Math 230 Calculus 1/Precalc Fall 11 Midterm 1b

Name: Solutions

- Do any 8 of the following 10 questions.
- You may use a calculator, but no notes.

1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
	90	

Midterm 1	
Overall	

(1) (10 points) Find $\cos 2\theta$ if $\sin\theta = 2/7$.

$\cos 2\theta = \cos^2\theta - \sin^2\theta$

$= 1 - 2\sin^2\theta = 1 - 2.\left(\frac{2}{7}\right)^2 = 1 - \frac{8}{49} = \frac{41}{49}$

(2) (10 points) Plot the points $(-4, -1)$ and $(3, -4)$ on the grid below, and draw the straight line through the two points. Find the equation of the straight line.

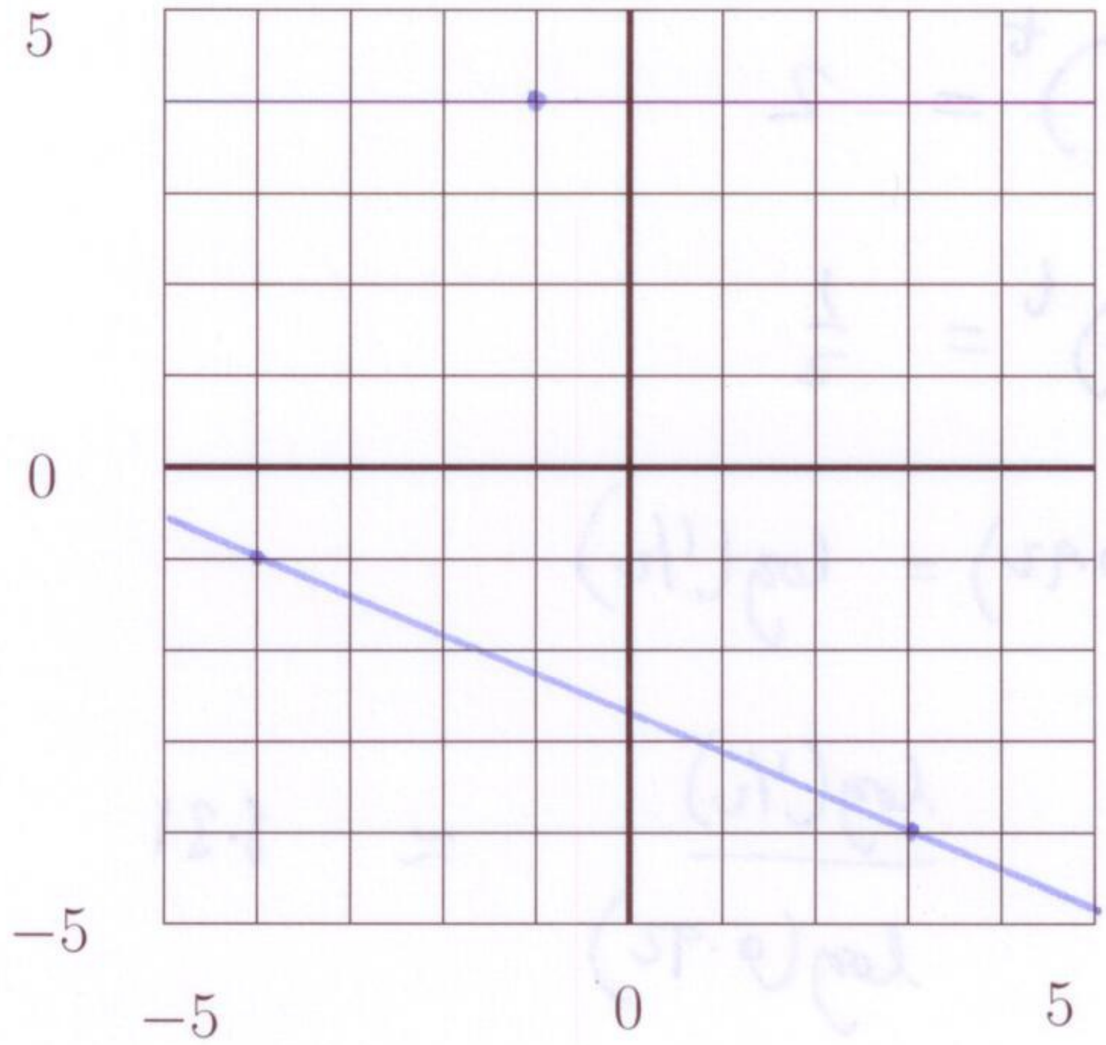


FIGURE 1

slope $= \dfrac{-1-(-4)}{-4-3} = \dfrac{3}{-7}$

$y - y_0 = m(x - x_0)$

$y + 1 = -\dfrac{3}{7}(x+4)$

(3) (10 points) Four kilograms of radioactive material decays as $M(t) = 4(0.92)^t$, where t is in hours. How long does it take to halve in mass?

$$4(0.92)^t = 2$$

$$(0.92)^t = \frac{1}{2}$$

$$t \log(0.92) = \log(1/2)$$

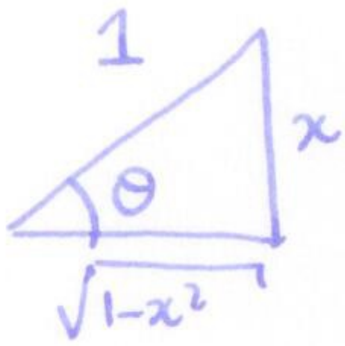
$$t = \frac{\log(1/2)}{\log(0.92)} \approx 8.31$$

(4) (10 points) Find $\tan(\sin^{-1}(x))$.

$\tan\theta$

$\theta = \sin^{-1}(x)$

$\sin\theta = x$



$$\tan\theta = \frac{x}{\sqrt{1-x^2}}$$

(5) (10 points) The graph of $y = f(x)$ is shown below. Evaluate each limit, or write DNE if the limit does not exist. No justifications are necessary.

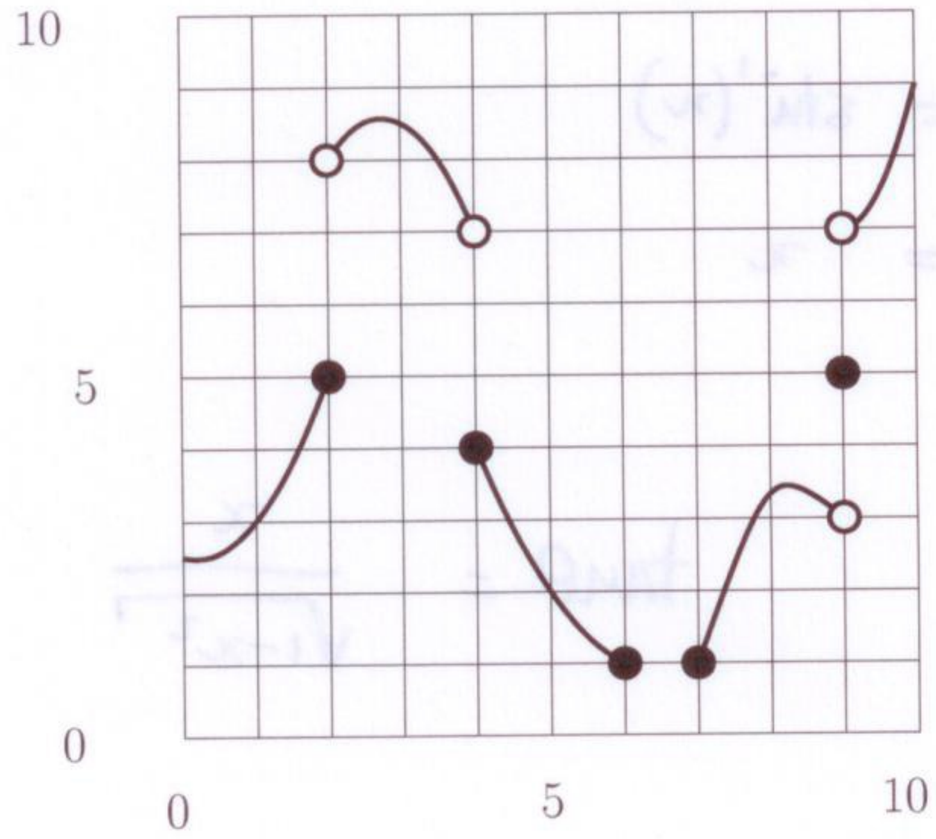


FIGURE 2. $f(x)$

(a) $\lim_{x\to 2-} f(x)$ 5

(b) $\lim_{x\to 4} f(x)$ DNE

(c) $\lim_{x\to 6-} f(x)$ 1

(d) $\lim_{x\to 6+} f(x)$ DNE

(e) $\lim_{x\to 9} f(x)$ DNE

(6) (10 points) The area of a disc is given by $A = \pi r^2$. What is the average rate of change of area when the radius increases from $r = 4$ to $r = 6$?

$$\frac{\pi 6^2 - \pi 4^2}{6-4} = 10\pi$$

(7) (10 points) Evaluate the limit algebraically. For an infinite limit, write $+\infty$ or $-\infty$. If a limit does not exist (DNE), you must justify why this is the case.

$$\lim_{x \to -3} \frac{x^2 + x - 6}{x + 3}$$

$$\lim_{x \to -3} \frac{(x+3)(x-2)}{x+3} = \lim_{x \to -3} x - 2 = -5$$

(8) (10 points) Evaluate the limit algebraically. For an infinite limit, write $+\infty$ or $-\infty$. If a limit does not exist (DNE), you must justify why this is the case.

$$\lim_{x \to 0} \frac{\sin 5x}{2x}$$

$5x = \theta$

$$\lim_{\theta \to 0} \frac{\sin\theta}{2\frac{\theta}{5}} = \frac{5}{2} \lim_{\theta \to 0} \frac{\sin\theta}{\theta} = \frac{5}{2}$$

(9) (10 points) Evaluate the limit algebraically. For an infinite limit, write $+\infty$ or $-\infty$. If a limit does not exist (DNE), you must justify why this is the case.

$$\lim_{x\to 16} \frac{1}{\sqrt{x}-4} - \frac{8}{x-16}$$

$$\lim_{x\to 16} \frac{1}{\sqrt{x}-4} - \frac{8}{(\sqrt{x}-4)(\sqrt{x}+4)} = \lim_{x\to 16} \frac{\sqrt{x}+4-8}{(\sqrt{x}-4)(\sqrt{x}+4)}$$

$$= \lim_{x\to 16} \frac{\sqrt{x}-4}{(\sqrt{x}-4)(\sqrt{x}+4)} = \lim_{x\to 16} \frac{1}{\sqrt{x}+4} = \frac{1}{8}$$

(10) (10 points) For what value of c (if any) is the function $f(x)$ continuous at $x = 1$? Justify your answer.

$$f(x) = \begin{cases} \frac{x+3}{x-2} & x < 1 \\ c & x = 1 \\ 4\cos(\pi x) & x > 1 \end{cases}$$

$\lim_{x \to 1} \frac{x+3}{x-2} = \frac{4}{-1} = -4$

$\lim_{x \to 1} 4\cos(\pi x) = -4$

choose $c = -4$ then $f(x)$ is continuous.