Math 130 Precalculus Spring 10 Midterm 2b

Name: Solutions

- You may use a graphing calculator.
- You may use a 3×5 index card of notes.

1	10	
2	10	
3	10	
4	10	
5	15	
6	15	
7	15	
8	15	
	100	

(1) (10 points) Let $f(x) = -4x^2 + 2x - 7$ and $g(x) = 2x - 3$. Compute and simplify $(f \circ g)(x)$.

$$f(g(x)) = -4(2x-3)^2 + 2(2x-3) - 7$$

$$= -4(4x^2 - 12x + 9) + 4x - 6 - 7$$

$$= -16x^2 + 52x - 49$$

(2) (10 points) If $f(x) = \frac{-2x}{x+7}$, find and simplify $f^{-1}(x)$=?

$y = \frac{-2x}{x+7}$ swap xy : $x = \frac{-2y}{y+7}$

$x(y+7) = -2y$

$xy + 7x = -2y$

$7x = -xy - 2y$

$-7x = y(x+2)$

$y = \frac{-7x}{x+2}$ $f^{-1}(x) = \frac{-7x}{x+2}$

(3) (10 points) Show the following identity:

$$\sin^3(x) = \sin(x) - \tfrac{1}{2}\sin(2x)\cos(x)$$

$\sin x - \frac{1}{2}\sin 2x \cos x = \sin x - \frac{1}{2}(2\sin x \cos x)\cos x$ (double angle)

$= \sin x - \sin x \cos^2 x$

$= \sin x (1 - \cos^2 x)$ (Pythagorean identity)

$= \sin x \cdot \sin^2 x$

$= \sin^3 x$

(4) (10 points) Show the following identities:

(a) $(\cos x + \sin x)(\cos x - \sin x) = \cos 2x$

(b)

$$\frac{2\sin x}{\sin 2x} = \sec x$$

a) $(\cos x + \sin x)(\cos x - \sin x) = \cos^2 x - \sin^2 x$

$= \cos 2x$ (double angle formula).

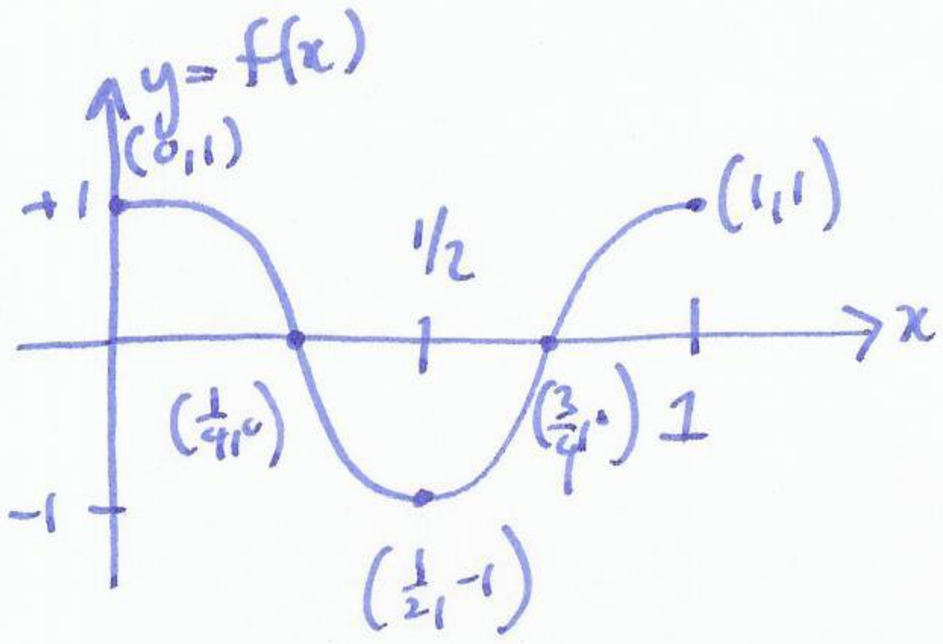
b) $\dfrac{2\sin x}{\sin 2x} = \dfrac{2\sin x}{2\sin x \cos x}$ (double angle formula)

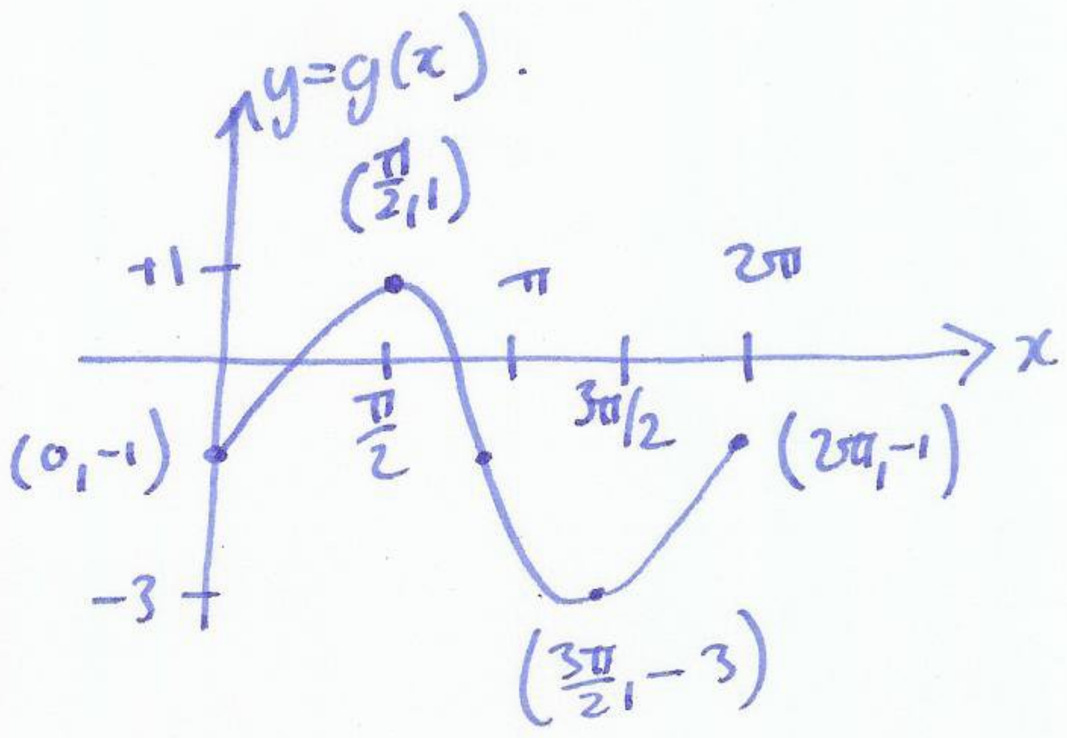
$= \dfrac{1}{\cos x}$

$= \sec x$

(5) (10 points) Sketch one period of the graphs of the following functions.

$$f(x) = \cos(2\pi x), \qquad g(x) = 2\sin(x) - 1$$





(6) (25 points) Consider the function $f(x) = -5\sin\left(2x + \frac{\pi}{2}\right)$.

(a) What is the amplitude?

(b) What is the period?

(c) What is the phase shift?

(d) Draw a picture of the graph for one period, labelling the x-intercepts and the maximum and minimum.

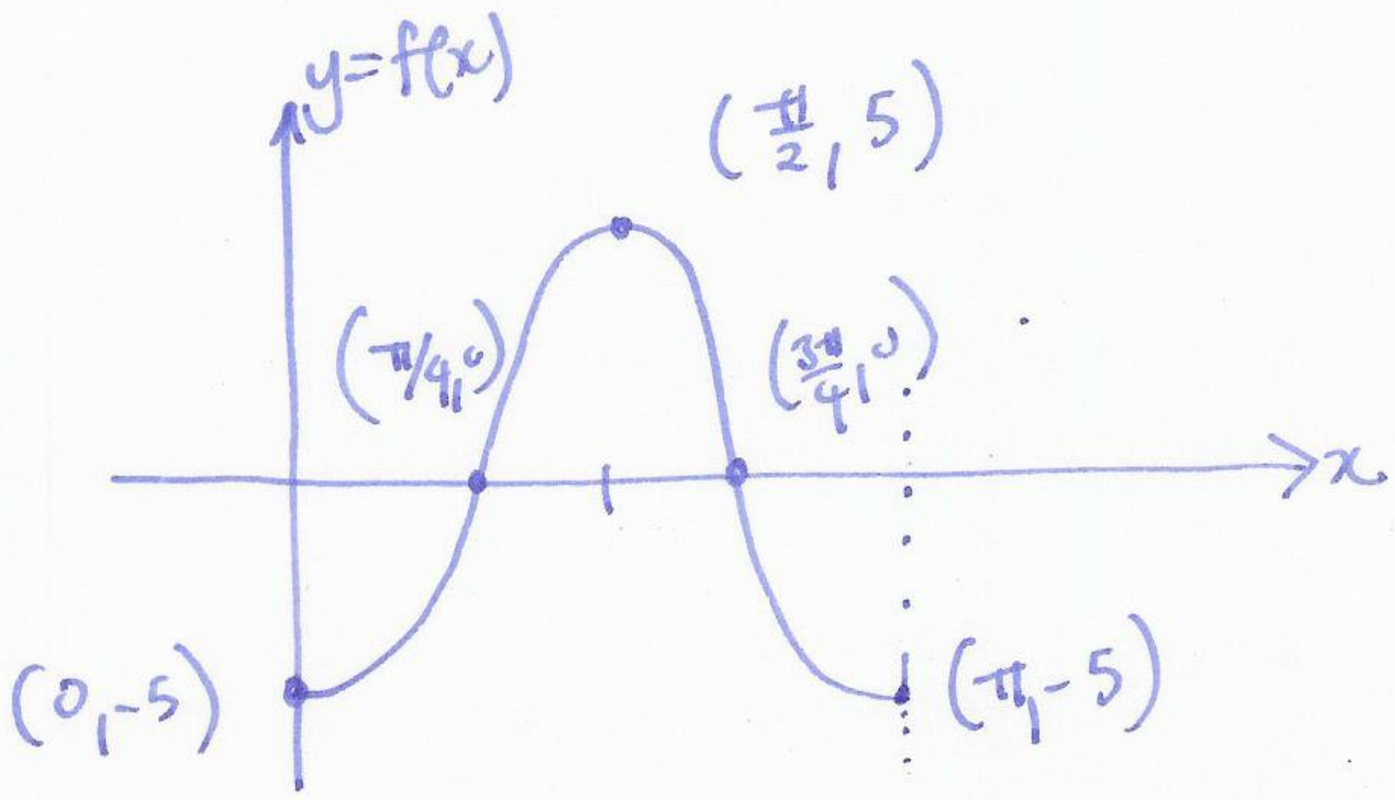
$f(x) = -5\sin\left(2\left(x + \frac{\pi}{4}\right)\right)$

a) amplitude $= 5$

b) period $= \frac{2\pi}{2} = \pi$

c) phase shift $-\pi/4$

d)



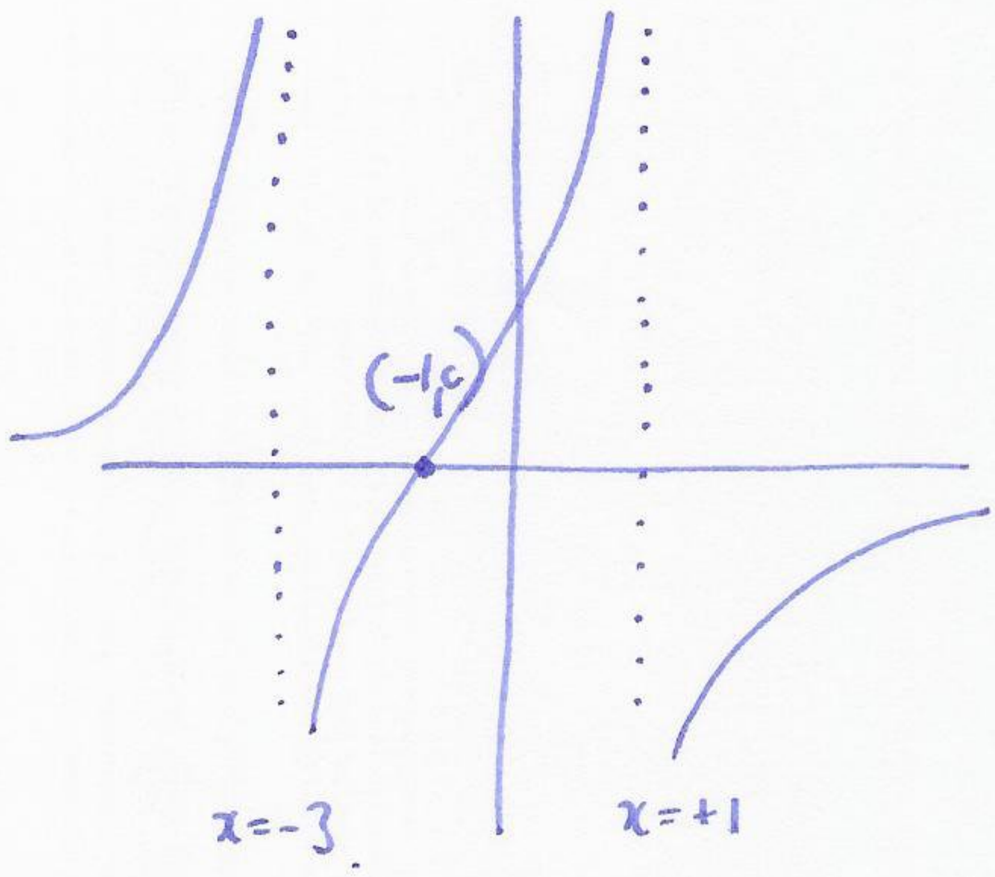
(7) (15 points) Solve the inequality $\frac{x+4}{x-1} \leqslant \frac{x-2}{x+3}$. Write your answer in interval notation.

$$\frac{x-2}{x+3} - \frac{x+4}{x-1} \geqslant 0 \qquad \frac{(x-2)(x-1)-(x+4)(x+3)}{(x+3)(x-1)} \geqslant 0$$

$$\frac{x^2-3x+2 \;\; -x^2-7x-12}{(x+3)(x-1)} \geqslant 0 \qquad \frac{-10x-10}{(x+3)(x-1)} \geqslant 0$$

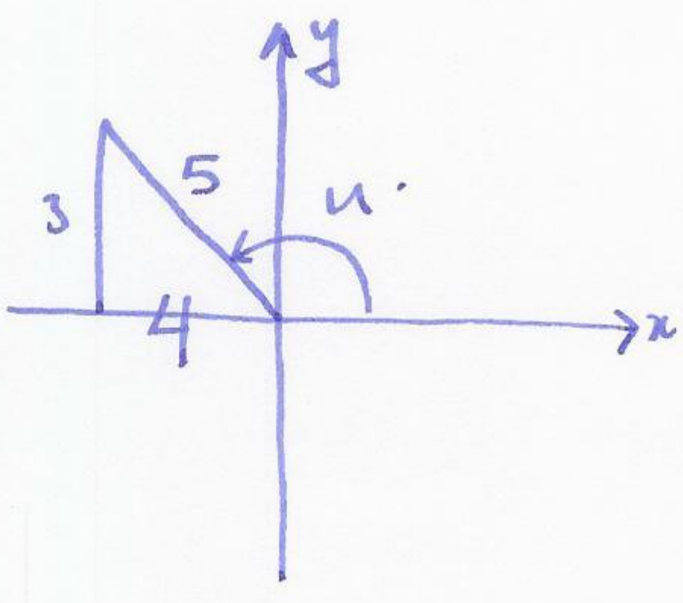
$$\frac{-10(x+1)}{(x+3)(x-1)} \geqslant 0.$$

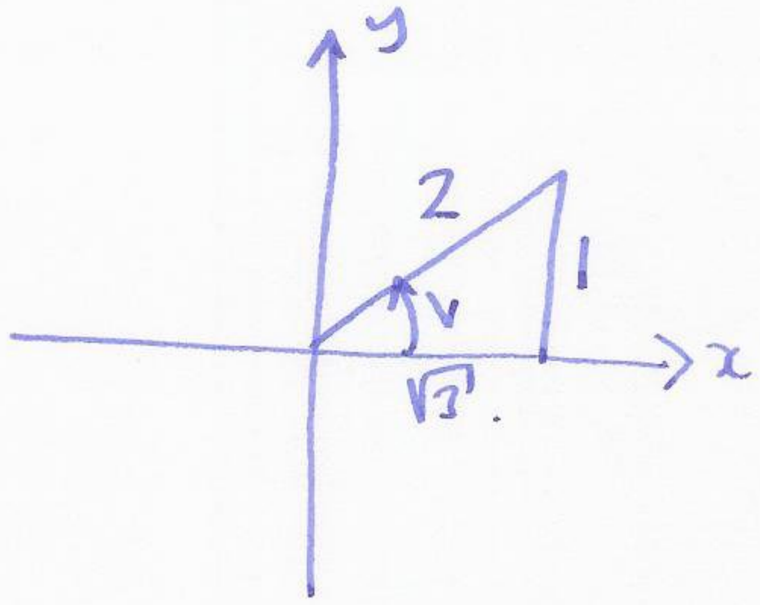
$x \geqslant 1$	$-/+ \; +$	$-$
$-1 \leqslant x < 1$	$-/+ \; -$	$+$
$-3 < x \leqslant -1$	$+/+ \; -$	$-$
$-3 < x$	$+/- \; -$	$+$



$$(-\infty, -3) \cup [-1, 1)$$

(8) (15 points) If $\cos(u) = -\frac{4}{5}$, (u in quadrant II), and $v = \frac{1}{2}$, (v in quadrant I), use properties of right angled triangles and suitable identities to find the values of the following: (a) $\sin(u)$, (b) $\cos(v)$, and (c) $\cos(u+v)$. Write all answers as fractions, possibly involving square roots.





a) $\sin(u) = \frac{3}{5}$

b) $\cos(v) = \frac{\sqrt{3}}{2}$

c) $\cos(u+v) = \cos(u)\cos(v) - \sin(u)\sin(v)$

$= -\frac{4}{5}\frac{\sqrt{3}}{2} - \frac{3}{5}\frac{1}{2} = \frac{-4\sqrt{3}-3}{10}$