Math 130 Precalculus Spring 10 Midterm 3b

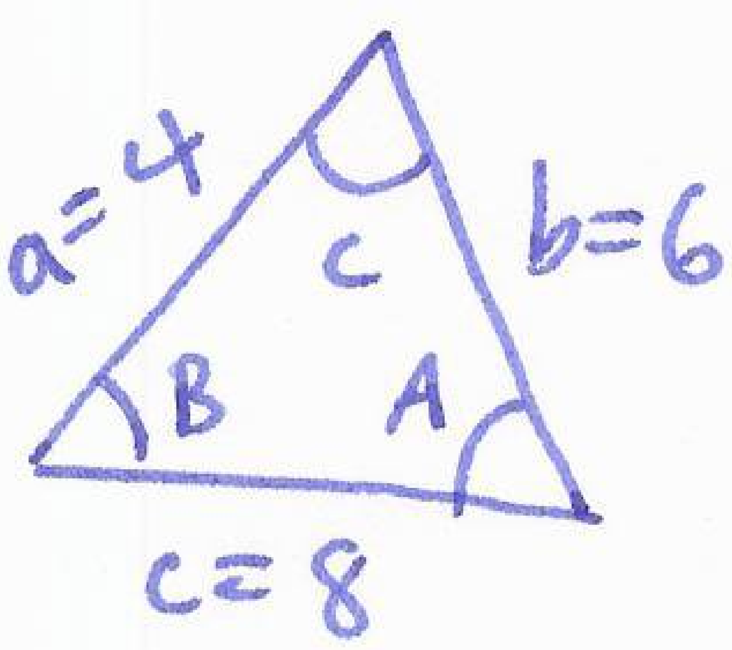
Name: Solutions

- You may use a graphing calculator.
- You may use a 3×5 index card of notes.

1	10	
2	15	
3	10	
4	15	
5	10	
6	15	
7	15	
	90	

Midterm 3	
Overall	

(1) (10 points) Solve the following triangle: $a = 4$cm, $b = 6$cm, $c = 8$cm.



$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$16 = 36 + 64 - 96\cos A$$

$$\cos A = \frac{84}{96} \qquad A = \cos^{-1}\left(\frac{84}{96}\right) \approx 0.505$$

$$28.96^\circ$$

$$b^2 = a^2 + c^2 - 2ac\cos B$$

$$36 = 16 + 64 - 64\cos B$$

$$\cos B = \frac{-44}{-64} \qquad B = \cos^{-1}\left(\frac{44}{64}\right) \approx 0.813$$

$$46.6^\circ$$

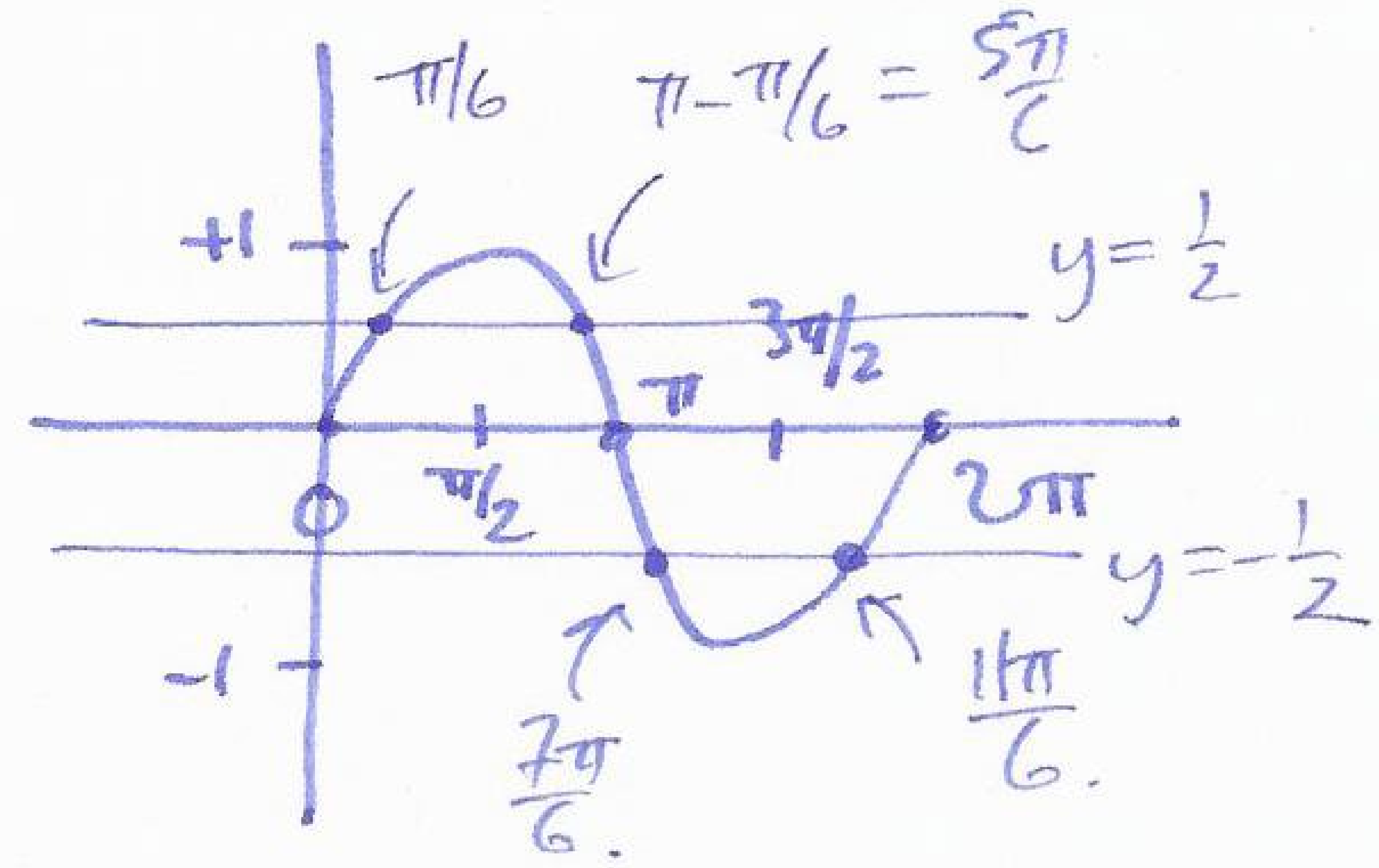
$$C = \pi - 0.505 - 0.813 \approx 1.824$$

$$104.7^\circ.$$

(2) (15 points) Find all solutions that are in $[0, 2\pi)$ of the equation

$$4\sin^2 x - 1 = 0$$

$\sin^2 x = \frac{1}{4}$ $\quad$ $\sin x = \pm\frac{1}{2}$



$\sin^{-1}\left(\frac{1}{2}\right) = \pi/6$ other sol$^{\underline{n}}$ $\pi - \frac{\pi}{6} = \frac{5\pi}{6}$

$\sin^{-1}\left(-\frac{1}{2}\right) = -\pi/6$ sol$^{\underline{n}}$ in $[0, 2\pi)$ is $2\pi - \pi/6 = \frac{11\pi}{6}$

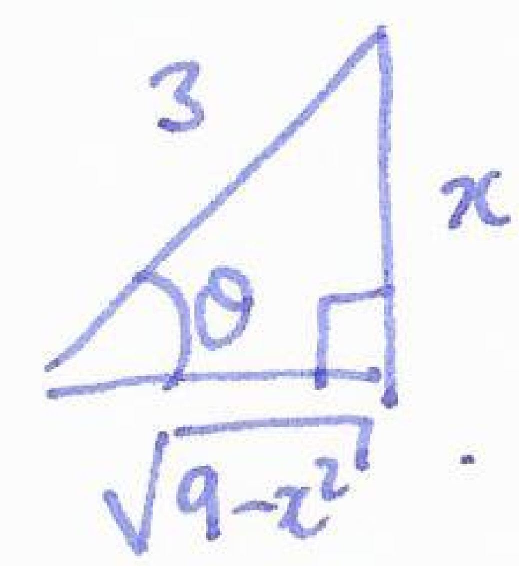
other solution is $\pi + \left(2\pi - \frac{11\pi}{6}\right)$

$\pi + \pi/6 = 7\pi/6.$

solutions are: $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

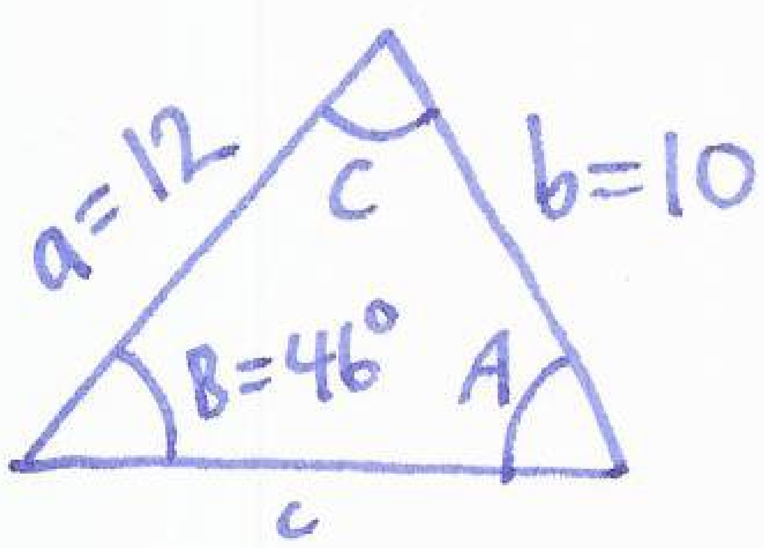
(3) (10 points) Simplify $\tan(\sin^{-1}(x/3))$.

$\tan\theta$ $\quad\theta = \sin^{-1}(x/3)$ $\quad\sin\theta = (x/3)$



$$\tan\theta = \frac{x}{\sqrt{9-x^2}}$$

(4) (15 points) Find all the angles in both triangles with the following properties:
$a = 12\text{cm}$, $b = 10\text{cm}$, $B = 46^\circ$.

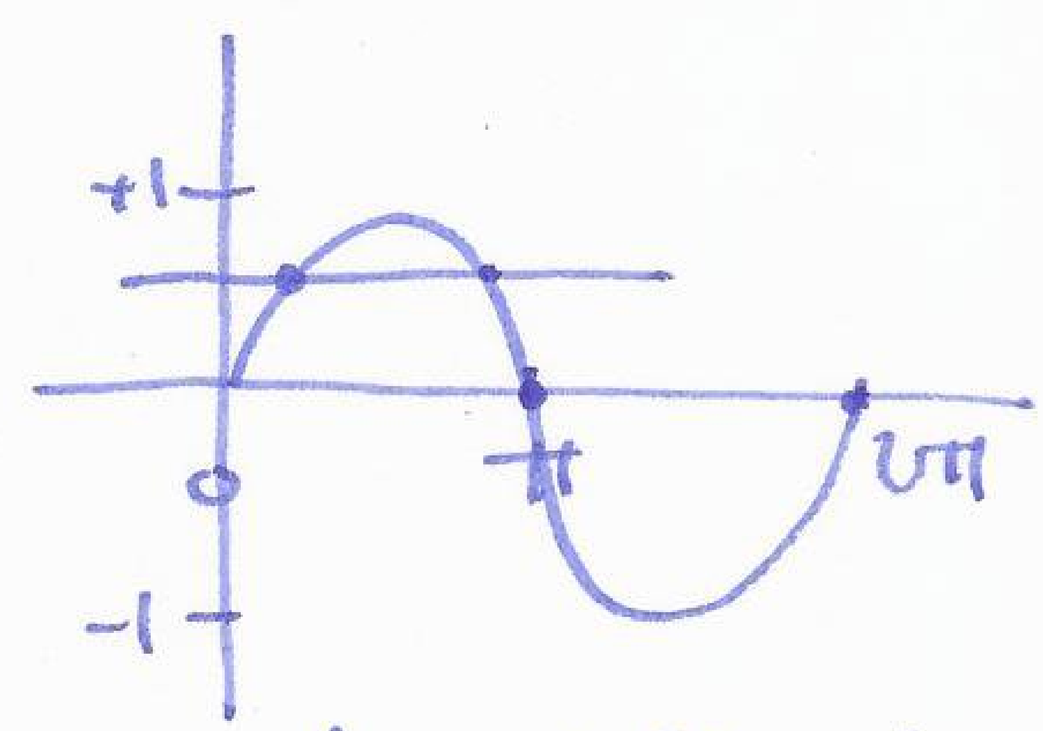


$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\sin A = \frac{12 \sin 46^\circ}{10} \approx 0.863$$

$$A \approx 1.042$$

59.7°

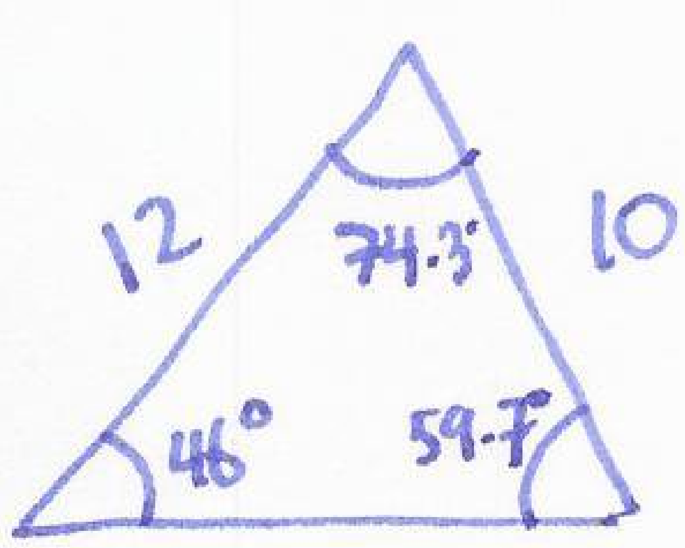


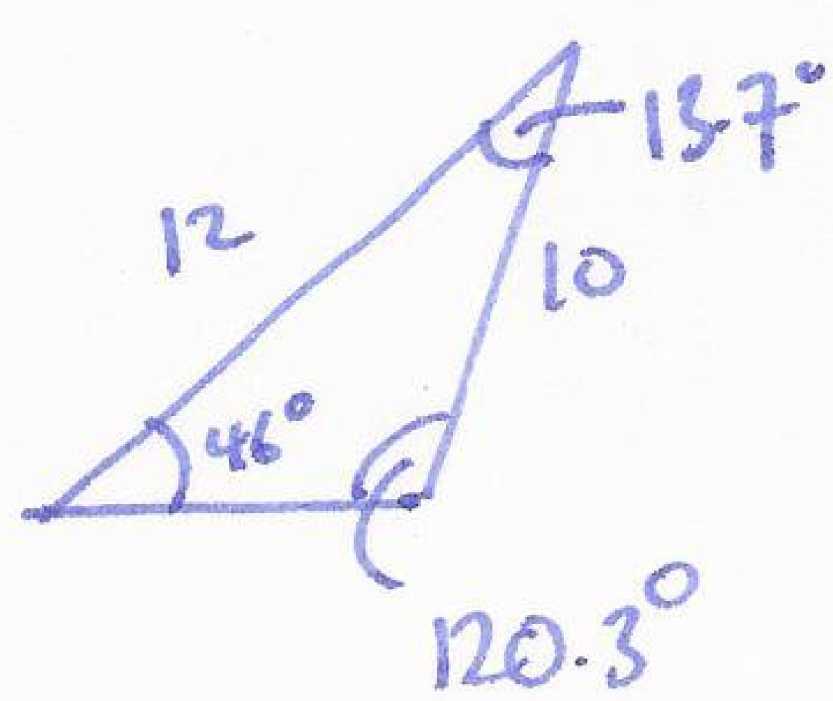
two solutions in $(0, \pi)$

other solution $\pi - 1.042$

≈ 2.100

120.3°





(5) (10 points) Prove the following identity:

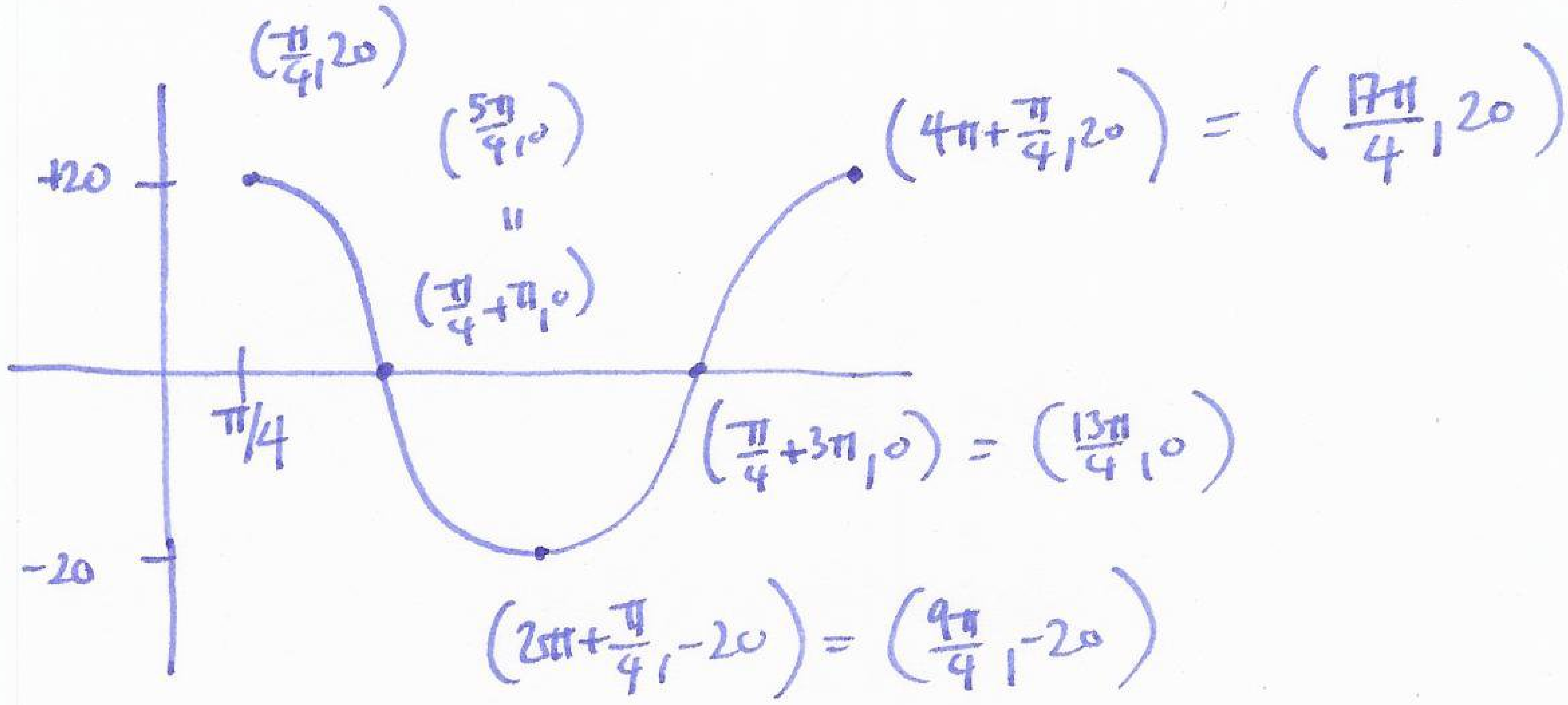
$$\frac{1}{\tan\theta + \cot\theta} = \sin\theta\cos\theta$$

$$\frac{1}{\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}} = \frac{1}{\frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}}$$

$$= \frac{1}{\frac{1}{\sin\theta\cos\theta}} = \sin\theta\cos\theta .$$

(6) (15 points) Sketch one period of the graph $y = 20\cos\left(\frac{x}{2} - \frac{\pi}{8}\right)$. Label the lowest points, the highest points and the x-intercepts of the graph with their coordinates.

$$y = 20\cos\left(\frac{1}{2}\left(x - \frac{\pi}{4}\right)\right)$$



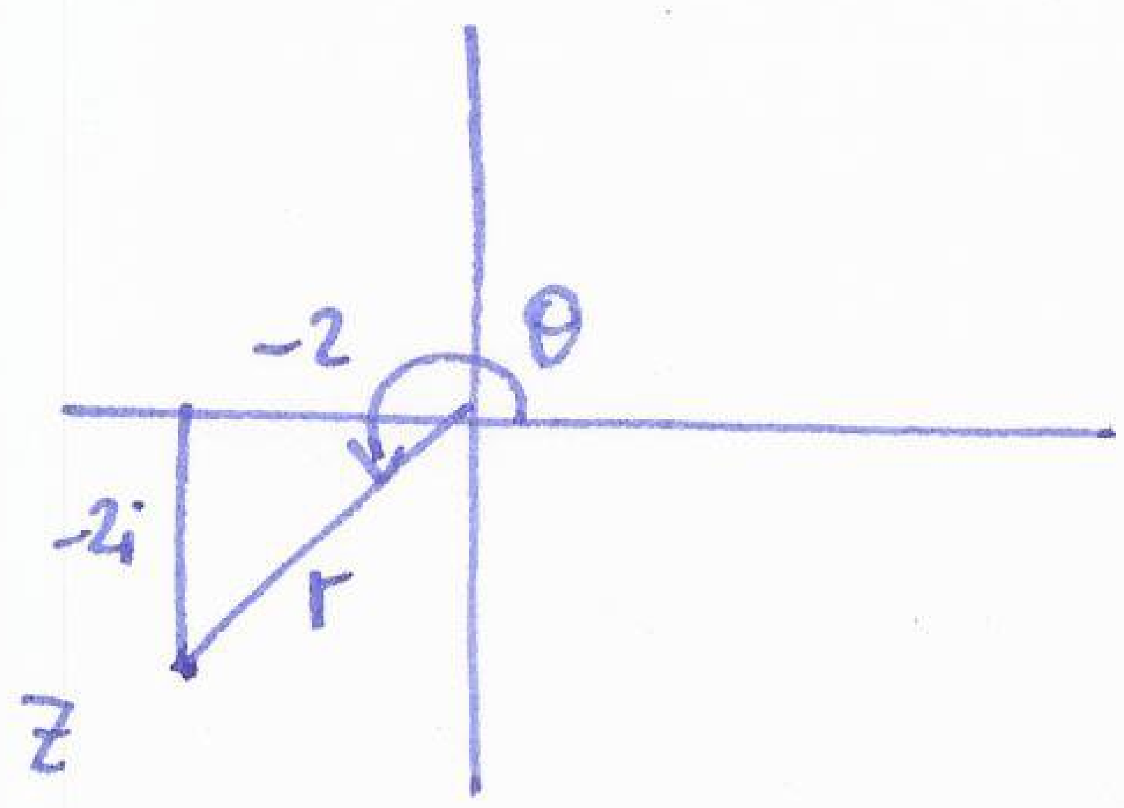
(7) (15 points) Consider the complex number $z = -2 - 2i$.

(a) Write z in trigonometric form as $z = r(\cos(\phi) + i\sin(\phi))$.

(b) Compute z^2 using the standard $a + bi$ form.

(c) Compute z^2 using the trigonometric form, e.g. by using De Moivre's theorem.

a)



$$r = \sqrt{2^2+2^2} = 2\sqrt{2}$$

$$\theta = \pi + \frac{\pi}{4} = \frac{5\pi}{4}.$$

$$z = 2\sqrt{2}\left(\cos\frac{5\pi}{4} + i\sin\frac{5\pi}{4}\right)$$

b) $$z^2 = (-2-2i)^2 = (-2)^2 + 2(-2)(-2i) + (-2i)^2$$
$$= 4 + 8i - 4 = 8i$$

c) $$z^2 = (2\sqrt{2})^2\left(\cos\frac{10\pi}{4} + i\sin\frac{10\pi}{4}\right)$$
$$= 8\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right) = 8i$$