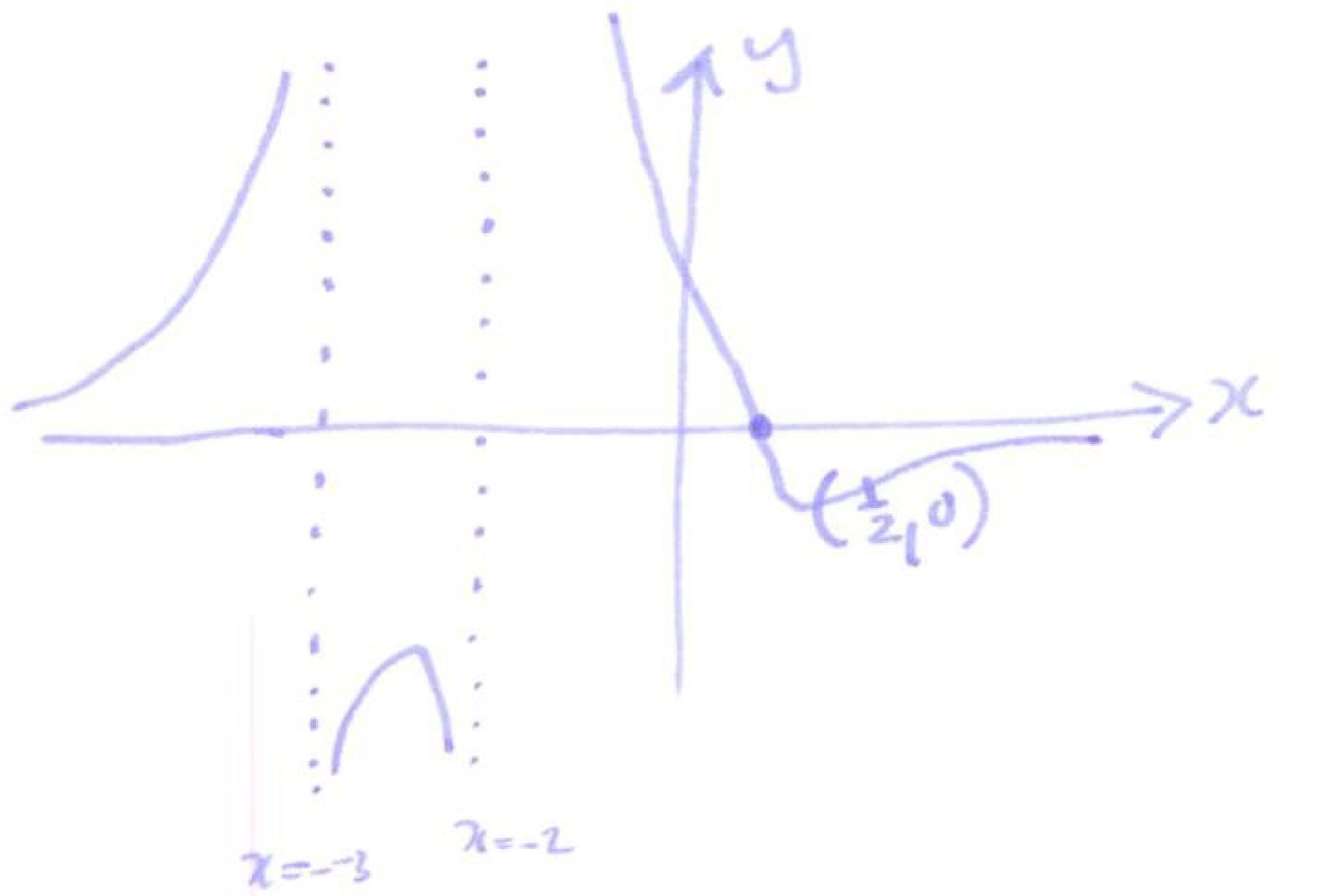
Math 130 Pre Calculus Sample midterm solutions

Q1 $\dfrac{x-3}{x+2} \leq \dfrac{x-4}{x+3}$ same as $\dfrac{x-4}{x+3} - \dfrac{x-3}{x+2} \geq 0$

$$\frac{(x-4)(x+2)-(x-3)(x+3)}{(x+3)(x+2)} \geq 0 \qquad \frac{x^2-2x-8-x^2+9}{(x+3)(x+2)} \geq 0$$

$$\frac{-2x+1}{(x+3)(x+2)} \geq 0$$



$x > \frac{1}{2}$	$-/+\,+$	$-$
$-2 < x < \frac{1}{2}$	$+/+\,+$	$+$
$-3 < x < -2$	$+/+\,-$	$-$
$x < -3$	$+/-\,-$	$+$

solution: $(-\infty,-3) \cup (-2, \frac{1}{2}]$

Q2 $f(x) = -3x^2-6x+1 \qquad g(x) = 2x-3$

$$f\circ g(x) = f(g(x)) = f(2x-3) = -3(2x-3)^2-6(2x-3)+1$$

$$= -3(4x^2-12x+9)-12x+18+1$$

$$= -12x^2+24x-8$$

Q3 $f(x) = \frac{2x}{x-3}$ $\quad y = \frac{2x}{x-3}$ $\quad$ swap x,y : $x = \frac{2y}{y-3}$

solve for y: $\quad xy - 3x = 2y$

$y(x-2) = 3x \qquad y = \frac{3x}{x-2}$

so $f^{-1}(x) = \frac{3x}{x-2}$

Q4 $\sin(x)\cos^2(x) + \cos(x)\cot(x) = \sin(x)\left[1-\sin^2 x\right] + \frac{\cos^2 x}{\sin x}.$

$= -\sin^3 x + \sin x + \frac{\cos^2 x}{\sin x}$

$= -\sin^3 x + \frac{1}{\sin x}\left(\cos^2 x + \sin^2 x\right) = -\sin^3 x + \frac{1}{\sin x}.$

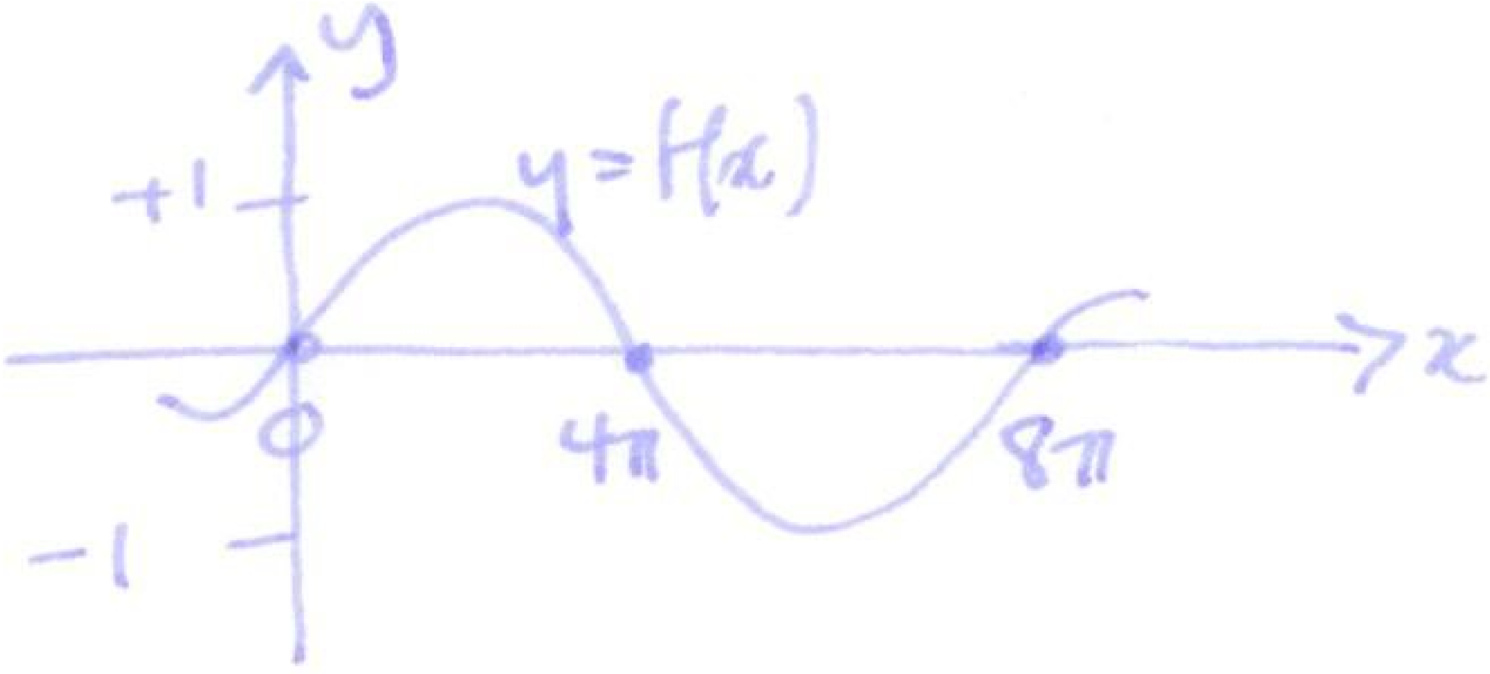
Q5 a) $2\sin^2 x + \cos 2x = 2\sin^2 x + \cos^2 x - \sin^2 x$

$= \sin^2 x + \cos^2 x = 1.$

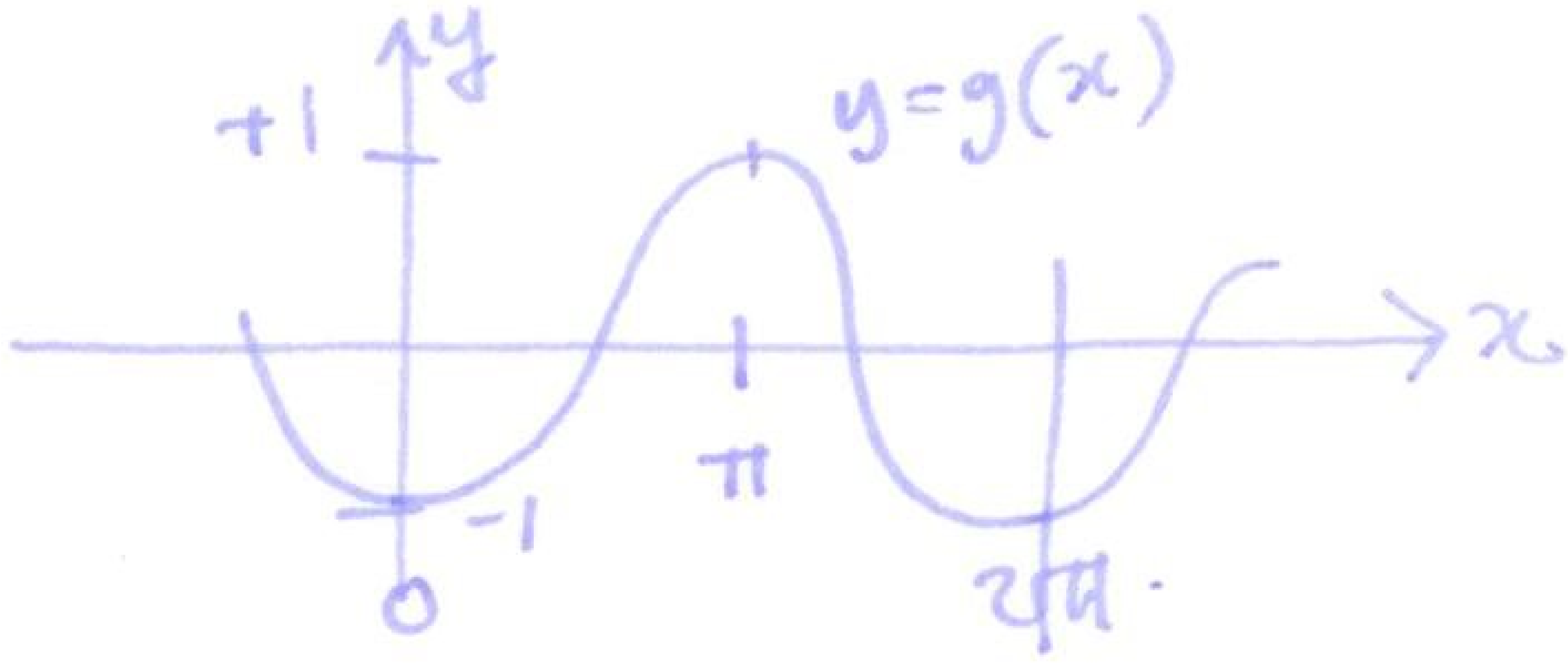
b) $(\sin x + \cos x)^2 = \sin^2 x + 2\sin x\cos x + \cos^2 x = 1 + 2\sin x\cos x$

$= 1 + \sin 2x.$

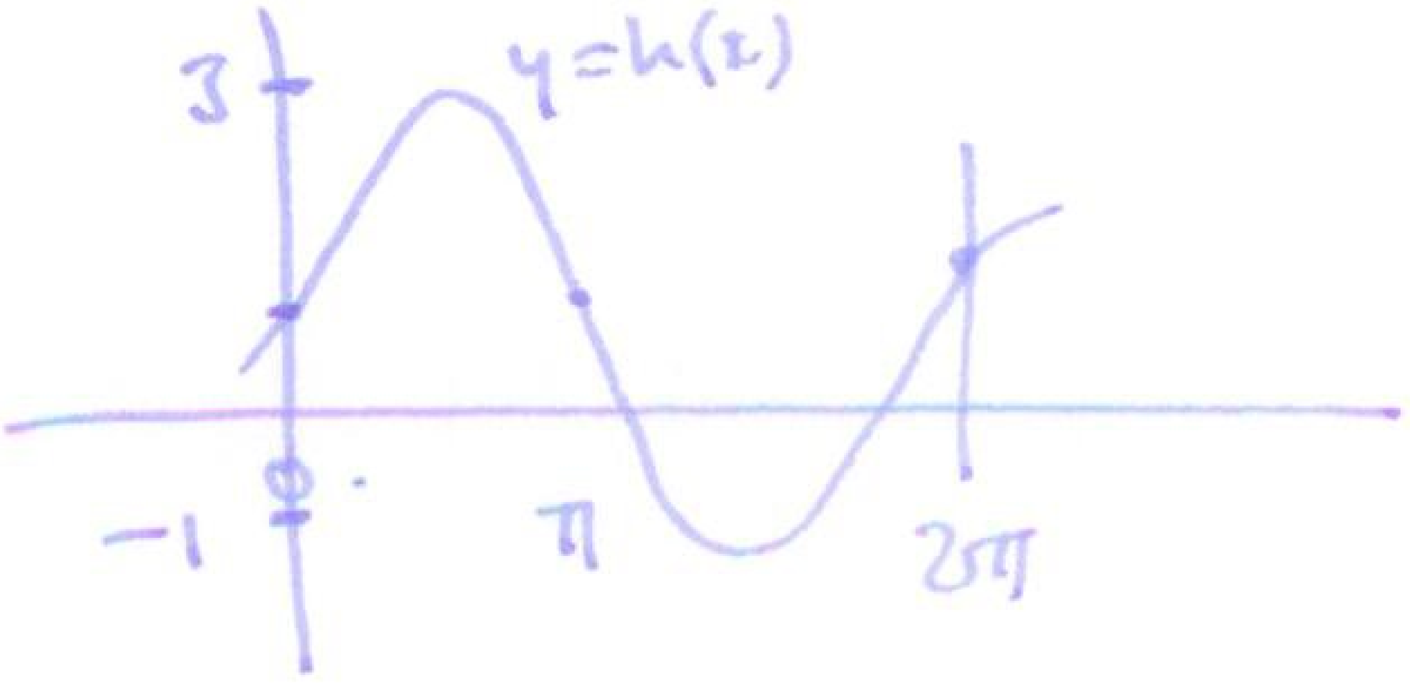
Q6 $f(x) = \sin(\frac{1}{4}x)$



$g(x) = \cos(x-\pi)$

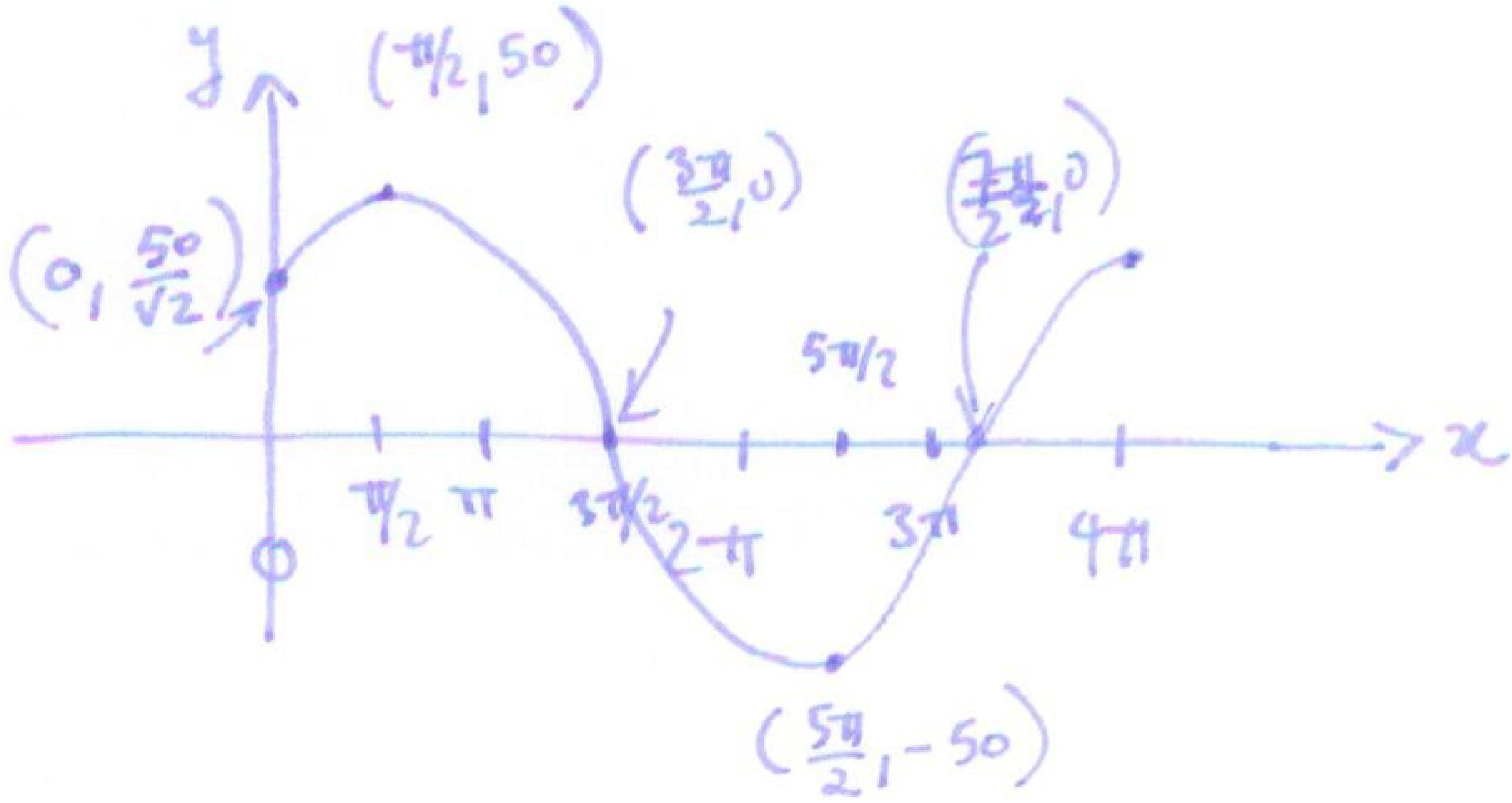


$h(x) = 2\sin(x)+1$



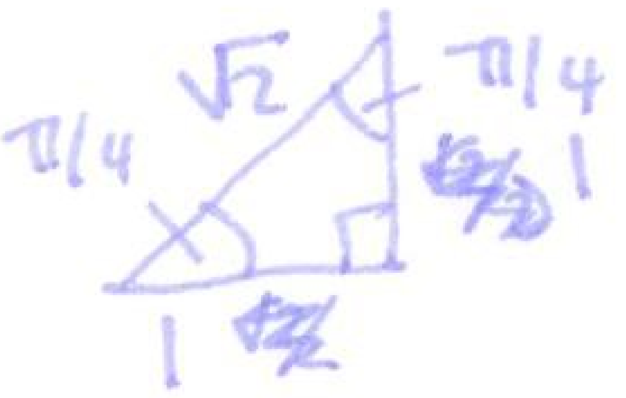
Q7 $y = 50\cos(\frac{1}{2}x - \frac{\pi}{4})$

$y = 50\cos(\frac{1}{2}(x - \frac{\pi}{2}))$



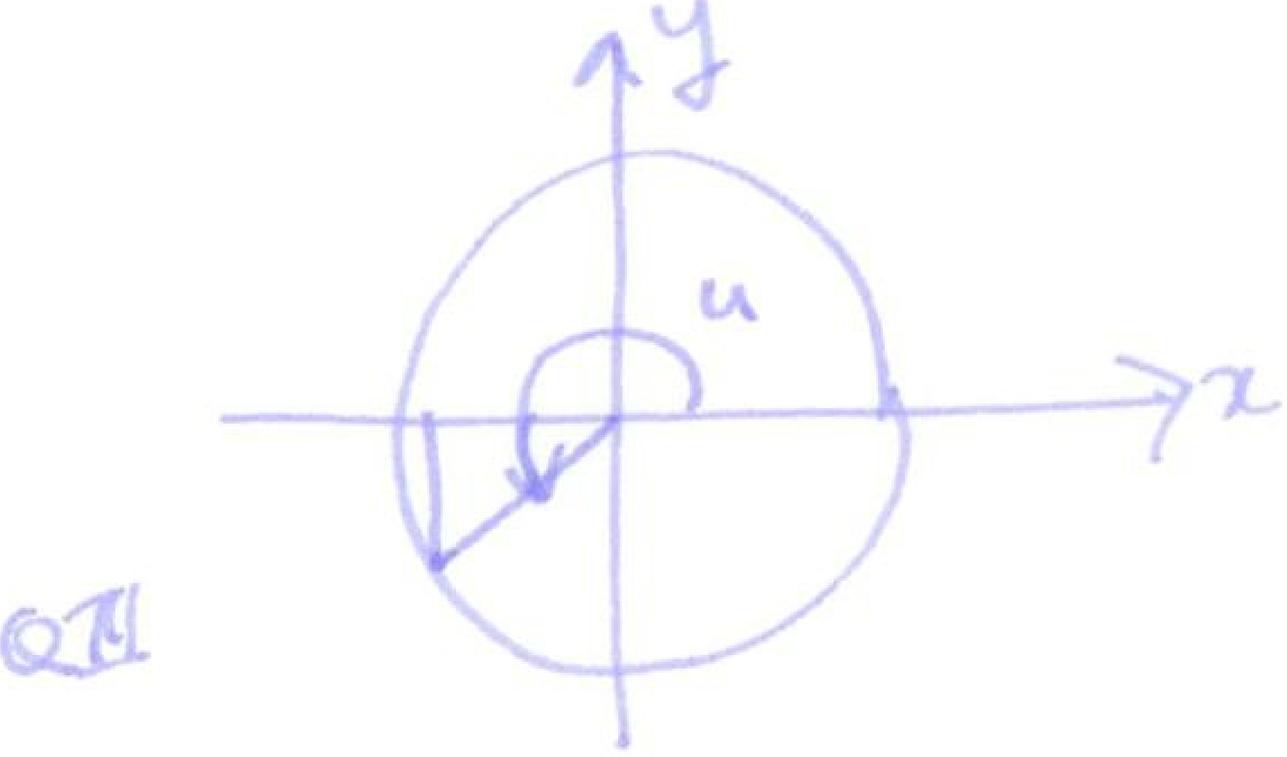
$x=0$: $y = 50\cos(-\frac{\pi}{4})$

$= 50\cos(\frac{\pi}{4})$

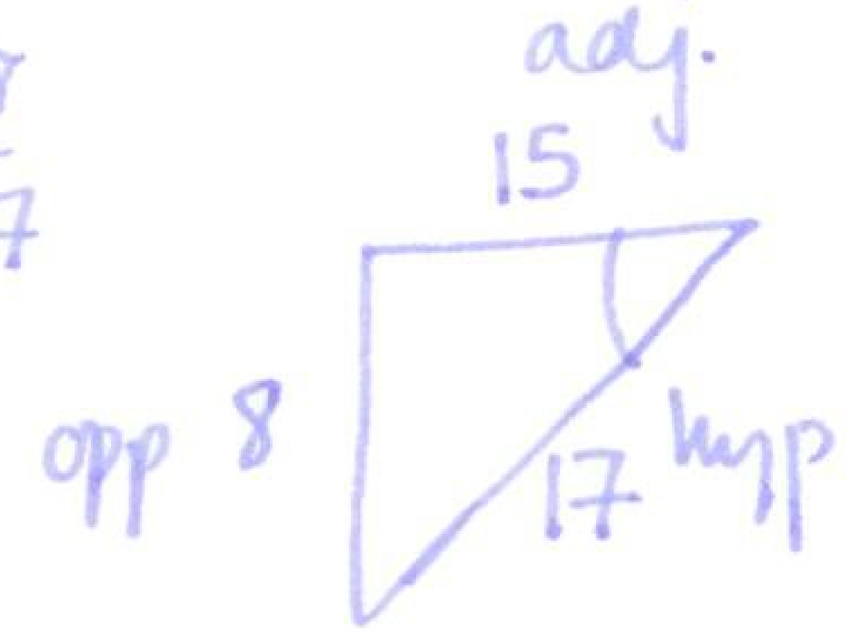


$\cos(\frac{\pi}{4}) = \frac{1}{\sqrt{2}}$.

Q8



$\sin(u) = \frac{-8}{17}$



$\cos(u) = \frac{-15}{17}$

a) $\sin(2u) = 2\sin(u)\cos(u) = 2 \cdot \frac{-8}{17} \cdot \frac{-15}{17} = \frac{240}{17^2} = \frac{240}{289}$

b) $\cos(2u) = \cos^2 u - \sin^2 u = \left(\frac{15}{17}\right)^2 - \frac{8^2}{17} = \frac{225-64}{17^2} = \frac{161}{289}$

c) $\tan(2u) = \frac{\sin 2u}{\cos 2u} = \frac{240}{289} \cdot \frac{289}{161} = \frac{240}{161}$

2u is in quadrant I

because ① $\sin 2u$, $\cos 2u$ both +ve.

② $\pi \leq u \leq \pi + \frac{\pi}{4}$ as opp < adj.