Math 230 Calculus 1/Precalc Fall 11 Final b

Name: Solutions

- Do any 8 of the following 10 questions.
- You may use a calculator, but no notes.

1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
	80	

Final	
Overall	

(1) (10 points) Differentiate the following functions. You do not need to simplify your answers.

(a) $f(x) = xe^{5x}$

3

$$e^{5x} + 5xe^{5x}$$

(b) $f(x) = \frac{\cos(2x)}{\ln(x)}$

3

$$\frac{-\ln(x)\, 2\sin(x) - \frac{1}{x}\cos 2x}{(\ln(x))^2}$$

(c) $f(x) = \frac{1}{\sqrt{1+3x^2}}$ $= (1+3x^2)^{-1/2}$

4

$$-\frac{1}{2}(1+3x^2)^{-3/2} \cdot 6x$$

(2) (10 points) Evaluate the following integrals.

(a) $\int -2e^{-4x}dx$

3

$\frac{1}{2}e^{-4x}+C$

4 (b) $\int \cos^3(x)\sin(x)dx$

$u = \cos(x)$ $\quad \frac{du}{dx} = -\sin(x)$

$\int u^3 \sin(x) \frac{1}{-\sin(x)} du = -\int u^3 du = -\frac{1}{4}u^4 + c$

$= -\frac{1}{4}\cos^4 x + c$

3 (c) $\int_1^2 \frac{x^2-1+x^{-2}}{x}dx = \int_1^2 x - \frac{1}{x} + \frac{1}{x^3}\, dx$

$= \left[\frac{1}{2}x^2 - \ln x - \frac{1}{2}\frac{1}{x^2}\right]_1^2 = 2 - \ln 2 - \frac{1}{8} - \frac{1}{2} + \frac{1}{2}$

$= \frac{15}{8} - \ln 2$

(3) (10 points) Find the following limits.

(a) $\lim_{x\to 0} \frac{\sin(3x)}{x}$

3

l'Hôpital: $\lim_{x \to 0} \frac{3\cos 3x}{1} = 3$

(b) $\lim_{x\to\infty} \frac{(x+2)^2}{3-3x-x^2}$ $= \frac{x^2+4x+4}{-x^2-3x+3} = -1$

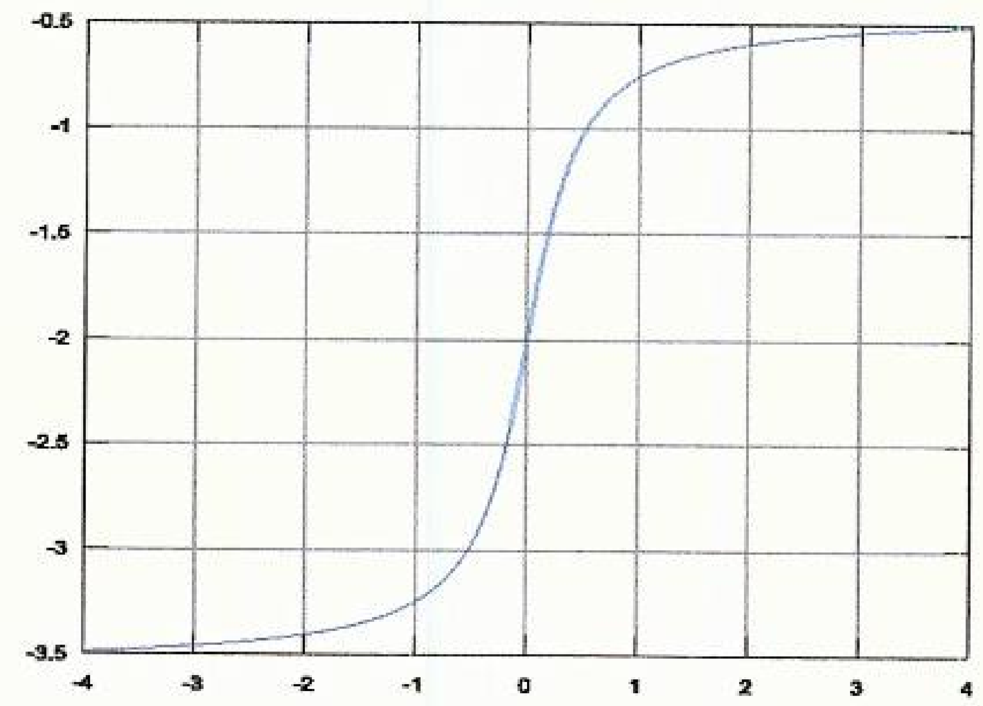
3

4 (c) $\lim_{x\to 0} x^{3x}$ $= e^{3x\ln(x)}$

$\lim_{x\to 0} 3x\ln(x) = \lim_{x\to 0} \frac{3\ln(x)}{1/x} \underset{\text{l'Hôpital}}{=} \lim_{x\to 0} \frac{3/x}{-x^{-2}} = \lim_{x\to 0} 3x = 0$

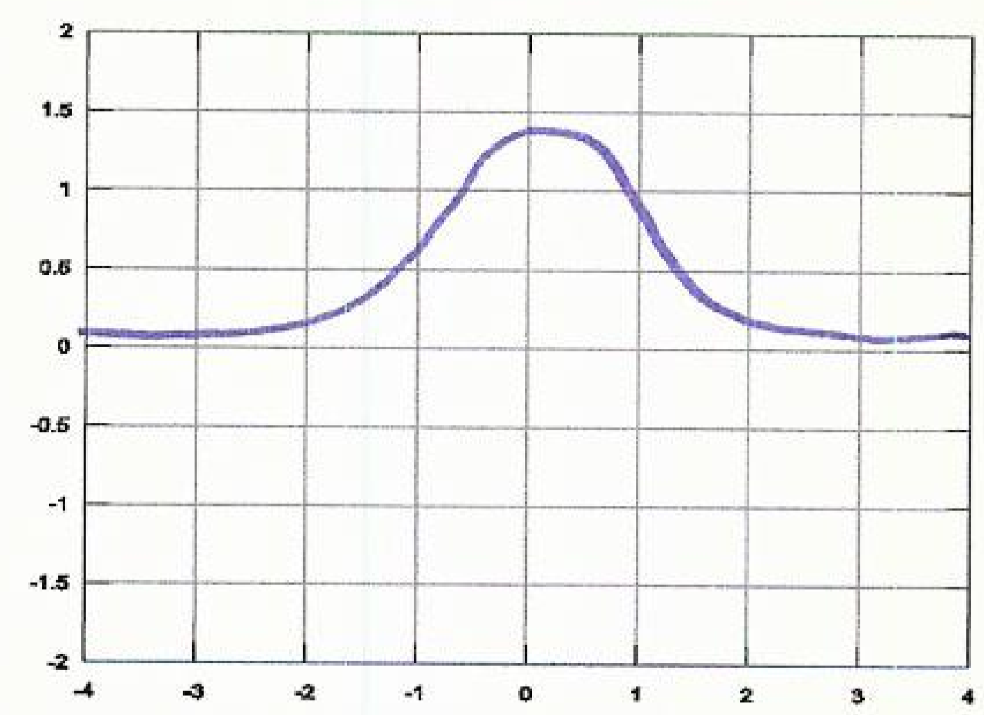
so $\lim_{x\to 0} x^{3x} = e^0 = 1$

(4) (10 points)



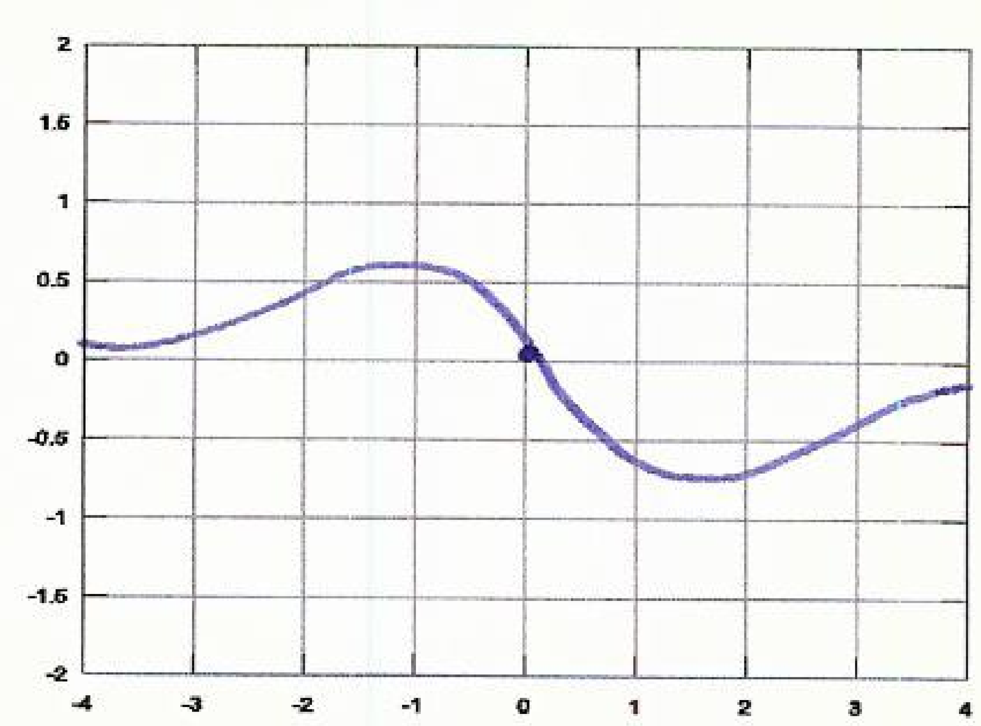
(a) Draw the first derivative of the function on the graph below.

4



(b) Draw the second derivative of the function on the graph below.

4



(c) Where is the function concave up?

2

$(-\infty, 0)$

(5) (10 points)

4 (a) Find the critical points of the function $f(x) = (x-4)e^{-x}$.

4 (b) Use the second derivative test to attempt to classify them as maxima or minima.

2 (c) Sketch the graph of the function.

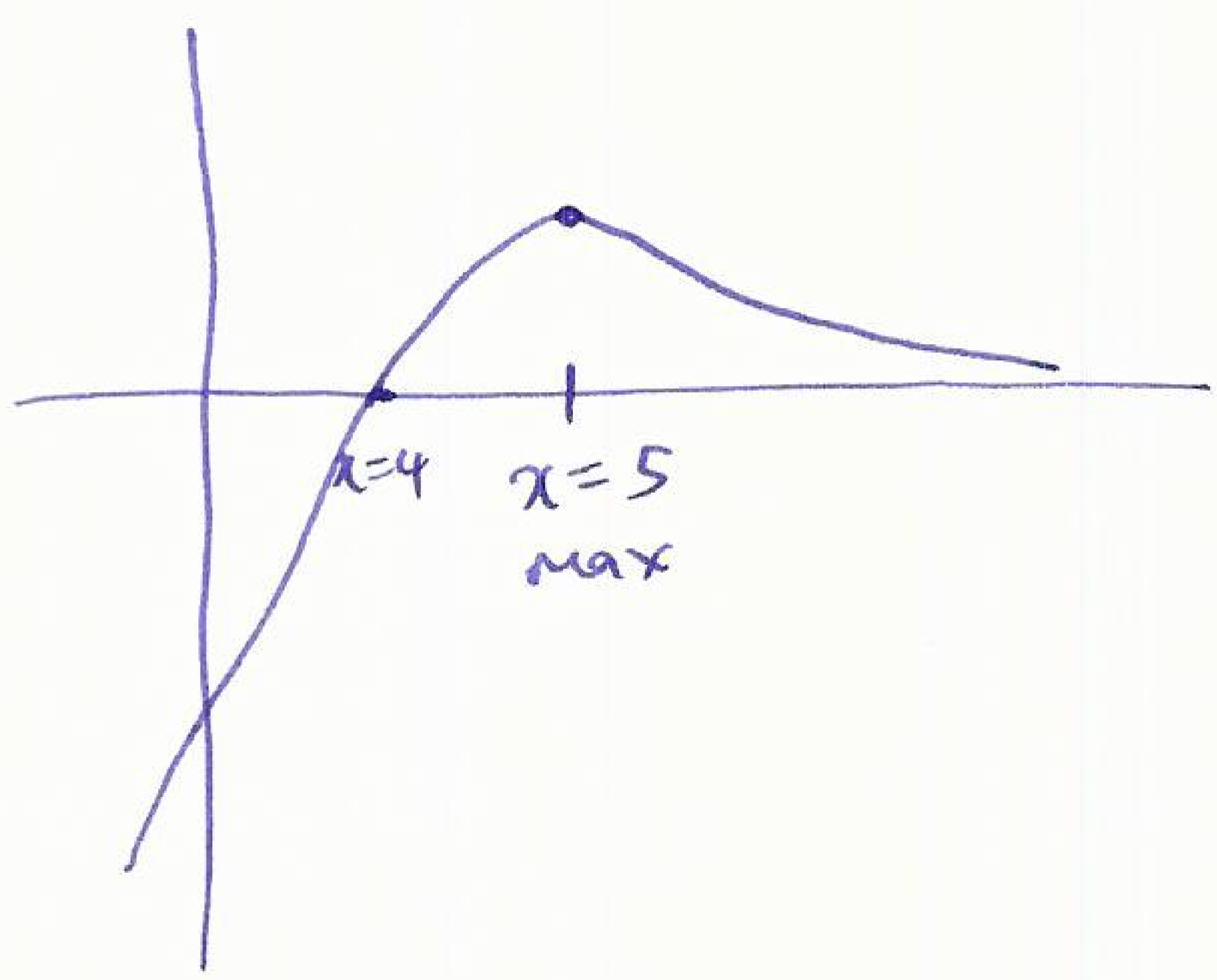
a) $f'(x) = e^{-x} + (x-4) \cdot -e^{-x} = (-x+5)\,e^{-x}$

solve $f'(x) = 0 \Rightarrow x = 5$

b) $f''(x) = -e^{-x} + (-x+5) \cdot -e^{-x} = e^{-x}(x-6)$

$f''(3) < 0$ So max

c)



(6) (10 points) Find the absolute maximum and minimum of the function $f(x) = x^2 - 6x + 1$ on the interval $[1, 5]$.

$f'(x) = 2x - 6$ critical points: $x = 3$

check $f(1) = -4$ max

$f(3) = -8$ min

$f(5) = -4$ max

(7) (10 points) Differentiate the function $f(x) = 3x^2 + 3x - 2$, *using the limit definition of the deriviative.*

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{3(x+h)^2 + 3(x+h) - 2 - 3x^2 - 3x + 2}{h}$$

$$\lim_{h \to 0} \frac{\cancel{3x^2} + 6xh + 3h^2 + \cancel{3x} + 3h - \cancel{2} - \cancel{3x^2} - \cancel{3x} + \cancel{2}}{h} = \lim_{h \to 0} 6x + 3h + 3 = 6x + 3.$$

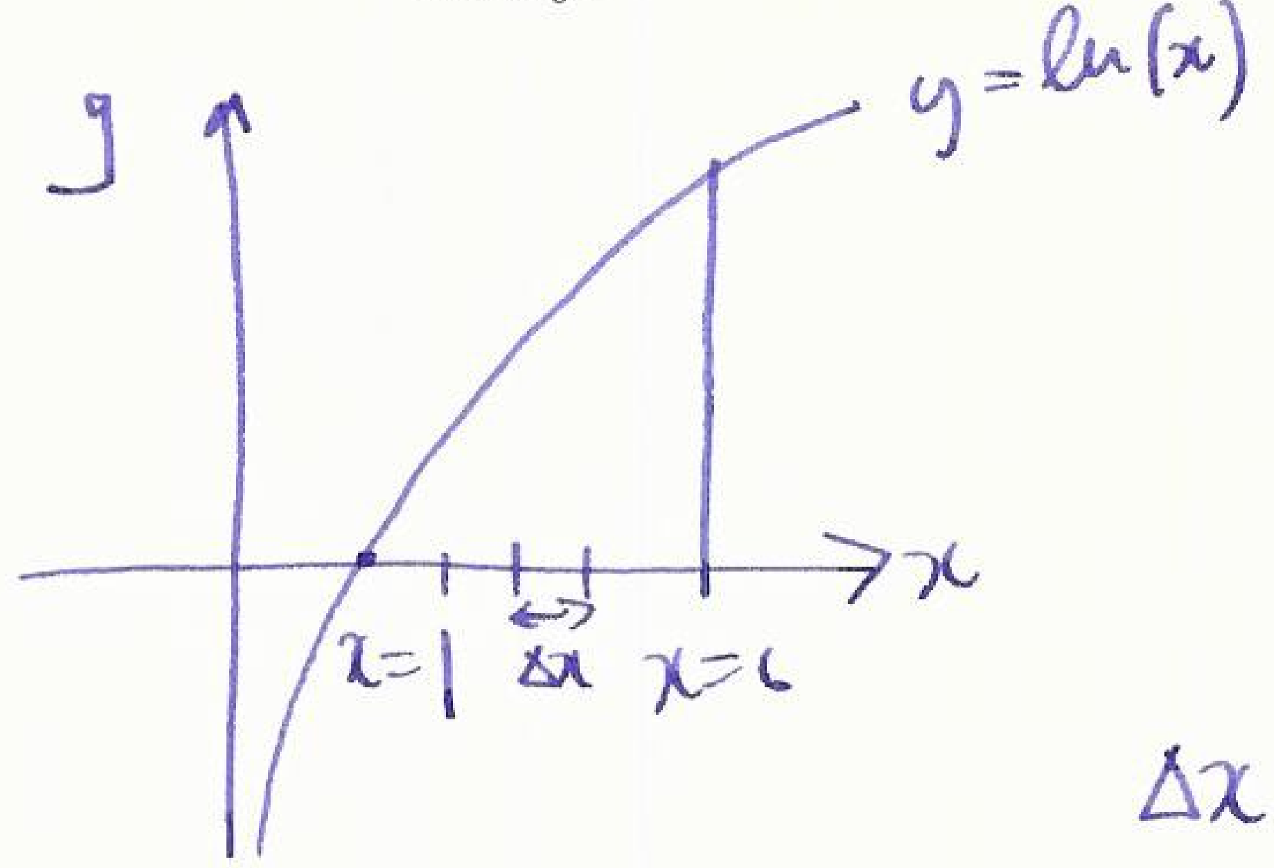
(8) (10 points) A region in the plane is bounded by the x-axis, and the curves $y = \ln(x)$ and $x = 6$.

3 (a) Draw a picture of this region.

3 (b) Write down an integral corresponding to this region.

4 (c) Write down an expression to approximate the integral for the left hand Riemann sum with 4 rectangles. Do not bother to compute this numerically.

a)

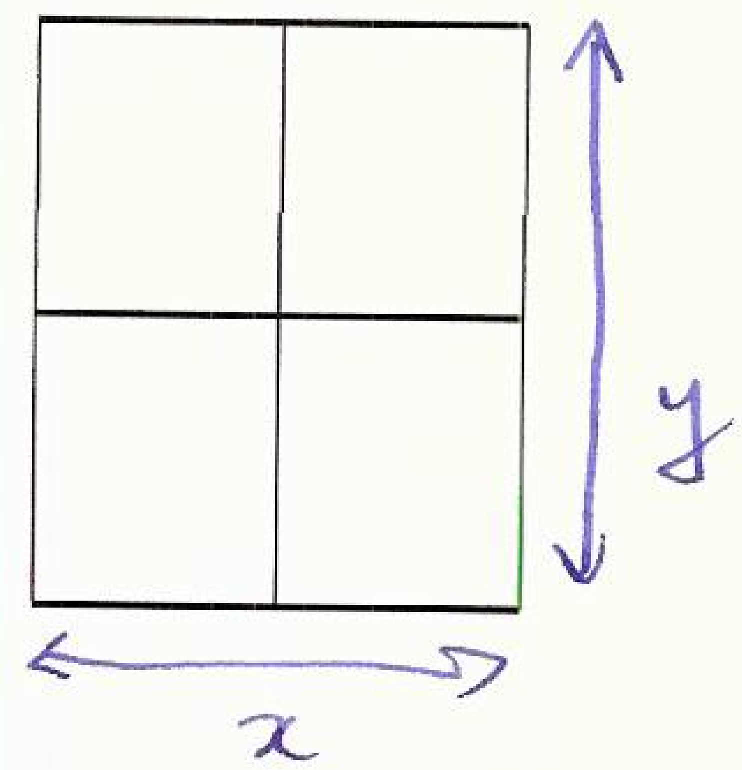


b) $\displaystyle\int_1^6 \ln(x)\,dx$

$\Delta x = \frac{6-1}{4} = \frac{5}{4}$.

c) $L_4 = \frac{5}{4}\left(\ln(1) + \ln\left(1+\frac{5}{4}\right) + \ln\left(1+\frac{10}{4}\right) + \ln\left(1+\frac{15}{4}\right)\right)$

(9) (10 points) A window frame with four panes, shown below, is to be made of horizontal pieces, which cost \$20/foot and vertical pieces, which cost \$30/foot. If the total area of the window should be 20 square feet, what are the dimensions of the cheapest window?



cost $C = 60x + 90y$

area $A = xy = 20 \Rightarrow y = \frac{20}{x}$.

$$C = 60x + \frac{1800}{x}$$

$$\frac{dC}{dx} = 60 - \frac{1800}{x^2} = 0 \Rightarrow x^2 = \frac{1800}{60} = 300 \quad x = \sqrt{300}$$

$$y = \frac{20}{\sqrt{300}}$$

(10) (10 points) A pebble is dropped into a calm pond, causing circular ripples to expand outward. If the radius of the outermost ripple is growing at 4ft/second, how fast is the area of disturbed water growing when the ripple is 2 feet across?

$$A = \pi r^2$$

$$\frac{dA}{dt} = 2\pi r \underbrace{\frac{dr}{dt}}_{4}$$

$$\frac{dA}{dt} = 2 \cdot \pi \cdot 2 \cdot 4 = 16\pi \ ft^2/sec.$$