Math 230 Calculus 1/Precalc Fall 11 Final a

Name: Solutions

- Do any 8 of the following 10 questions.
- You may use a calculator, but no notes.

1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
	80	

Final	
Overall	

(1) (10 points) Differentiate the following functions. You do not need to simplify your answers.

(a) $f(x) = xe^{4x}$

3

$e^{4x} + x.4e^{4x}$

(b) $f(x) = \frac{\sin(3x)}{\ln(x)}$

3

$$\frac{\ln(x)\cos(3x).3 - \frac{1}{x}\sin(3x)}{(\ln(x))^2}$$

(c) $f(x) = \frac{1}{\sqrt{1+2x^2}} = (1+2x^2)^{-1/2}$

4

$-\frac{1}{2}(1+2x^2)^{-3/2} . 4x$

(2) (10 points) Evaluate the following integrals.

(a) $\int -4e^{-3x}dx$

3

$$+\frac{4}{3}e^{-3x} + c$$

(b) $\int \sin^3(x)\cos(x)dx$

$$u = \sin(x) \quad \frac{du}{dx} = \cos(x)$$

4

$$\int u^3 \cos(x) \frac{1}{\cos(x)}\, du = \int u^3 du = \frac{1}{4}u^4 + c$$

$$= \frac{1}{4}\sin^4 x + c$$

(c) $\int_1^2 \frac{x^2+1-x^{-2}}{x}dx = \int_1^2 x + \frac{1}{x} - \frac{1}{x^3}dx = \left[\frac{1}{2}x^2 + \ln|x| + \frac{1}{2}x^{-2}\right]_1^2$

3

$$= 2 + \ln(2) + \frac{1}{8} - \frac{1}{2} - \frac{1}{2} = \frac{9}{8} + \ln(2)$$

(3) (10 points) Find the following limits.

(a) $\lim_{x\to 0} \frac{\sin(4x)}{x}$

3

l'Hôpital: $\lim_{x\to 0} \frac{4\cos(4x)}{1} = 4.$

(b) $\lim_{x\to\infty} \frac{(x+1)^2}{4-4x-x^2}$ $= \lim_{x\to\infty} \frac{x^2+2x+1}{-x^2-4x+4} = -1$

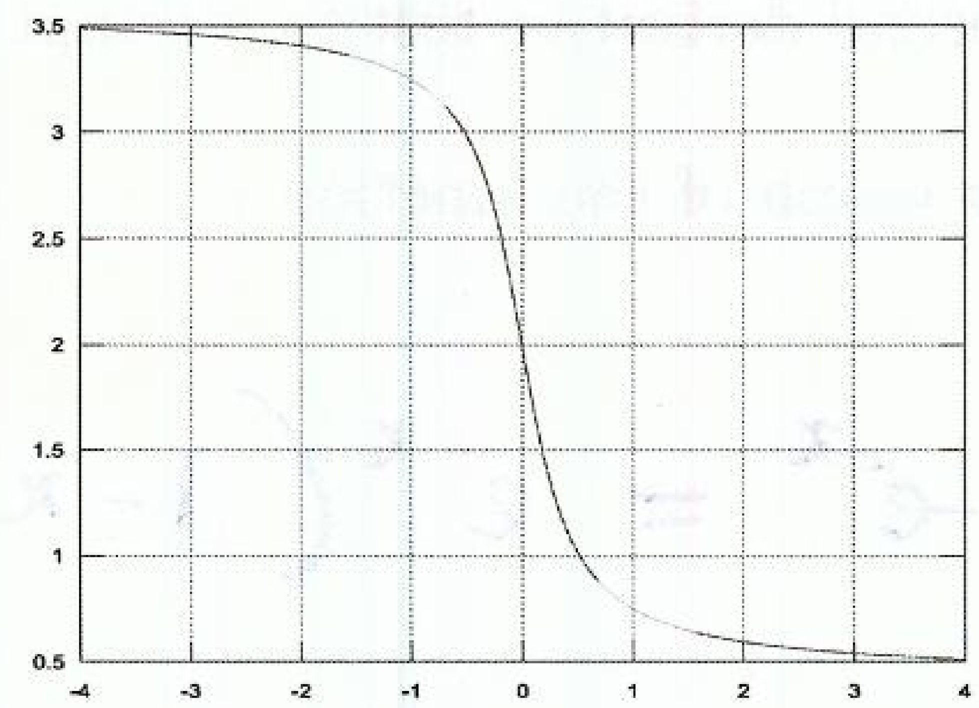
3

4 (c) $\lim_{x\to 0} x^{2x}$ $= \lim_{x\to 0} e^{2x\ln(x)}$

$\lim_{x\to 0} 2x\ln(x) = \lim_{x\to 0} \frac{\ln(x)}{1/2x} \underset{\text{l'H}}{=} \lim_{x\to 0} \frac{2\cdot 1/2}{-x^{-2}} = \lim_{x\to 0} -x^2 = 0$

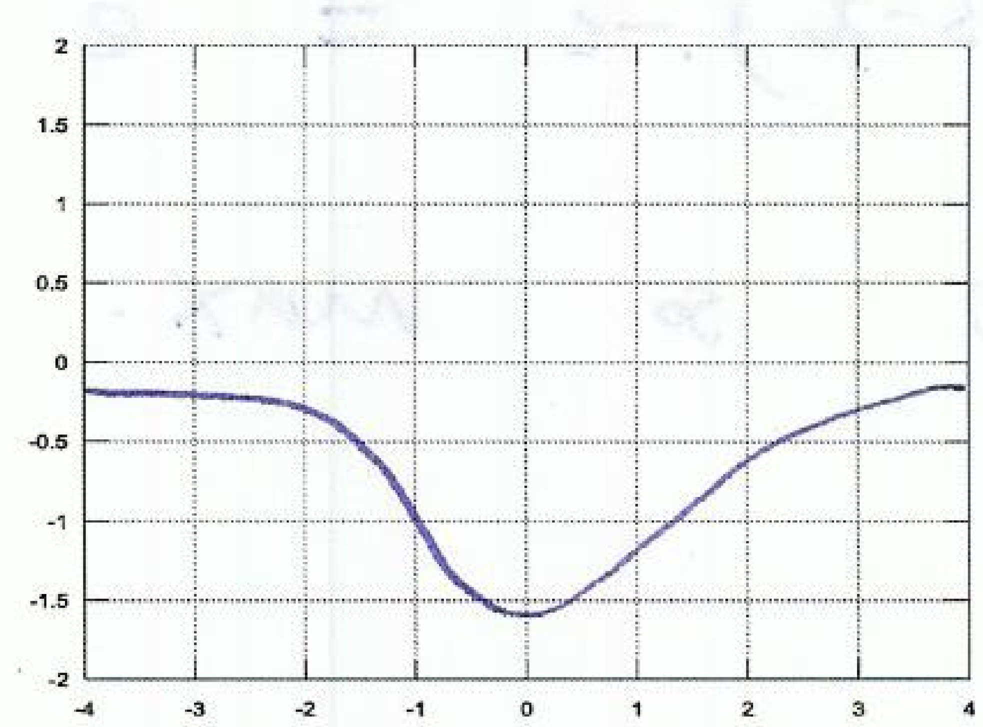
so $\lim_{x\to 0} x^{2x} = e^0 = 1$

(4) (10 points)



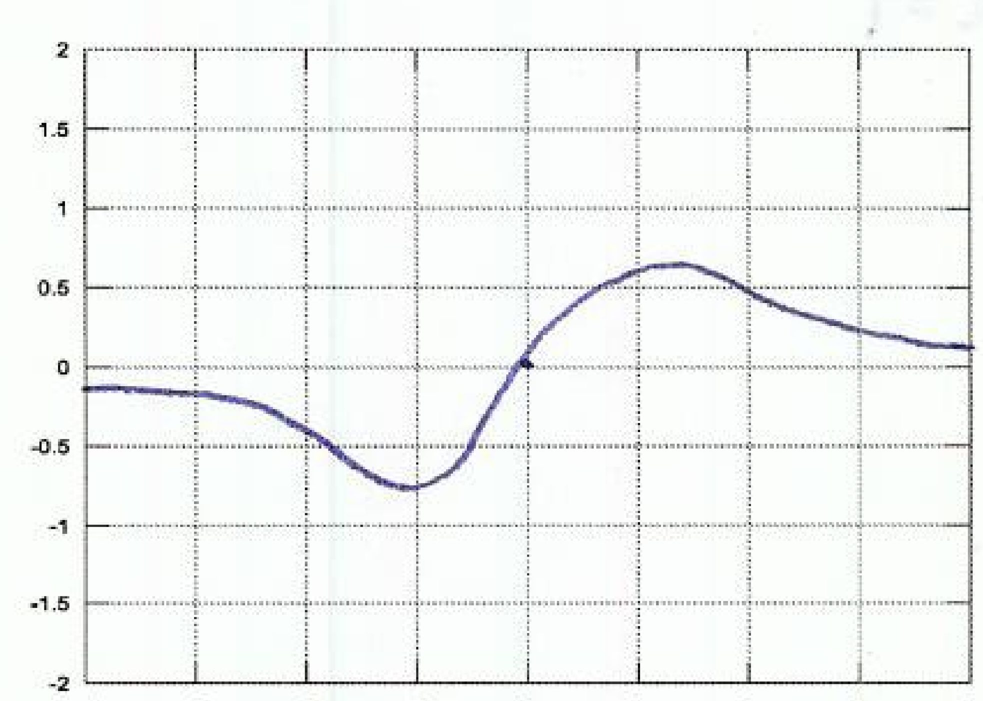
(a) Draw the first derivative of the function on the graph below.

34



(b) Draw the second derivative of the function on the graph below.

34



(c) Where is the function concave up? $(0, \infty)$

2

(5) (10 points)

4 (a) Find the critical points of the function $f(x) = (x-2)e^{-x}$.

4 (b) Use the second derivative test to attempt to classify them as maxima or minima.

2 (c) Sketch the graph of the function.

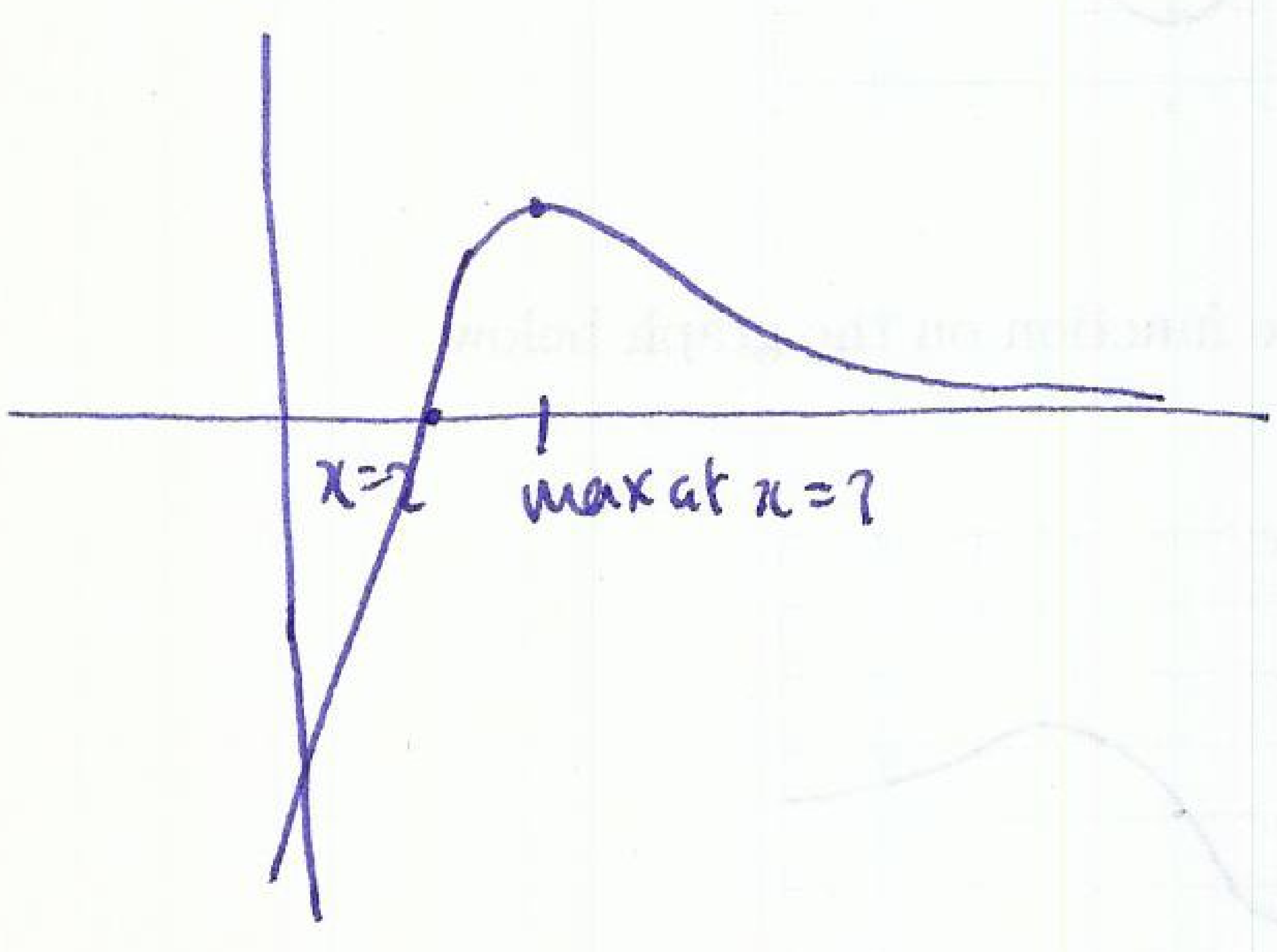
a) $f'(x) = e^{-x} + (x-2) \cdot -e^{-x} = e^{-x}(1-x+2) = (3-x)e^{-x}$

$f'(x) = 0 \Rightarrow x = 3.$

b) $f''(x) = -e^{-x} + (3-x) \cdot -e^{-x} = e^{-x}(-1-3+x) = (x-4)e^{-x}$

$f''(3) = -e^{-3} < 0$ so max.

c)



(6) (10 points) Find the absolute maximum and minimum of the function $f(x) = x^2 - 5x + 2$ on the interval $[2, 4]$.

$f'(x) = 2x - 5$ critical points : $x = \frac{5}{2}$.

check $f(2) = 4 - 10 + 2 = -4$ ~~max~~

$f(\frac{5}{2}) = \frac{25}{4} - \frac{25}{2} + 2 = \frac{-17}{4}$ ~~max~~ min

$f(4) = 16 - 20 + 2 = -2$ max

(7) (10 points) Differentiate the function $f(x) = 4x^2 - 6x + 2$, *using the limit definition of the deriviative.*

$$\lim_{x\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{x\to 0} \frac{4(x+h)^2 - 6(x+h) + 2 - 4x^2 + 6x - 2}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{4x^2} + 8xh + 4h^2 - \cancel{6x} - 6h + \cancel{2} - \cancel{4x^2} + \cancel{6x} - \cancel{2}}{h} = \lim_{h\to 0} 8x + 4h - 6 = 8x - 6.$$

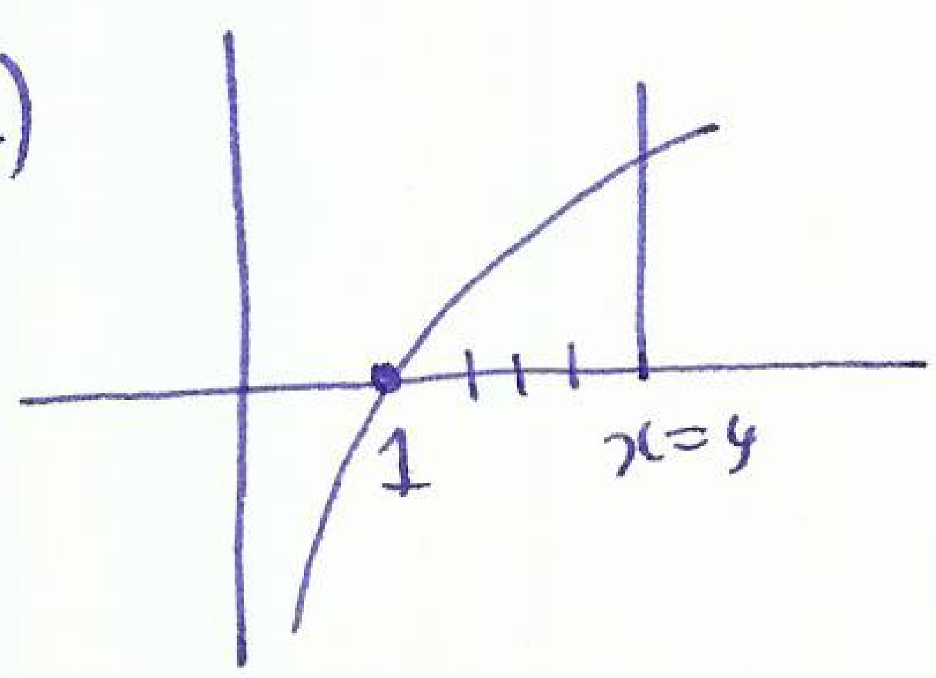
(8) (10 points) A region in the plane is bounded by the x-axis, and the curves $y = \ln(x)$ and $x = 4$.

3 (a) Draw a picture of this region.

3 (b) Write down an integral corresponding to this region.

4 (c) Write down an expression to approximate the integral for the left hand Riemann sum with 4 rectangles. Do not bother to compute this numerically.

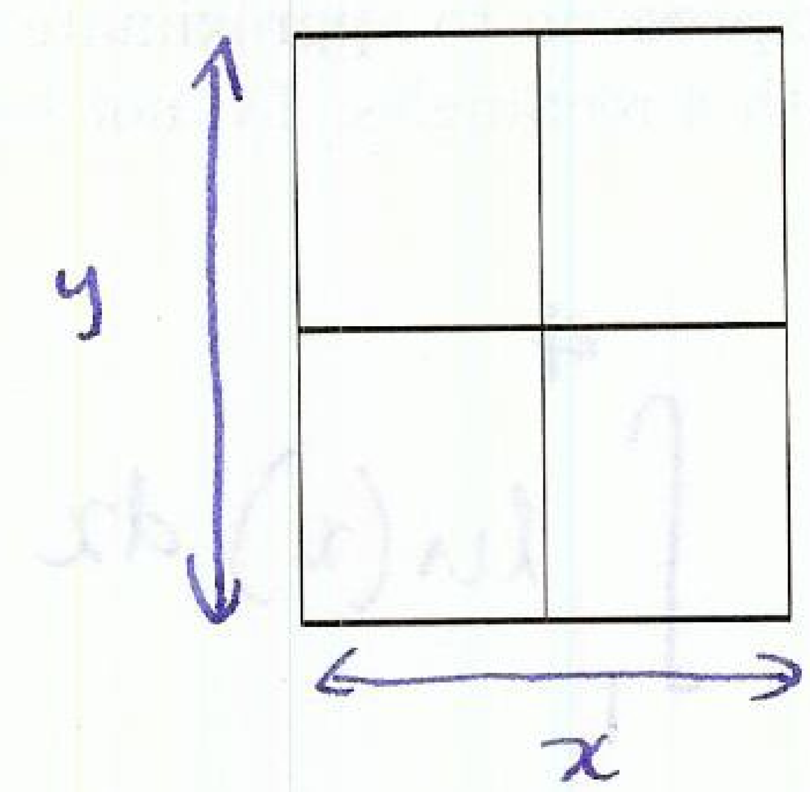
a)



b) $$\int_1^4 \ln(x)\,dx$$

$$L_4 = \frac{3}{4}\left(\ln(1) + \ln\left(1+\frac{3}{4}\right) + \ln\left(1+\frac{6}{4}\right) + \ln\left(\frac{9}{4}\right) \right)$$

(9) (10 points) A window frame with four panes, shown below, is to be made of horizontal pieces, which cost \$10/foot and vertical pieces, which cost \$20/foot. If the total area of the window should be 10 square feet, what are the dimensions of the cheapest window?



$C = 30x + 60y$

$A = xy = 10 \qquad y = \frac{10}{x}$

$C = 30x + \frac{600}{x}$

$\frac{dC}{dx} = 30 + -\frac{600}{x^2} = 0 \qquad x^2 = \frac{600}{30} = 20 \qquad x = \sqrt{20}$

$y = \frac{10}{\sqrt{20}}$

(10) (10 points) A pebble is dropped into a calm pond, causing circular ripples to expand outward. If the radius of the outermost ripple is growing at 3ft/second, how fast is the area of disturbed water growing when the ripple is 4 feet across?

$A = \pi r^2 \qquad \frac{dr}{dt} = 3$

$\frac{dA}{dt} = \pi \cdot 2r \frac{dr}{dt} = \pi \cdot 2 \cdot 4 \cdot 3 = 24\pi \ ft^2/sec.$