Math 230 Calculus 1/Precalc Fall 11 Midterm 1a

Name: Solutions

- Do any 8 of the following 10 questions.
- You may use a calculator, but no notes.

1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
	90	

Midterm 1	
Overall	

(1) (10 points) Plot the points $(-3, 4)$ and $(-1, -3)$ on the grid below, and draw the straight line through the two points. Find the equation of the straight line.

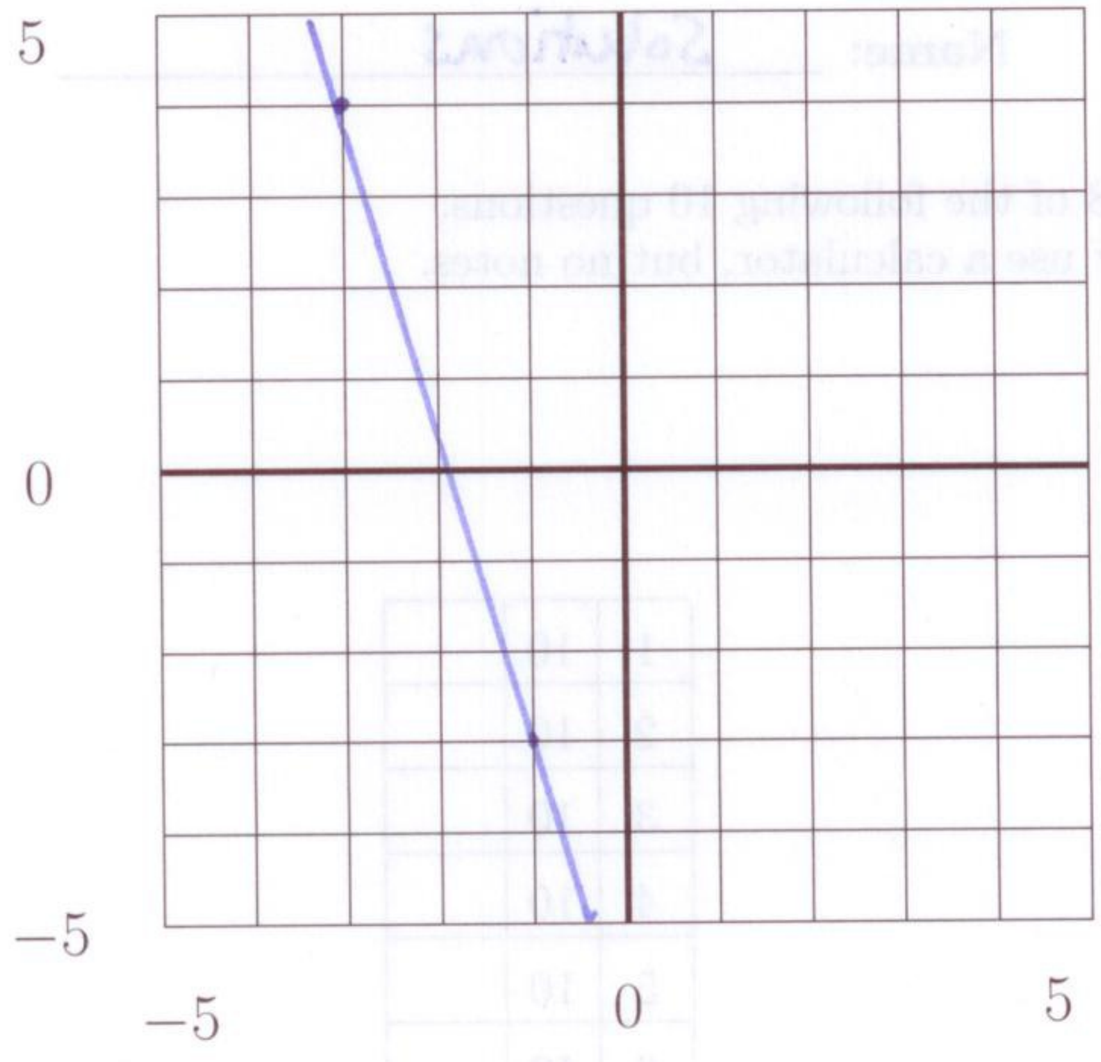


FIGURE 1

slope $= \frac{4-(-3)}{-3-(-1)} = \frac{7}{-2}$

$y - y_0 = m(x - x_0)$

$y - 4 = -\frac{7}{2}(x+3)$

(2) (10 points) Find $\cos 2\theta$ if $\sin\theta = 3/5$.

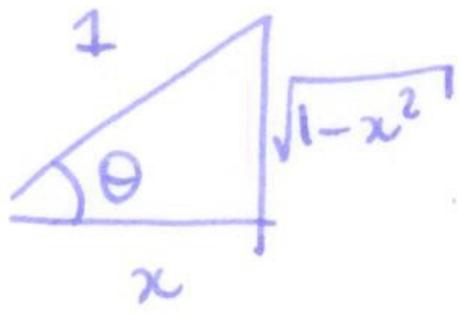
$\cos 2\theta = \cos^2\theta - \sin^2\theta$

$= 1 - 2\sin^2\theta = 1 - 2\cdot\frac{9}{25} = \frac{7}{25}$

(3) (10 points) Find $\tan(\cos^{-1}(x))$.

$\tan\theta$ $\quad \theta = \cos^{-1}(x)$

$\cos\theta = x$



$\tan\theta = \dfrac{\sqrt{1-x^2}}{x}$

(4) (10 points) Four kilograms of radioactive material decays as $M(t) = 4(0.87)^t$, where t is in hours. How long does it take to halve in mass?

$4(0.87)^t = 2$

$(0.87)^t = \frac{1}{2}$

$t \log(0.87) = \log(1/2)$

$t = \frac{\log(1/2)}{\log(0.87)} \approx 4.98$ hours

(5) (10 points) The area of a disc is given by $A = \pi r^2$. What is the average rate of change of area when the radius increases from $r = 3$ to $r = 5$?

$$\frac{\pi \cdot 5^2 - \pi \cdot 3^2}{5-3} = \frac{\pi}{2}(25-9) = 7\pi$$

(6) (10 points) The graph of $y = f(x)$ is shown below. Evaluate each limit, or write DNE if the limit does not exist. No justifications are necessary.

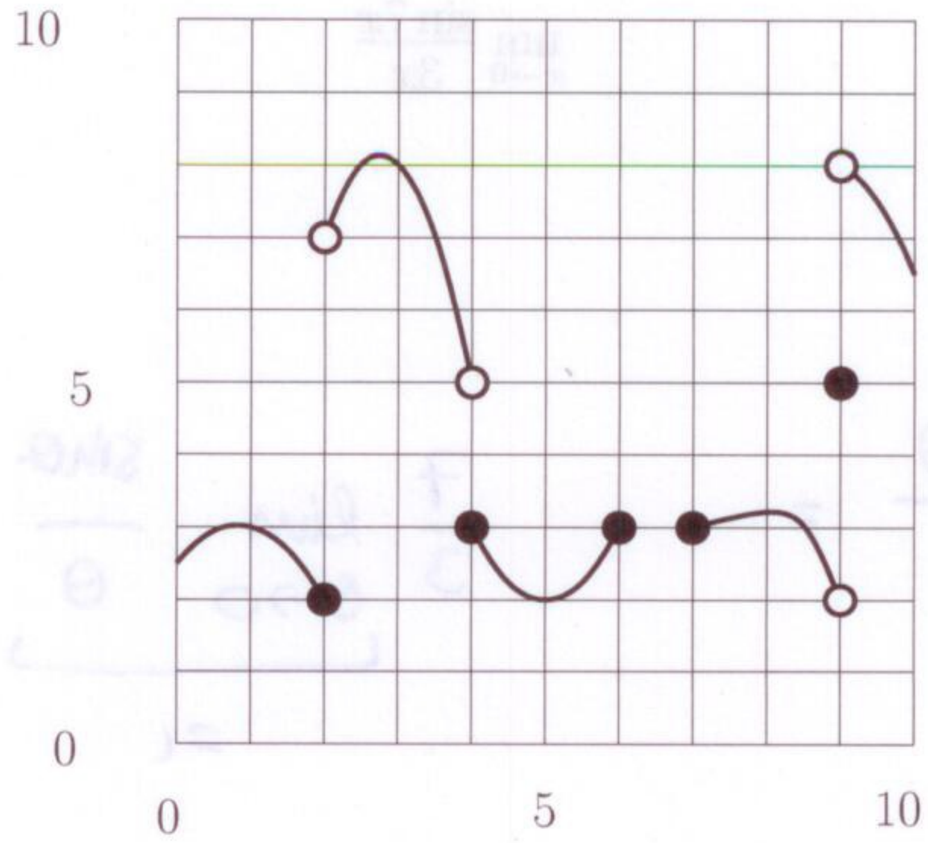


FIGURE 2. $f(x)$

(a) $\lim_{x\to 2-} f(x)$ 2

(b) $\lim_{x\to 4} f(x)$ DNE

(c) $\lim_{x\to 6-} f(x)$ 3

(d) $\lim_{x\to 6+} f(x)$ DNE

(e) $\lim_{x\to 9} f(x)$ DNE

(7) (10 points) Evaluate the limit. For an infinite limit, write $+\infty$ or $-\infty$. If a limit does not exist (DNE), you must justify why this is the case.

$$\lim_{x\to 0} \frac{\sin 7x}{3x}$$

$\theta = 7x$

$$\lim_{\theta\to 0} \frac{\sin\theta}{3\frac{\theta}{7}} = \frac{7}{3} \underbrace{\lim_{\theta\to 0} \frac{\sin\theta}{\theta}}_{=1} = \frac{7}{3}$$

(8) (10 points) Evaluate the limit. For an infinite limit, write $+\infty$ or $-\infty$. If a limit does not exist (DNE), you must justify why this is the case.

$$\lim_{x \to -2} \frac{x^2 - x - 6}{x + 2}$$

$$\lim_{x \to -2} \frac{(x+2)(x-3)}{x+2} = \lim_{x \to -2} x - 3 = -5$$

(9) (10 points) Evaluate the limit. For an infinite limit, write $+\infty$ or $-\infty$. If a limit does not exist (DNE), you must justify why this is the case.

$$\lim_{x\to 25} \frac{1}{\sqrt{x}-5} - \frac{10}{x-25}$$

$$\lim_{x\to 25} \frac{1}{\sqrt{x}-5} - \frac{10}{(\sqrt{x}-5)(\sqrt{x}+5)} = \lim_{x\to 25} \frac{\sqrt{x}+5-10}{(\sqrt{x}-5)(\sqrt{x}+5)}$$

$$= \lim_{x\to 25} \frac{\sqrt{x}-5}{(\sqrt{x}-5)(\sqrt{x}+5)} = \lim_{x\to 25} \frac{1}{\sqrt{x}+5} = \frac{1}{10}$$

(10) (10 points) For what value of c (if any) is the function $f(x)$ continuous at $x = 1$? Justify your answer.

$$f(x) = \begin{cases} \frac{x-3}{x+3} & x < 1 \\ c & x = 1 \\ \frac{1}{2}\cos(\pi x) & x > 1 \end{cases}$$

$\lim_{x \to 1} \frac{x-3}{x+3} = \frac{1-3}{1+3} = \frac{-2}{4} = -\frac{1}{2}$

$\lim_{x \to 1} \frac{1}{2}\cos(\pi x) = -\frac{1}{2}$

choose $c = -\frac{1}{2}$.