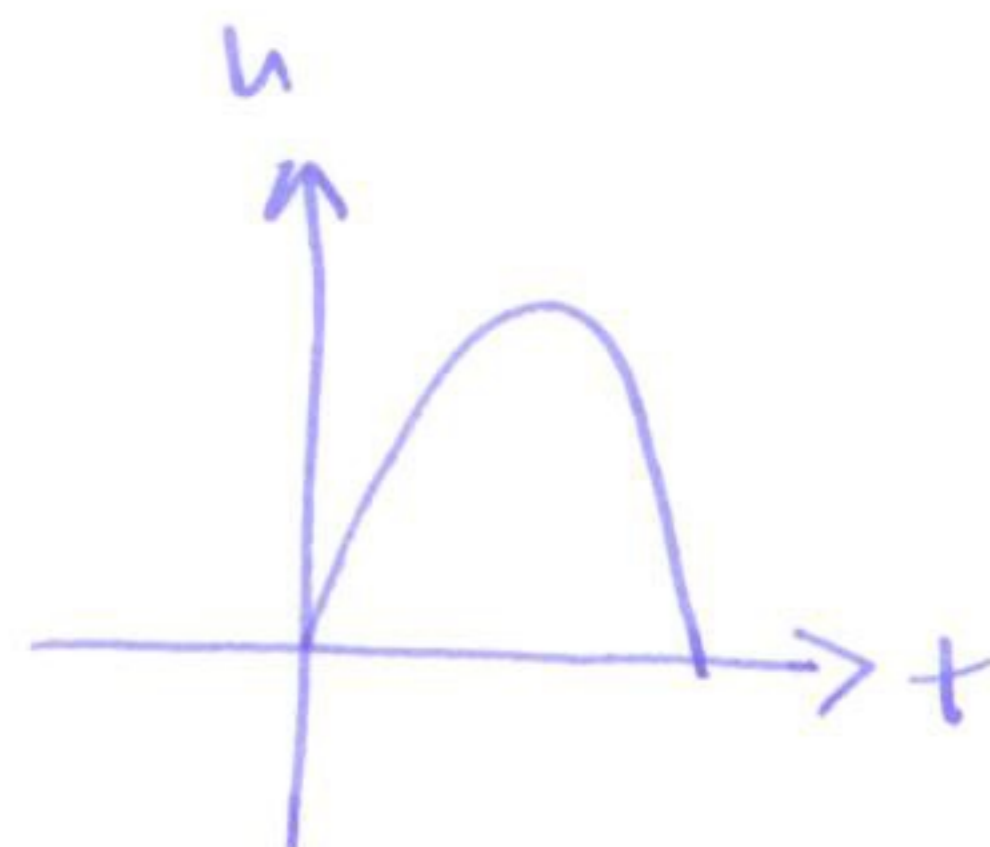


(10) (10 points) How fast do you need to throw a ball from the ground to get it to the top of a 10m building?

$$h(t) = -\frac{1}{2}gt^2 + v_0t + h_0$$

$$h_0 = 0$$

$$h'(t) = -gt + v_0$$



max height when $h'(t) = 0 \Rightarrow v_0 - gt = 0 \Rightarrow t = \frac{v_0}{g}$

height at $t = \frac{v_0}{g}$: $h\left(\frac{v_0}{g}\right) = -\frac{1}{2}g\left(\frac{v_0}{g}\right)^2 + v_0\left(\frac{v_0}{g}\right) = \frac{1}{2}\frac{v_0^2}{g}$

set height = 10m:

$$\frac{1}{2}\frac{v_0^2}{g} = 10$$

$$v_0^2 = 20g$$

$$v_0 = \sqrt{20g}$$