Solutions

Math 230 Calculus 1/Precalc Fall 11 Sample midterm 1

(1) Plot the points $(-4, 3)$ and $(3, -2)$ on the grid below, adn draw the straight line through the two points. Find the equation of the straight line.

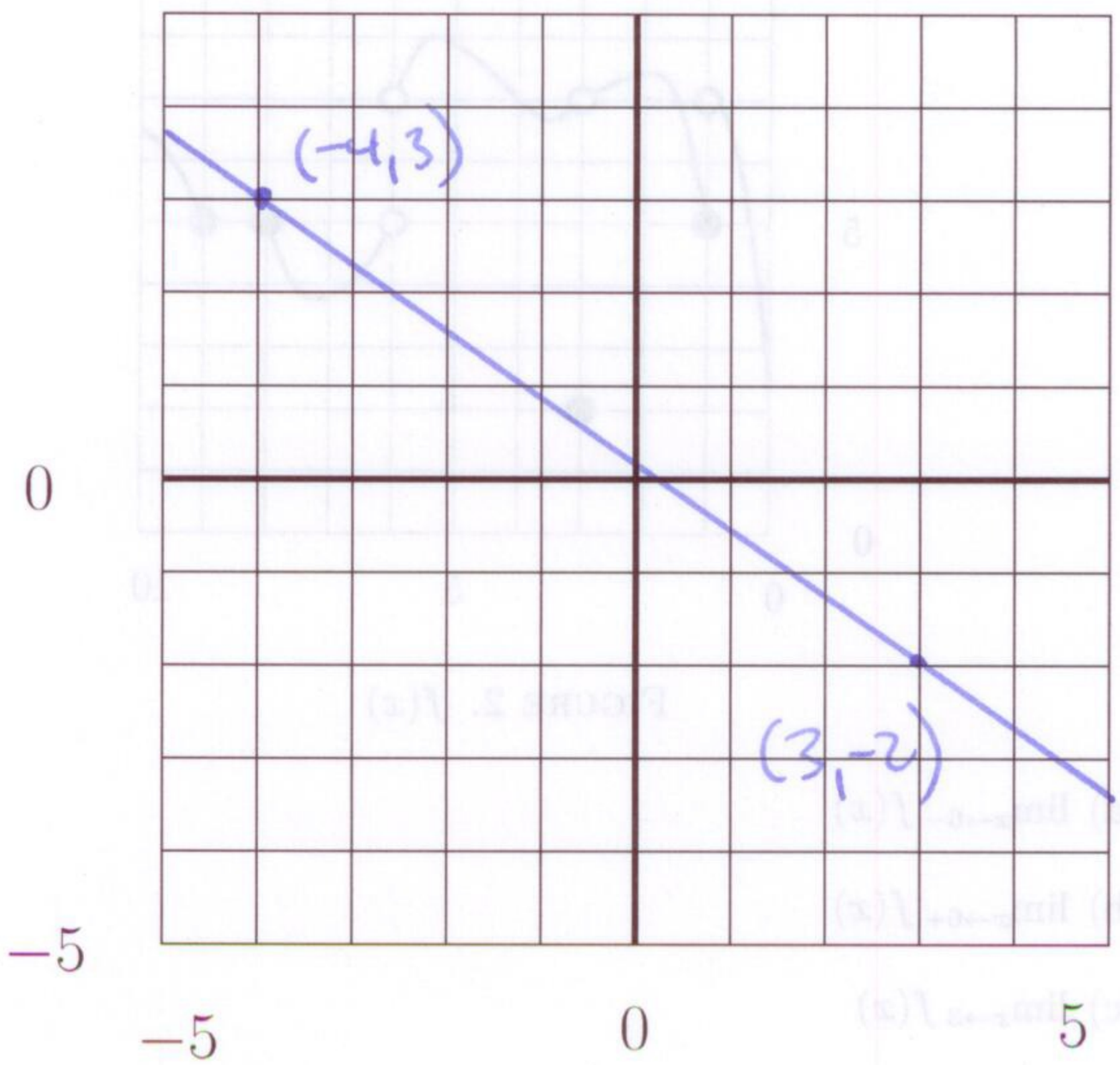


FIGURE 1

(2) What interval corresponds to $|3x - 4| \leq 12$?

(3) Find $\cos^{-1}(\cos(37\pi/3))$.

(4) Find $\sin 2\theta$ if $\sin\theta = 2/5$.

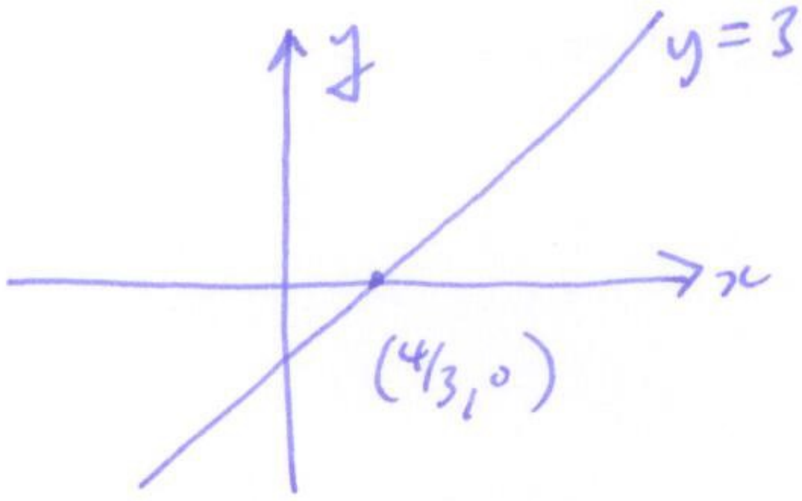
(5) Find $\sin(\cos^{-1}(x))$.

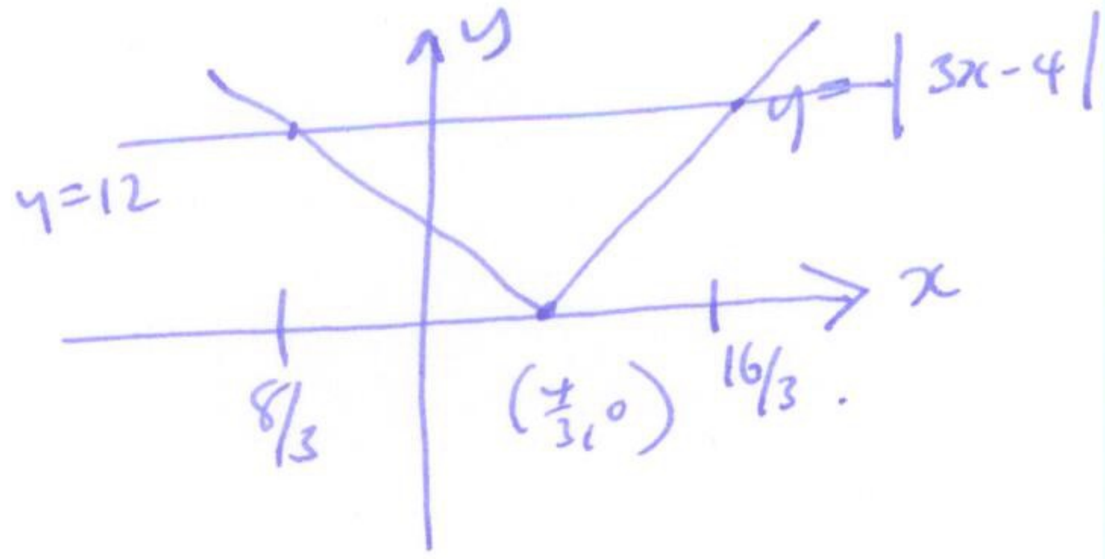
(6) A population of bacteria grows according to the equation $B(t) = 12(1.17)^t$, where t is in hours. How long does it take to double?

Q1 slope $m = \frac{3-(-2)}{-4-3} = -\frac{5}{7}$

$y - y_0 = m(x - x_0)$ $\quad y - 3 = -\frac{5}{7}(x+4)$

Q2





$3x - 4 = 12$ $\quad -3x + 4 = 12$

$3x = 16$ $\quad -3x = 8$

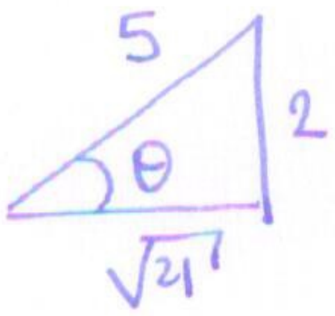
$x = 16/3.$ $\quad x = -8/3$

interval $\left[-\frac{8}{3}, \frac{16}{3}\right]$.

Q3 $\cos\left(\frac{37\pi}{3}\right) = \cos\left(12\pi + \frac{\pi}{3}\right) = \cos\left(\frac{\pi}{3}\right)$.

$\cos^{-1}\left(\cos\left(\frac{\pi}{3}\right)\right) = \frac{\pi}{3}$.

Q4

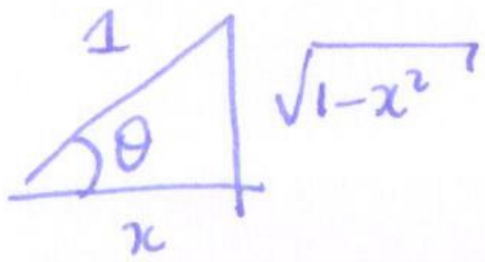


$\sin 2\theta = 2\sin\theta\cos\theta$ $\quad \cos\theta = \frac{\sqrt{21}}{5}$

$= 2 \cdot \frac{2}{5} \cdot \frac{\sqrt{21}}{5} = \frac{4\sqrt{21}}{5}$

Q5 $\sin\theta$ $\quad \theta = \cos^{-1}(x)$

$\cos\theta = x$



$\sin\theta = \sqrt{1-x^2}$

Q6

$B(0) = 12$

$B(t) = 12(1.17)^t$ solve $B(t) = 12(1.17)^t = 24$

$(1.17)^t = 2$

$t \log(1.17) = \log 2$ $\quad t = \dfrac{\log 2}{\log 1.17} \approx 4.41$ hours.

Q7 a) 7

b) 5

c) 7

d) 7

e) 5

f) DNE

Q8 a) $\lim_{x \to 5} \dfrac{\sqrt{x-1}-2}{x-5} \cdot \dfrac{\sqrt{x-1}+2}{\sqrt{x-1}+2} = \lim_{x \to 5} \dfrac{x-1-4}{(x-5)(\sqrt{x-1}+2)}$

$= \lim_{x \to 5} \dfrac{1}{\sqrt{x-1}+2} = \dfrac{1}{4}$

b) $\lim_{x \to -3+} \dfrac{x^2+5x+6}{|x+3} = \lim_{x \to -3+} \dfrac{(x+3)(x+2)}{(x+3)} = \lim_{x \to -3+} x+2 = -1$

$\lim_{x \to -3-} \dfrac{x^2+5x+6}{-x-3} = \lim_{x \to -3-} -(x+2) = +1$

$\lim_{x \to -3+} f(x) \neq \lim_{x \to -3-} f(x)$ so $\lim_{x \to -3} f(x)$ DNE.

c) $\lim_{x\to 0} \frac{\tan 3x}{5x}$ $\quad 3x=\theta$.

$= \lim_{x\to 0} \frac{\sin\theta}{\frac{5\theta}{3}\cos\theta} = \lim_{\theta\to 0} \frac{3}{5}\frac{1}{\cos\theta}\frac{\sin\theta}{\theta}$

$\lim_{\theta\to 0}\frac{1}{\cos\theta} = 1 \quad \lim_{\theta\to 0}\frac{\sin\theta}{\theta} = 1$ so $\lim_{x\to 0}\frac{\tan 3x}{5x} = \frac{3}{5}$.

d) $\lim_{x\to 0}\frac{x^2}{\sin(\frac{\pi}{x})}$ DNE ~~note~~ can choose sequences $x_n \to 0$, $y_n \to 0$ s.t. $f(x_n)\to +\infty$, $f(y_n)\to -\infty$

e.g. set $x_n = \frac{1}{2n+\theta_n}$ where $\sin(2n\theta_n) = 1/(2n+\pi/2)^2$ $(0\le\theta_n\le\frac{\pi}{2})$

$y_n = \frac{1}{2n+\phi_n}$ where $\sin(2n\phi_n) = -1/(2n+\pi/2)^2$ $(-\frac{\pi}{2}\le\phi_n\le 0)$

e) $\lim_{x\to 0+}\frac{1}{\sqrt{x+1}} - \frac{1}{\sqrt{x^2+x}} = \lim_{x\to 0+}\frac{\sqrt{x}-\sqrt{x+1}}{\sqrt{x}\sqrt{x+1}}$

$\lim_{x\to 0+}\frac{1-1/\sqrt{x}}{\sqrt{x+1}} = -\infty$. $\left(\lim_{x\to 0+}\frac{1}{\sqrt{x}} = \infty\right)$.

Q9 $\lim_{x\to 1^-}\frac{2-x}{x+2} = \frac{1}{3}$.

$\lim_{x\to 1+} x\cos(\pi x) = -1$

} no value of c makes f cts.

Q10

$$\frac{\frac{f(4)-f(2)}{4-2}}{2} = \frac{4\pi 4^2 - 4\pi 2^2}{2} = 2\pi(16-4)$$

$$= 24\pi .$$