Math 130 Precalculus Sample midterm 3 Solutions

Q1

$$\frac{\cos^3 x - \sin^3 x}{\cos x - \sin x} = \frac{\cos x(1-\sin^2 x) - \sin x(1-\cos^2 x)}{\cos x - \sin x}$$

$$= \frac{\cos x - \sin x \quad - \cos x \sin^2 x + \sin x \cos^2 x}{\cos x - \sin x}$$

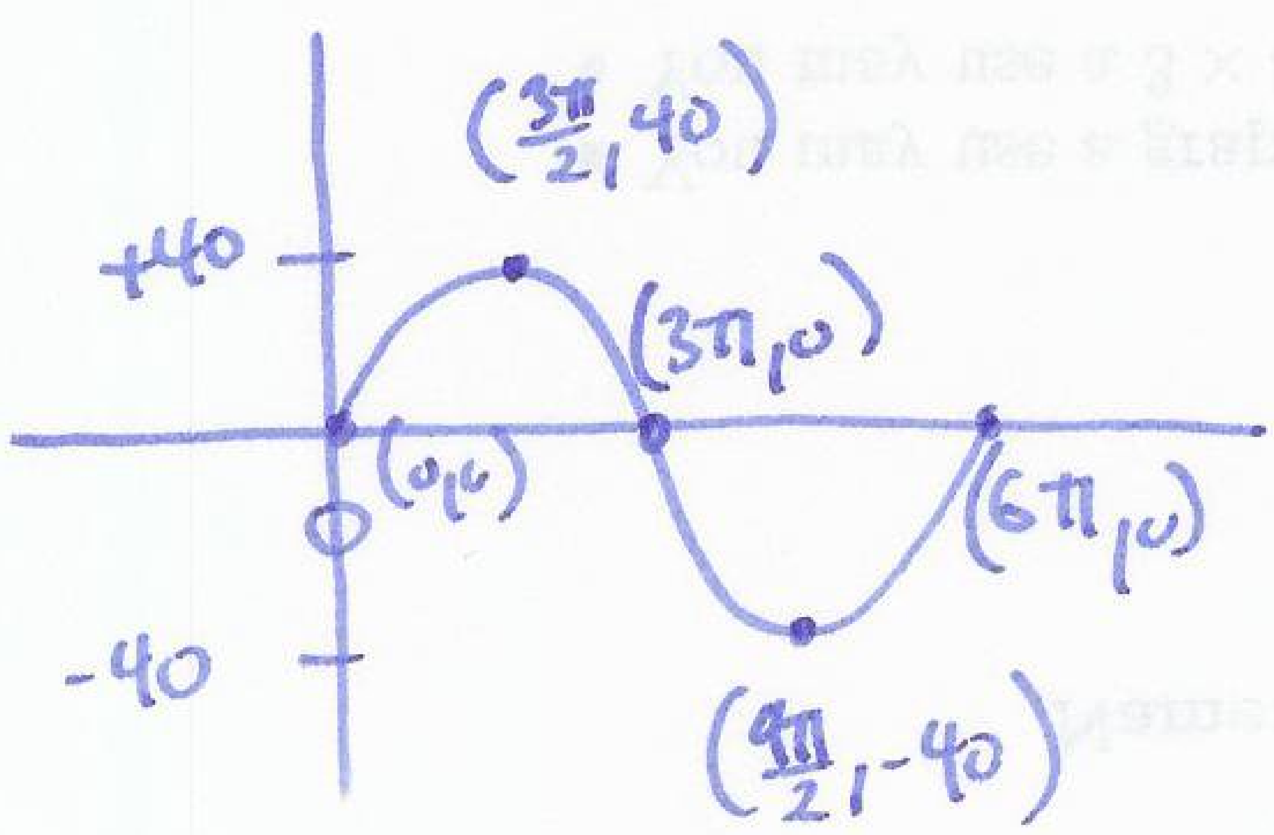
$$= 1 + \frac{\sin x \cos x\,(\cos x - \sin x)}{\cos x - \sin x} = 1 + \sin x \cos x$$

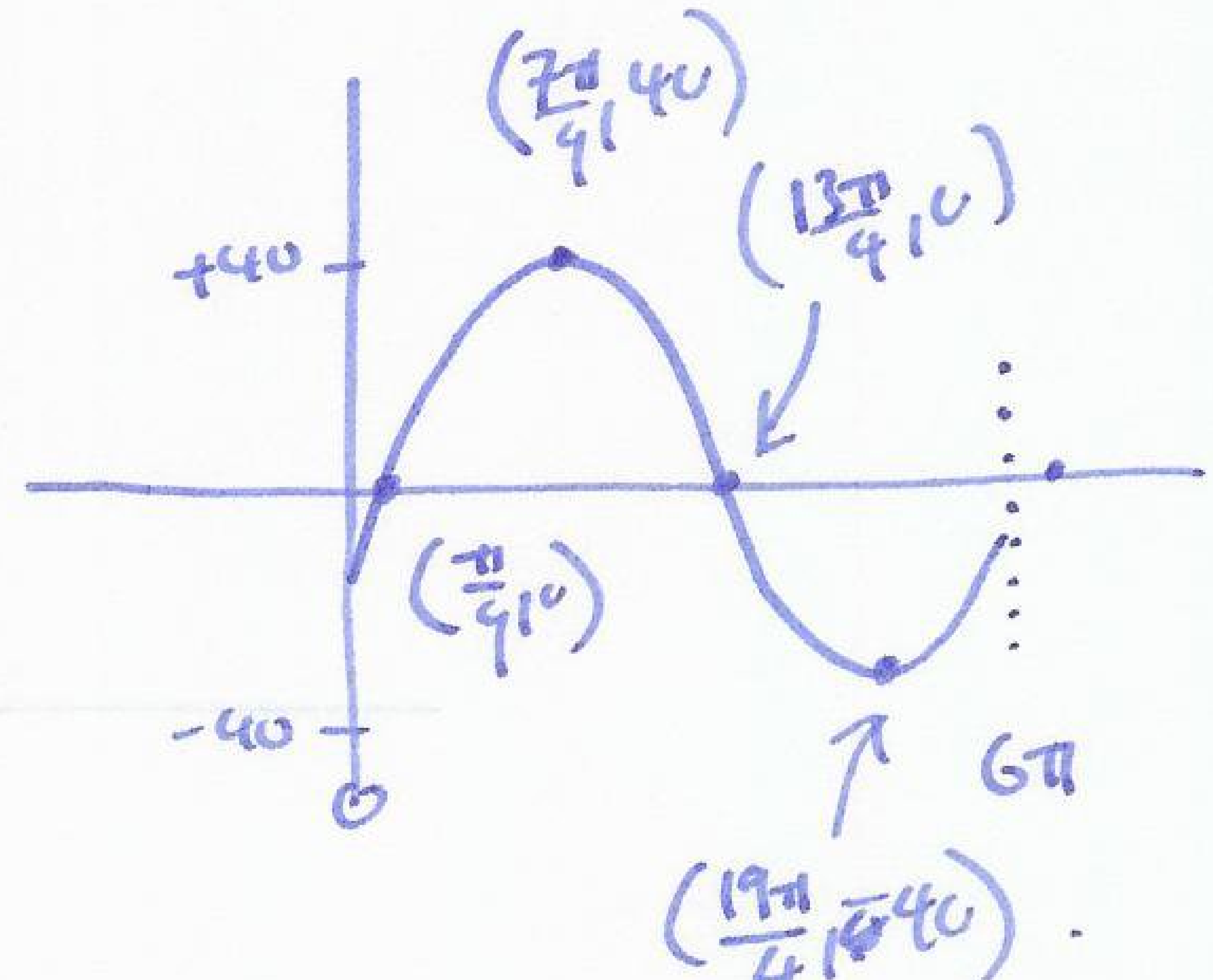
$$= 1 + \frac{1}{2}\sin 2x = \frac{2+\sin 2x}{2}.$$

Q2 $y = 40\sin\left(\frac{1}{3}x - \frac{\pi}{12}\right) = 40\sin\left(\frac{1}{3}\left(x - \frac{\pi}{4}\right)\right)$ period 6π.

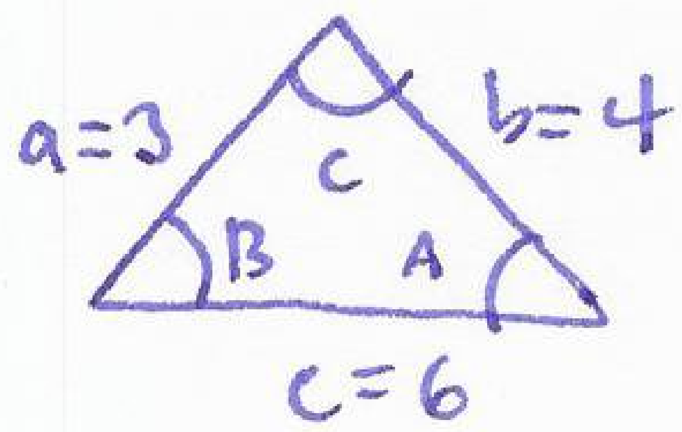
$y = 40\sin(\frac{1}{3}x)$

shift $\frac{\pi}{4}$





Q3



$$c^2 = a^2 + b^2 - 2ab\cos C$$

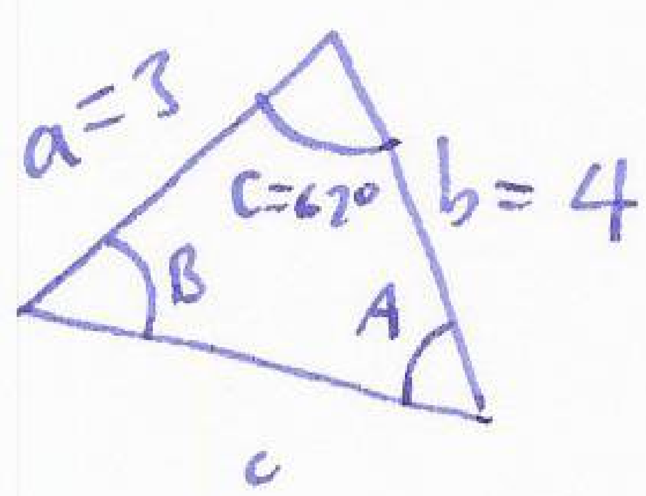
$$36 = 9 + 16 - 24\cos C \qquad \cos C = \frac{-11}{24} \qquad C \simeq 2.05$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$9 = 16 + 36 - 48\cos A \qquad \cos A = \frac{43}{48} \qquad A \simeq 0.46$$

$$B = \pi - A - C \simeq 0.63.$$

Q4



$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$c^2 = 9 + 16 - 24\cos 62^\circ \simeq 13.73$$

$$c \simeq 3.71.$$

$$\frac{\sin 62^\circ}{3.71} = 0.24 \qquad \frac{\sin A}{3} = 0.24 \qquad \sin A = 0.71 \qquad A \simeq 0.79 = 45.6^\circ$$

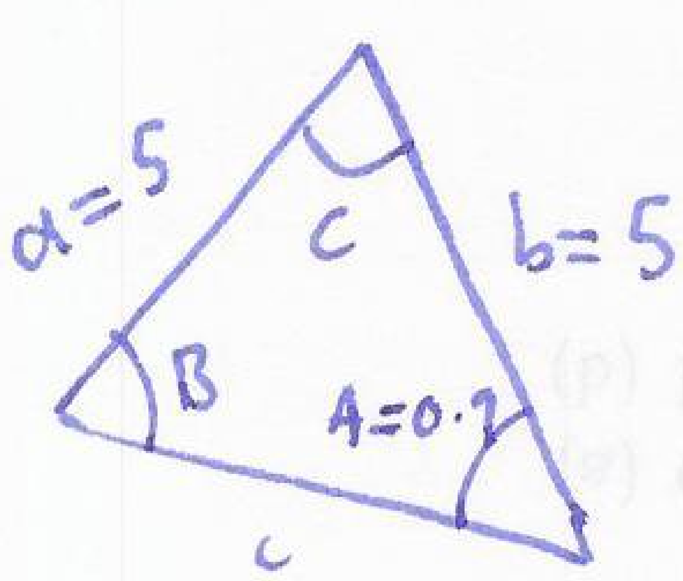
$$\text{or } \pi - 0.79 \simeq 2.35.$$

$$\text{too big!} \longrightarrow 134.4^\circ$$

$$\text{so } A \simeq 45.6^\circ.$$

$$\text{so then} \quad B = 180 - A - C \simeq 72.4.$$

Q5



$$\cancel{a^2 = b^2 + c^2 - 2bc\cos A}.$$

$$\cancel{25 = 25 + c^2}$$

$$\frac{\sin A}{a} = \frac{\sin 0.9}{5} = \frac{\sin B}{5}$$

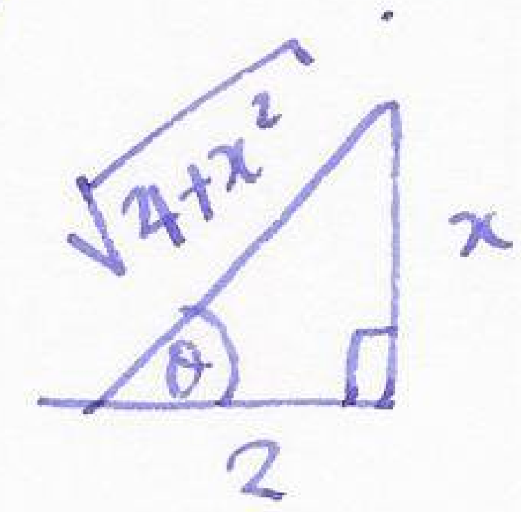
$$\Rightarrow \sin B = \sin 0.9 \quad \text{so} \quad B = 0.9 \quad \text{or} \quad \underbrace{\pi - 0.9}_{\text{too big! would imply } C=0}.$$

$$\text{then} \quad C = \pi - 0.9 - 0.9 = 1.34$$

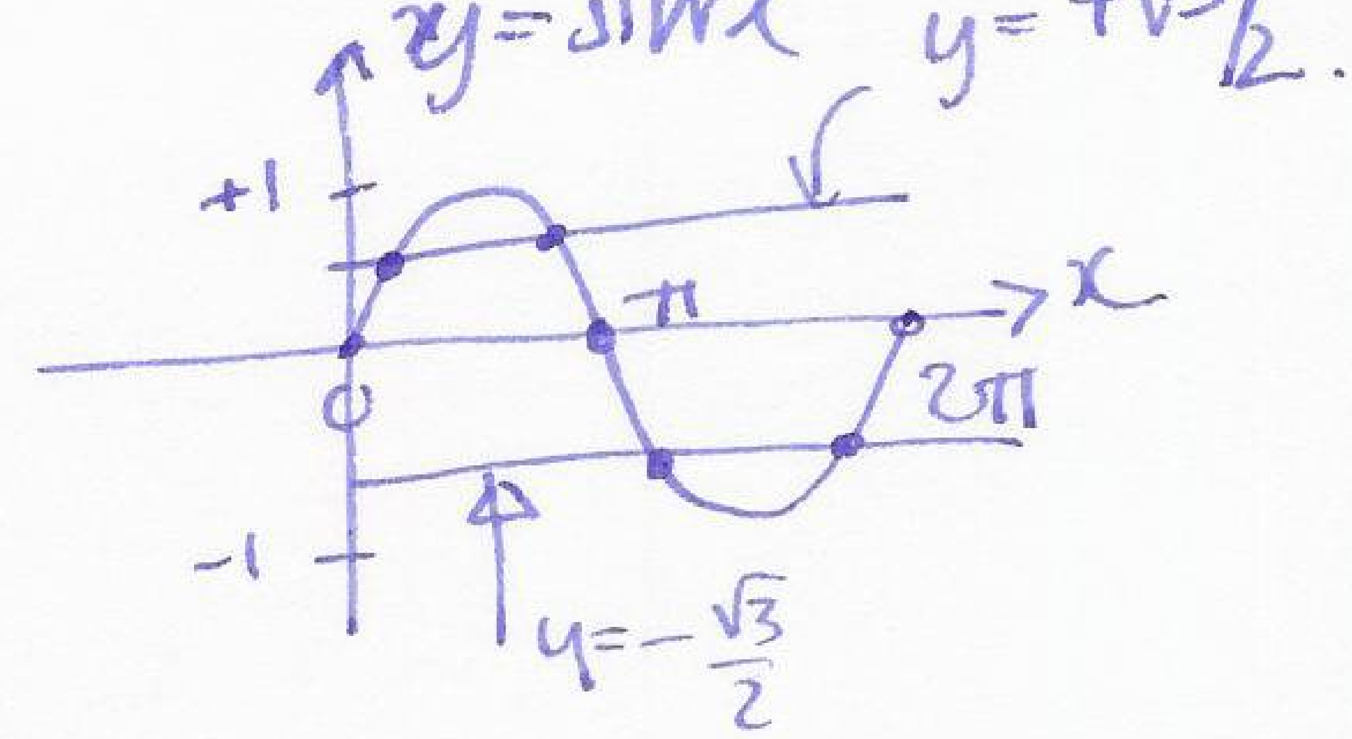
$$c^2 = 25 + 25 - 50\cos(1.34) \simeq 38.56 \quad \text{so} \quad c \simeq 6.21.$$

Q6 $\sin\left(\tan^{-1}(x/2)\right)$ $\quad \sin\theta \quad \theta = \tan^{-1}(x/2)$.

$\tan\theta = x/2$



$$\sin\theta = \frac{x}{\sqrt{4+x^2}}.$$



Q7 $\sin^2 x = \frac{3}{4}$ $\qquad \sin x = \pm \frac{\sqrt{3}}{2}$

$\sin^{-1}(\sqrt{3}/2) = \frac{\pi}{3}$. so solutions are

$\frac{\pi}{3}, \pi - \frac{\pi}{3} = \frac{2\pi}{3}$.

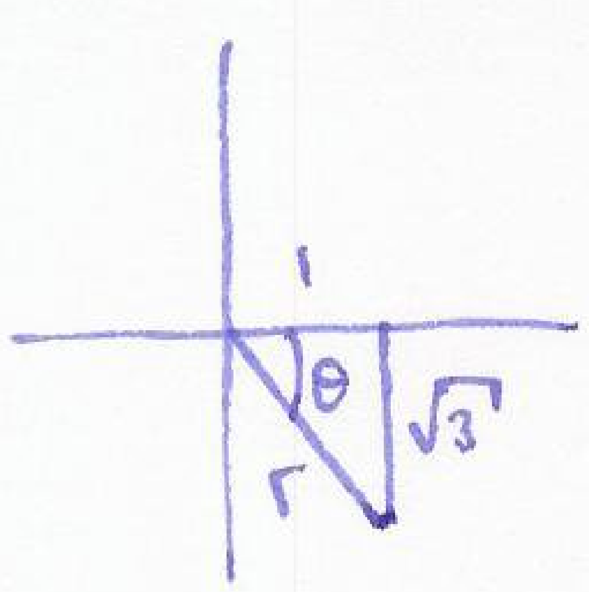
$\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) = -\frac{\pi}{3}$ so solutions are $2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$.

and $\pi + \left(2\pi - \frac{5\pi}{3}\right) = \pi + \frac{\pi}{3} = \frac{4\pi}{3}$.

so solutions are : $\boxed{\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}}$

Q8 a) $z = 1 - \sqrt{3}i = r(\cos\theta + i\sin\theta)$, where $r^2 = 1^2 + (\sqrt{3})^2 = 4$.

$\theta = \tan^{-1}(-\sqrt{3}) = -\pi/3$.



so $z = 2\left(\cos(-\pi/3) + i\sin(-\pi/3)\right)$.

b) $z^2 = (1-\sqrt{3}i)(1-\sqrt{3}i) = 1 - 2\sqrt{3}i + 3i^2 = -2 - 2\sqrt{3}i$

$z^2 = r^2(\cos 2\theta + i\sin 2\theta) = 4\left(\cos\left(-\frac{2\pi}{3}\right) + i\sin\left(-\frac{2\pi}{3}\right)\right)$

$= 4\left(-\frac{1}{2} + i \cdot -\frac{\sqrt{3}}{2}\right) = -2 - 2\sqrt{3}i$ ✓.

c) $z^{10} = r^{10}(\cos 10\theta + i\sin 10\theta)$

$= 2^{10}\left(\cos\left(-\frac{10\pi}{3}\right) + i\sin\left(-\frac{10\pi}{3}\right)\right)$

$= 1024\left(-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right) = -512 + i\,512\sqrt{3}.$