Sample midterm 2 Solutions

Q1 a) $y = xe^x \sin(x)$

$$\frac{dy}{dx} = e^x \sin(x) + x\left(e^x \sin(x) + e^x \cos(x)\right)$$

b) $y = \dfrac{e^{-2x}}{x + \tan(2x)}$ $\qquad \dfrac{dy}{dx} = \dfrac{(x+\tan(2x)).e^{-2x}.-2 - e^{-2x}.\left(1+\sec^2(2x).2\right)}{(x+\tan(2x))^2}$

c) $y = \left((2x)^{1/4} + (x^2+2)^{1/2}\right)^{11}$

$$\frac{dy}{dx} = 11\left((2x)^{1/4} + (x^2+2)^{1/2}\right)^{10} . \left(\frac{1}{4}(2x)^{-3/4}.2 + \frac{1}{2}(x^2+2)^{-1/2}.2x\right)$$

d) $y = \ln(x + \cos(3-x))$

$$\frac{dy}{dx} = \frac{1}{x+\cos(3-x)} . \left(1 - \sin(3-x).-1\right) = \frac{1+\sin(3-x)}{x+\cos(3-x)}$$

e) $xe^{x+y} = y - 1$

$$x.e^{x+y}\left(1 + \frac{dy}{dx}\right) + e^{x+y} = \frac{dy}{dx}$$

$$xe^{x+y} + e^{x+y} = \frac{dy}{dx} - \frac{dy}{dx}xe^{x+y}$$

$$e^{x+y}(x+1) = \frac{dy}{dx}\left(1 - xe^{x+y}\right)$$

$$\frac{dy}{dx} = \frac{(x+1)e^{x+y}}{(1 - xe^{x+y})}$$

Q2 $f(x) = \dfrac{1}{3-2x}$

a) $f'(x) = \lim\limits_{h\to 0} \dfrac{f(x+h)-f(x)}{h} = \lim\limits_{h\to 0} \dfrac{\dfrac{1}{3-2(x+h)} - \dfrac{1}{3-2x}}{h}$

$= \lim\limits_{h\to 0} \dfrac{1}{h} \dfrac{3-2x \quad -3+2x+2h}{(3-2x-2h)(3-2x)} = \lim\limits_{h\to 0} \dfrac{2}{(3-2x-2h)(3-2x)} = \dfrac{2}{(3-2x)^2}$

$= 2(3-2x)^{-2}$

b) $f''(x) = -4(3-2x)^{-3} \cdot -2 = \dfrac{8}{(3-2x)^2}$

Q3 a) $A'(x) = f'(x)g(x) + f(x)g'(x)$

$A'(1) = \underbrace{f'(4)}_{-1}\underbrace{g(1)}_{2} + \underbrace{f(1)}_{2}\underbrace{g'(1)}_{2} = -2+4 = 2$

b) $B'(x) = f'(g(x)).g'(x)$

$B'(1) = f'(\underbrace{g(1)}_{2}).\underbrace{g'(1)}_{2} = 2\underbrace{f'(2)}_{-1} = -2$

c) $C(4) = 2$

$C'(x) = \dfrac{1}{g'(C(x))}$ $\qquad C'(4) = \dfrac{1}{g'(2)} = \dfrac{1}{2}$

Q4 $x^2y^3 + 2y = 3x$

$2xy^3 + x^2 3y^2 \dfrac{dy}{dx} + 2\dfrac{dy}{dx} = 3$

$x=2, y=1$: $\quad 4 + 12\dfrac{dy}{dx} + 2\dfrac{dy}{dx} = 3 \qquad 14\dfrac{dy}{dx} = -1 \quad \dfrac{dy}{dx} = \dfrac{-1}{14}.$

tangent line : $y-1 = -\frac{1}{14}(x-2)$

Q5 $h(t) = -\frac{1}{2}\cdot 10t^2 + 20t = -5t^2+20t$

a) $h'(t) = -10t+20$

solve $h'(t)=0$: $-10t+20=0$ $t = 2$ secs.

height at $t=2$: $h(t) = -5.4+20.2 = 20$m.

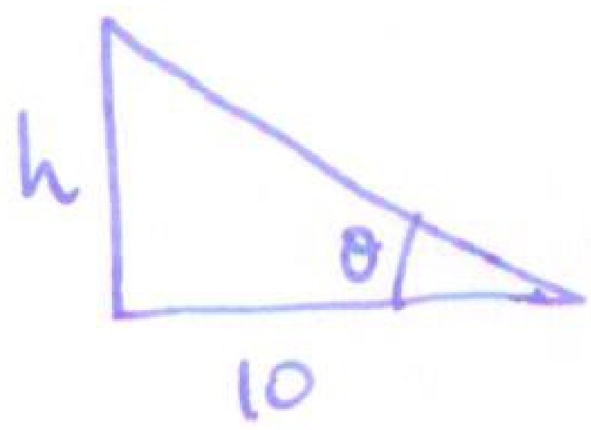
b) solve $h(t)=5$: $-5t^2+20t=5$

$t^2-4t+1=0$ $t = \frac{4 \pm \sqrt{16-4}}{2}$

first t is $t = 2-\sqrt{3}$ $t = 2 \pm \sqrt{3}$

$h'(2-\sqrt{3}) = -10(2-\sqrt{3})+20 = 10\sqrt{3}$ m/s.

Q6



$\frac{h}{10} = \tan\theta$

$\frac{1}{10}\frac{dh}{dt} = \sec^2\theta \frac{d\theta}{dt}$

$t = 3$ mins $\Rightarrow h = \frac{800\times 3}{60} = 40$ so $\tan\theta = 4$

$\sin^2\theta + \cos^2\theta = 1$

$\tan^2\theta + 1 = \sec^2\theta$ $\Rightarrow$ $\sec^2\theta = 5$.

so $\frac{1}{10}\cdot 800 = 5\cdot\frac{d\theta}{dt}$ $\frac{d\theta}{dt} = 16$ rad/hour.