

Q1

$$\frac{\cos^3 x - \sin^3 x}{\cos x - \sin x} = \frac{\cos x(1 - \sin^2 x) - \sin x(1 - \cos^2 x)}{\cos x - \sin x}$$

$$= \frac{\cos x - \sin x - \cos x \sin^2 x + \sin x \cos^2 x}{\cos x - \sin x}$$

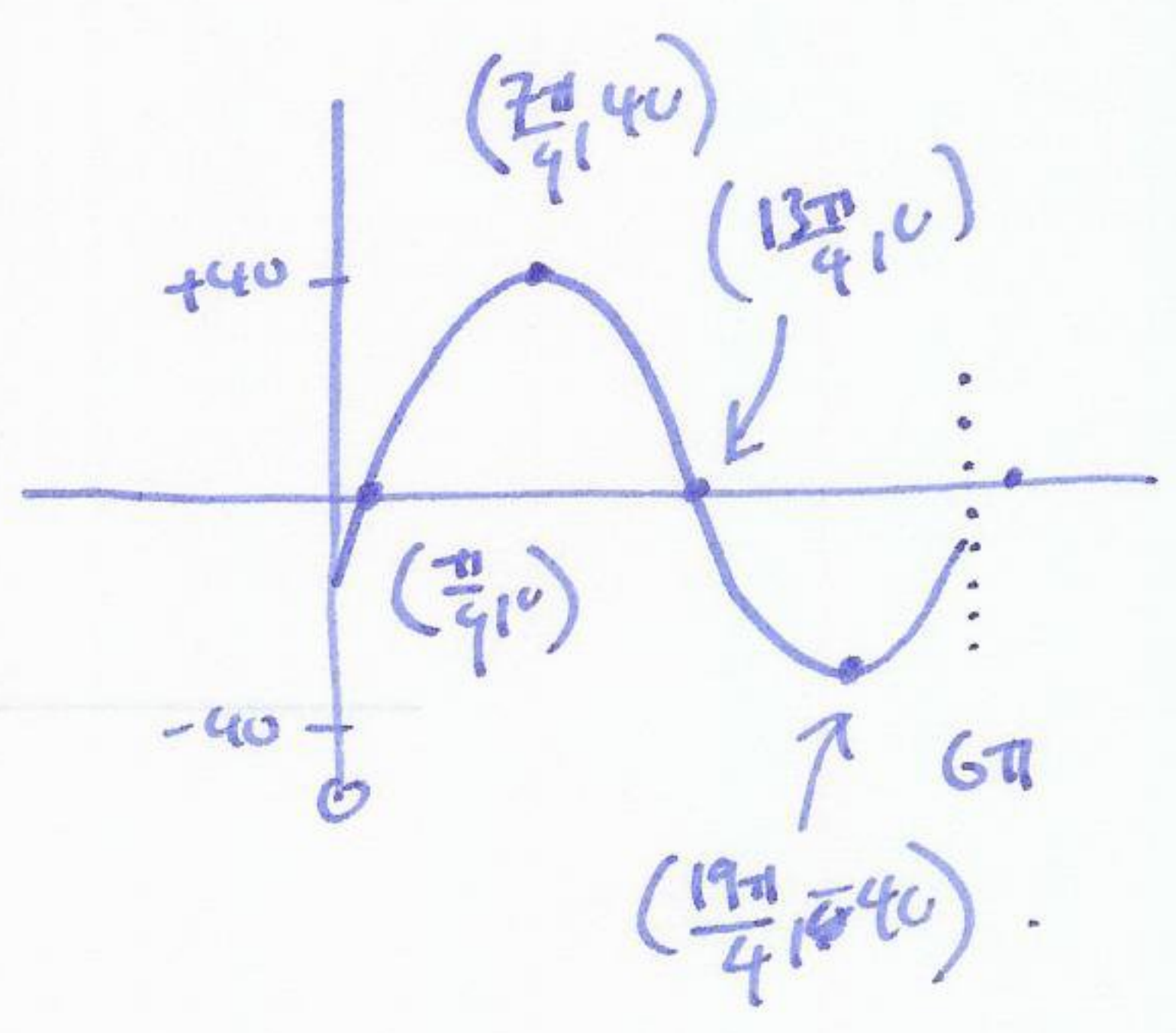
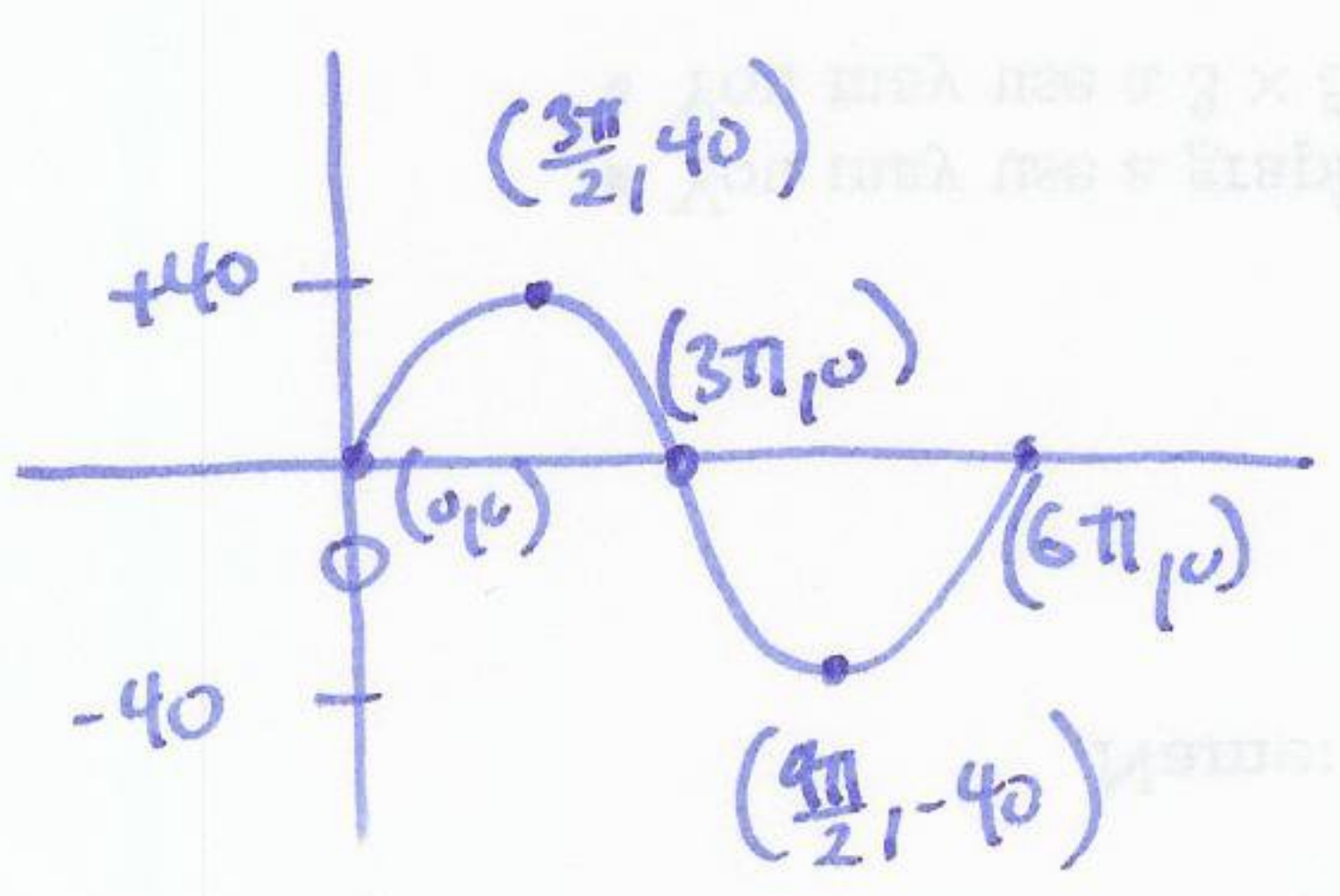
$$= 1 + \frac{\sin x \cos x (\cos x - \sin x)}{\cos x - \sin x} = 1 + \sin x \cos x$$

$$= 1 + \frac{1}{2} \sin 2x = \frac{2 + \sin 2x}{2}$$

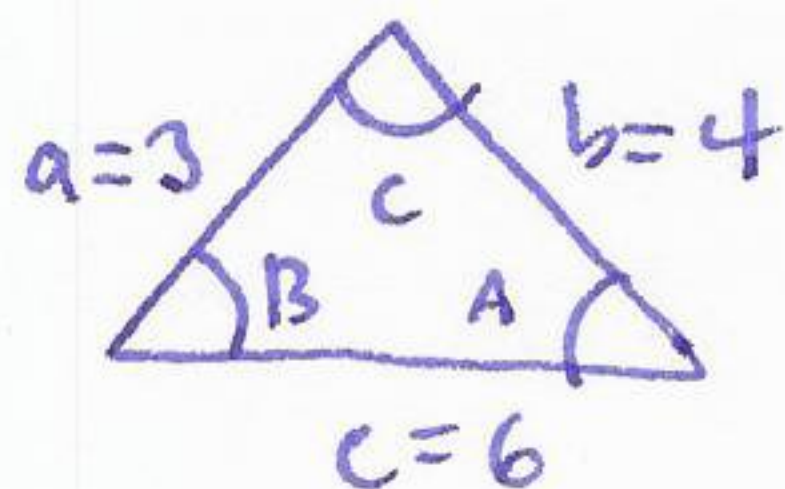
Q2 $y = 40 \sin\left(\frac{1}{3}x - \frac{\pi}{12}\right) = 40 \sin\left(\frac{1}{3}\left(x - \frac{\pi}{4}\right)\right)$ period 6π .

$y = 40 \sin\left(\frac{1}{3}x\right)$

shift $\frac{\pi}{4}$



Q3



$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$36 = 9 + 16 - 24 \cos C$$

$$\cos C = \frac{-11}{24} \approx -0.46$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

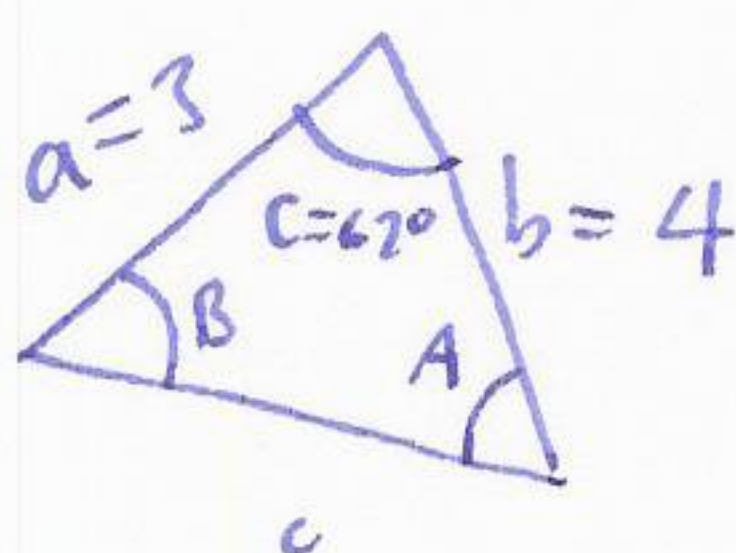
$$9 = 16 + 36 - 48 \cos A$$

$$\cos A = \frac{43}{48} \approx 0.896$$

$$B = \pi - A - C \approx 0.63$$

(2)

Q4



$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$c^2 = 9 + 16 - 24 \cos 62^\circ \approx 13.73$$

$$c \approx 3.71$$

$$\frac{\sin 62^\circ}{3.71} = 0.24$$

$$\frac{\sin A}{3} = 0.24$$

$$\sin A = 0.71$$

$$A \approx 0.79 = 45.6^\circ$$

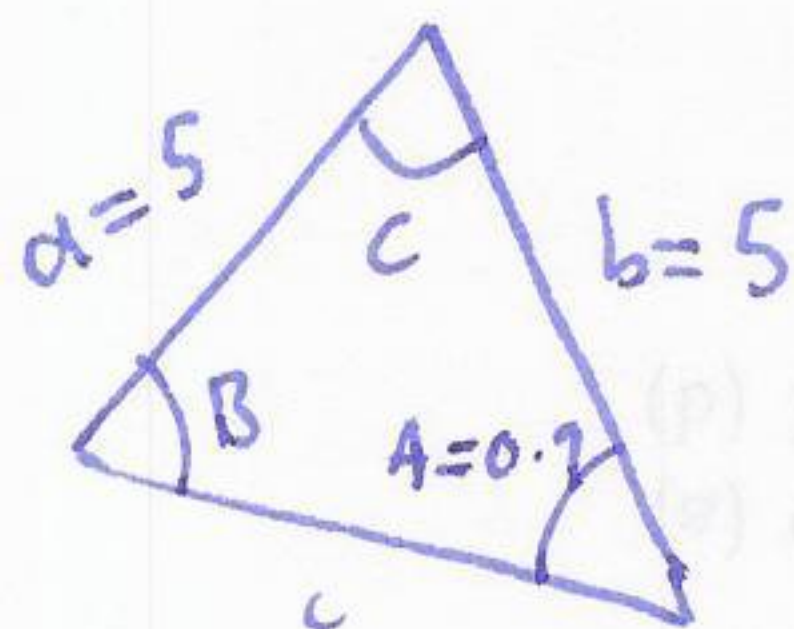
$$\text{or } \pi - 0.79 \approx 2.35$$

too big! $\rightarrow 134.4^\circ$

$$\text{so } A \approx 45.6^\circ$$

$$\text{so then } B = 180 - A - C \approx 72.4$$

Q5



~~$$a^2 = b^2 + c^2 - 2bc \cos A$$~~

~~$$25 = 25 + c^2$$~~

$$\frac{\sin A}{a} = \frac{\sin 0.9}{5} = \frac{\sin B}{5}$$

$$\Rightarrow \sin B = \sin 0.9$$

$$\text{so } B = 0.9 \text{ or } \pi - 0.9$$

too big! would imply $C = 0$

$$\text{then } C = \pi - 0.9 - 0.9 = 1.34$$

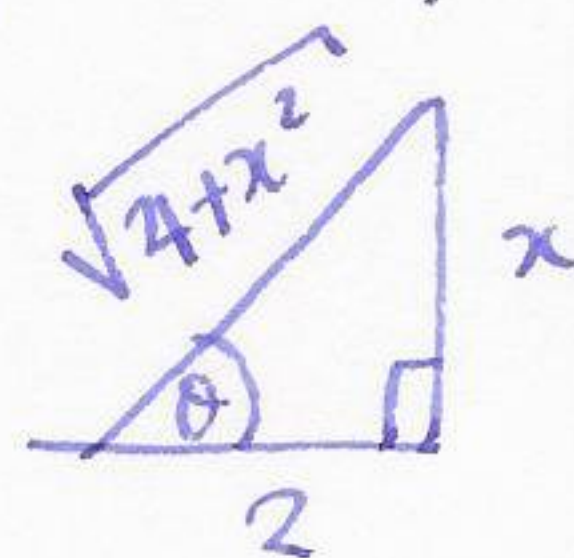
$$c^2 = 25 + 25 - 50 \cos(1.34) \approx 38.56 \text{ so } c \approx 6.21$$

Q6

$$\sin(\tan^{-1}(x/2))$$

$$\sin \theta \quad \theta = \tan^{-1}(x/2)$$

$$\tan \theta = x/2$$



$$\sin \theta = \frac{x}{\sqrt{4+x^2}}$$

(3)

Q7

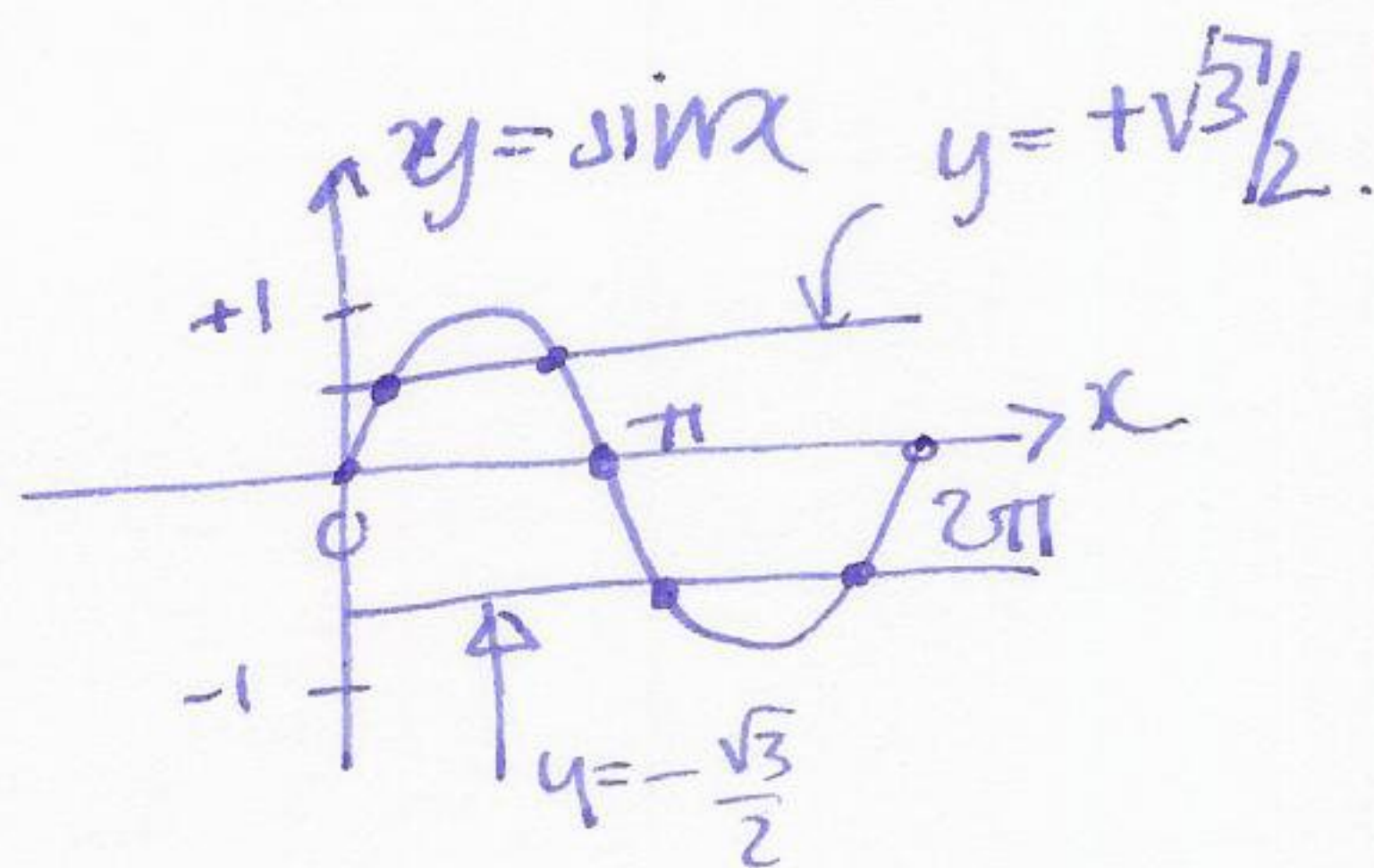
$$\sin^2 x = \frac{3}{4}$$

$$\sin x = \pm \frac{\sqrt{3}}{2}$$

$$\sin^{-1}(\sqrt{3}/2) = \frac{\pi}{3}$$

so solutions are

$$\frac{\pi}{3}, \pi - \frac{\pi}{3} = \frac{2\pi}{3}$$



$$\sin^{-1}(-\frac{\sqrt{3}}{2}) = -\frac{\pi}{3}$$

so solutions are $2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$

$$\text{and } \pi + (2\pi - \frac{5\pi}{3}) = \pi + \frac{\pi}{3} = \frac{4\pi}{3}$$

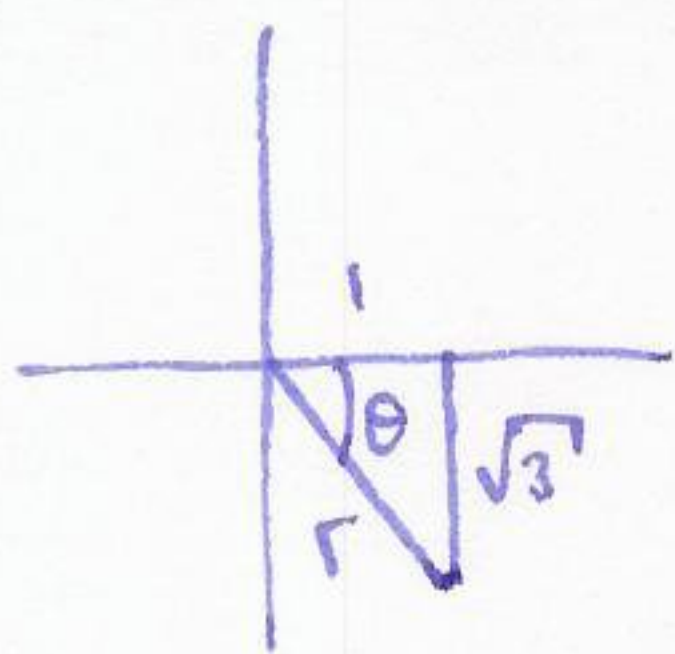
so solutions are :

$$\boxed{\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}}$$

$$\text{Q8 a) } z = 1 - \sqrt{3}i = r(\cos \theta + i \sin \theta)$$

$$\text{where } r^2 = 1^2 + (\sqrt{3})^2 = 4$$

$$\theta = \tan^{-1}(\sqrt{3}) = -\frac{\pi}{3}$$



so

$$z = 2 \left(\cos(-\frac{\pi}{3}) + i \sin(-\frac{\pi}{3}) \right)$$

$$\text{b) } z^2 = (1 - \sqrt{3}i)(1 - \sqrt{3}i) = 1 - 2\sqrt{3}i + 3i^2 = -2 - 2\sqrt{3}i$$

$$z^2 = r^2(\cos 2\theta + i \sin 2\theta) = 4 \left(\cos(-\frac{2\pi}{3}) + i \sin(-\frac{2\pi}{3}) \right)$$

$$= 4 \left(-\frac{1}{2} + i \cdot -\frac{\sqrt{3}}{2} \right) = -2 - 2\sqrt{3}i \quad \checkmark$$

$$\begin{aligned}
 c) \quad z^{10} &= r^{10} (\cos 10\theta + i \sin 10\theta) \\
 &= 2^{10} \left(\cos\left(-\frac{10\pi}{3}\right) + i \sin\left(-\frac{10\pi}{3}\right) \right) \\
 &= 1024 \left(-\frac{1}{2} + i \frac{\sqrt{3}}{2} \right) = -512 + i 512\sqrt{3} .
 \end{aligned}$$

- (a) Factor the cubic of b
- (b) Write b as a product of linear factors
- (c) Find all remaining zeros
- (d) Check using either long division or synthetic division that $x = 3$ is a zero
- (e) State a complete list of all complex zeros
- (f) (50 points) For $b(x) = 3x^2 + 12x - 27x - 8$