Sample midterm solutions

Q1 a) $\dfrac{(t+5)^{1/2}(4t^3) - \frac{1}{2}(t+5)^{-1/2}(t^4-1)}{t+5}$ b) $5e^{5x}\sin^2 x + e^{5x}\cdot 2\sin x\cos x$

~~Q2~~ c) $g'(x) = \frac{1}{3x^2}\cdot 6x = \frac{2}{x}$ $g''(x) = -\frac{2}{x^2}$

Q2 a) $\frac{x^{-2}}{2} - 10x + \frac{1}{5}x^5 + C$ b) $u = \cos\theta$ $\frac{du}{d\theta} = -\sin\theta$

$\int 4u^2 \sin\theta \frac{1}{-\sin\theta}\,du = -4\int u^2\,du = -\frac{4}{3}u^3 + c$

$= -\frac{4}{3}\cos^3\theta + c$.

c) $\int_0^{\ln 5} 2e^{-t}\,dt$

$\left[-2e^{-t}\right]_0^{\ln 5} = -2e^{-\ln 5} + 2$

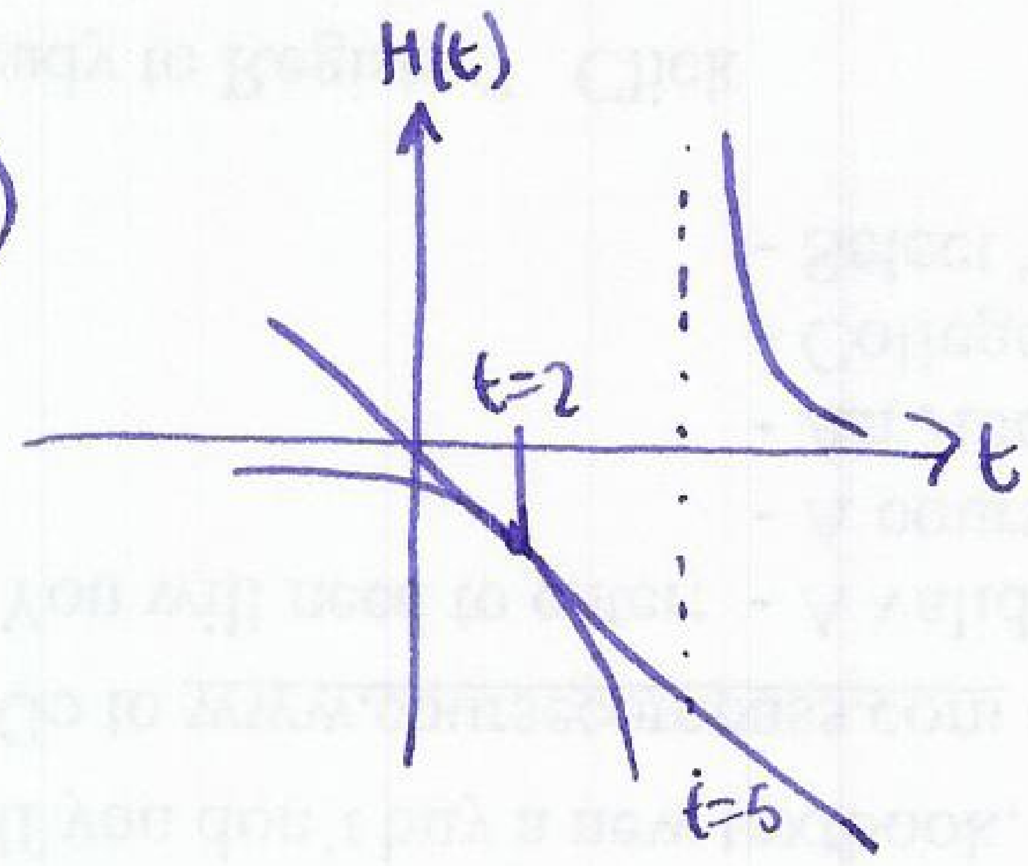
$= -\frac{2}{5} + 2 = \frac{8}{5}$

Q3 a) $\lim_{x\to 4} \frac{(x-4)(x+1)}{(x-4)} = \lim_{x\to 4} x+1 = 5$

b) no. $\lim_{x\to 4} f(x) = 5$ so function tends to 5 not $\pm\infty$.

c) $y = -\frac{1}{3}$.

Q4 a)



b) c) $\frac{dH}{dt} = -3(t-5)^{-2}$

$H'(2) = -\frac{3}{3^2} = -\frac{1}{3}$.

$12x^3 - 18x^2 = 6x^2(2x-3)$ so +ve on $(\frac{3}{2}, \infty)$

Q5 a) increasing $\Leftrightarrow p'(x) > 0$

decreasing $\Leftrightarrow p'(x) < 0$ $(-\infty, \frac{3}{2})$

	0		3/2	
−	|	−	|	+

critical points: $0, \frac{3}{2}$ first derivative test:

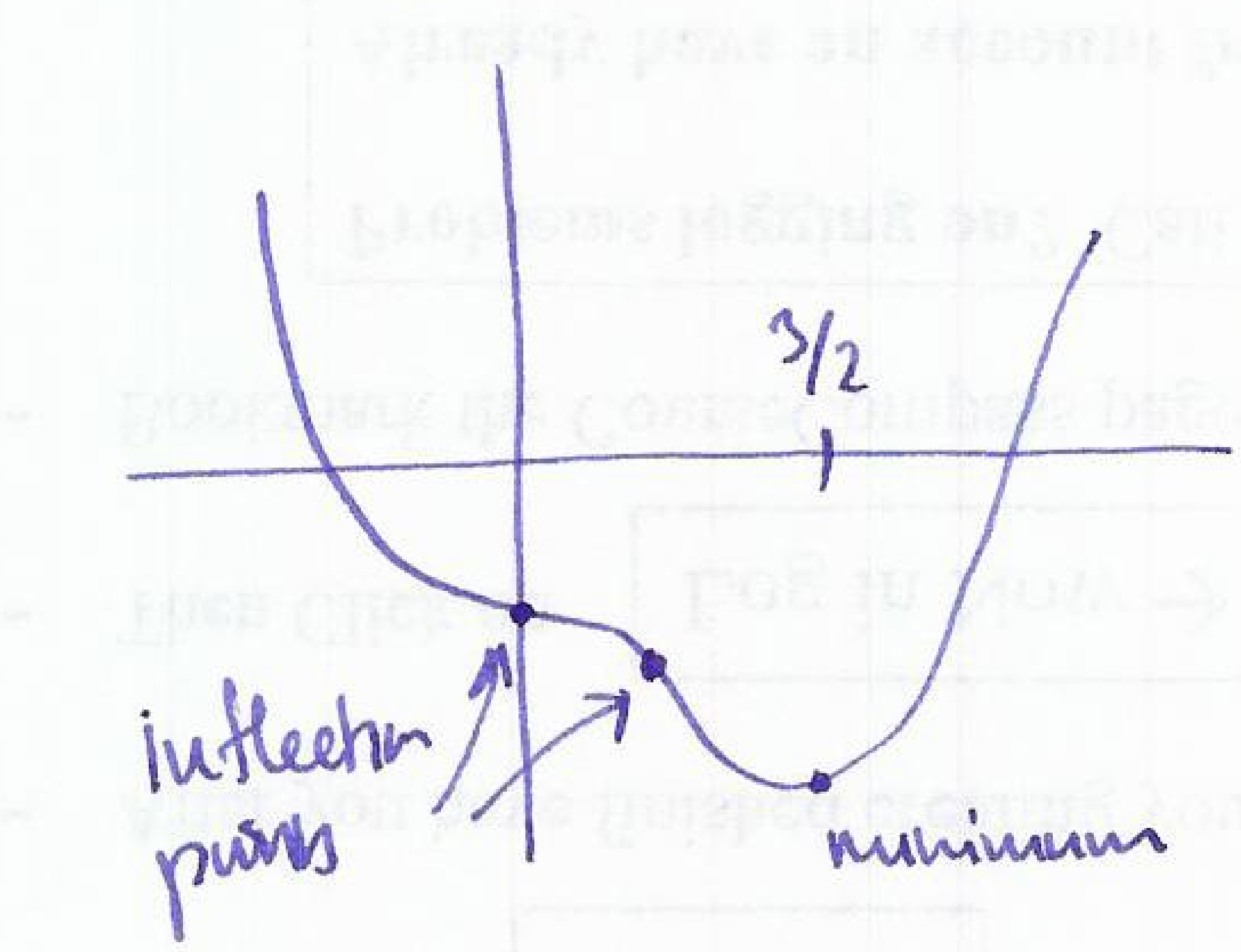
no maxima, minima at $3/2$.

b) concave up $\Leftrightarrow p''(x) > 0$ $\quad 36x^2 - 36x = 36x(x-1)$ +ve on $(-\infty,0) \cup (1,\infty)$

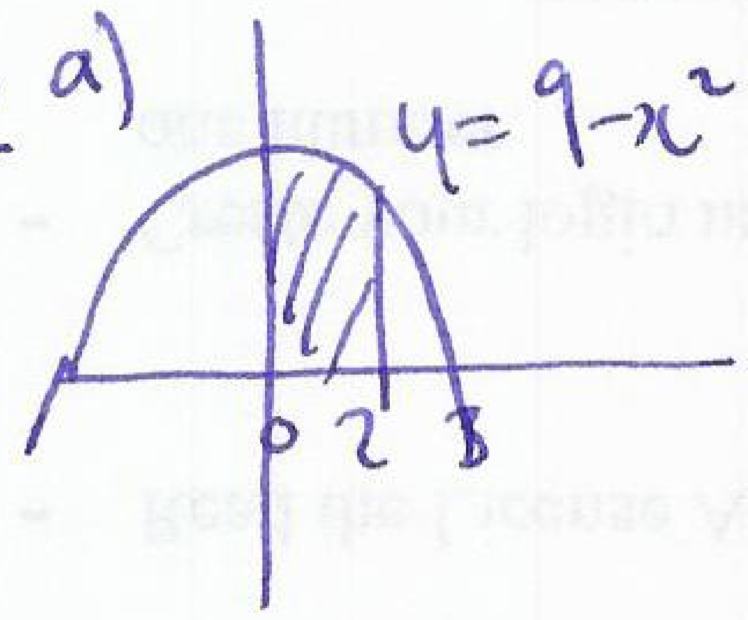
concave down $\Leftrightarrow p''(x) < 0$ $\quad$ -ve on $(0,1)$

inflection points : 0, 1

c)



Q6 a)



b) $\displaystyle\int_0^2 9 - x^2\,dx = \left[9x - \tfrac{1}{3}x^3\right]_0^2 = 18 - \frac{8}{3}$

Q7 $\displaystyle\lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{3(x+h)^2 + 5 - 3x^2 - 5}{h} = \lim_{h\to 0} \frac{\cancel{3x^2} + 6xh + h^2 + \cancel{5} - \cancel{3x^2} - \cancel{5}}{h}$

$\displaystyle = \lim_{h\to 0} 6x + h = 6x$

Q8 a) $\displaystyle\frac{1}{4}\left(f\left(1+\tfrac{1}{4}\right) + f\left(1+\tfrac{1}{2}\right) + f\left(1+\tfrac{3}{4}\right) + f(2)\right) = \frac{1}{4}\left(3 - \frac{4}{5} + 3 - \frac{2}{3} + 3 - \frac{4}{7} + 3 - \frac{1}{2}\right)$

b) smaller: increasing function, so left endpoints always less than right endpoints.

Q9 max $A = 3xy$ subject to $4x + 6y = 60$

$y = \dfrac{60 - 4x}{6} = 10 - \frac{2}{3}x$

$A = 3x\left(10 - \frac{2}{3}x\right) = 30x - 2x^2$

$\dfrac{dA}{dx} = 30 - 4x = 0$

$x = \dfrac{30}{4} = 7\tfrac{1}{2}$, $y = 5$ (so height = 15)

Q10 a) $A = \pi r^2$

$$\frac{dA}{dt} = 2\pi r \frac{dr}{dt} \qquad \frac{dr}{dt} = \frac{1}{2\pi r}\frac{dA}{dt}$$

b) $\frac{1}{2\pi \cdot 50} \cdot 500 = \frac{5}{\pi}$ m/hr.

c) decreasing as $\frac{1}{2\pi r} \to 0$ as $r \to \infty$ and $\frac{dA}{dt}$ is constant.

Q11 a) $3x^2 + 2y\frac{dy}{dx} = 2y + 2x\frac{dy}{dx}$

$$\frac{dy}{dx}(2y - 2x) = 2y - 3x^2 \qquad \frac{dy}{dx} = \frac{2y - 3x^2}{2y - 2x}.$$

b) $\frac{2.2 - 3}{2.2 - 2} = +\frac{1}{2}$

c) $y - 2 = \frac{1}{2}(x - 1)$