Math 130 Precalculus Spring 10 Midterm 1b

Name: Solutions

- You may use a graphing calculator.
- You may use a 3 × 5 index card of notes.

1	10	
2	10	
3	10	
4	10	
5	10	
6	20	
7	20	
8	15	
	105	

(1) (10 points) Given $u = 3 + 4i$ and $v = 5 - 2i$, compute $u + v$, uv and u/v.

$u+v = 8+2i$

$uv = (3+4i)(5-2i) = 15-6i+20i-8i^2 = 23+14i$

$$\frac{u}{v} = \frac{(3+4i)}{(5-2i)}\frac{(5+2i)}{(5+2i)} = \frac{15+6i+20i+8i^2}{25+4} = \frac{7}{29} + \frac{26}{29}i$$

(2) (10 points)

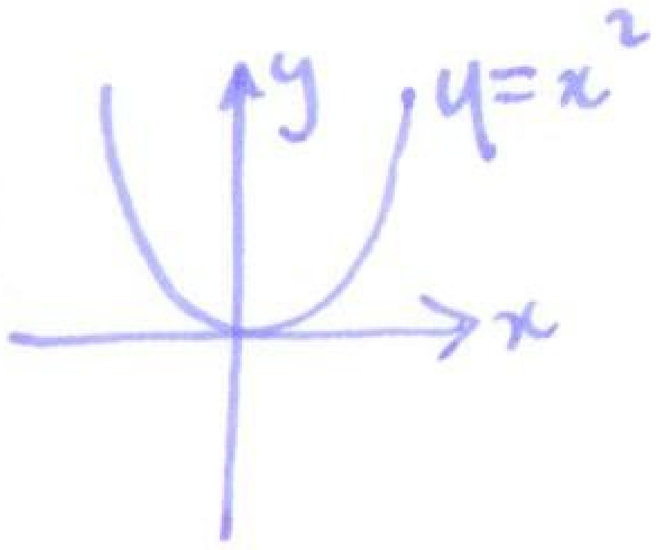
(a) What is the maximum number of distinct roots a cubic polynomial can have? Write down a cubic polynomial with this number of roots.

(b) What is the smallest number of distinct roots a cubic polynomial can have? Write down a cubic polynomial with this number of roots.

a) 3 $(x-1)(x-2)(x+3)$

b) 1 x^3

(3) (10 points) Find the equation of the graph obtained by taking the graph of $f(x) = x^2$ and shifting it 4 units to the right, then reflecting it across the y-axis, and finally shifting it up 6 units.



shift 4 units to the right : $(x-4)^2$

reflect across y-axis : $(-x-4)^2$

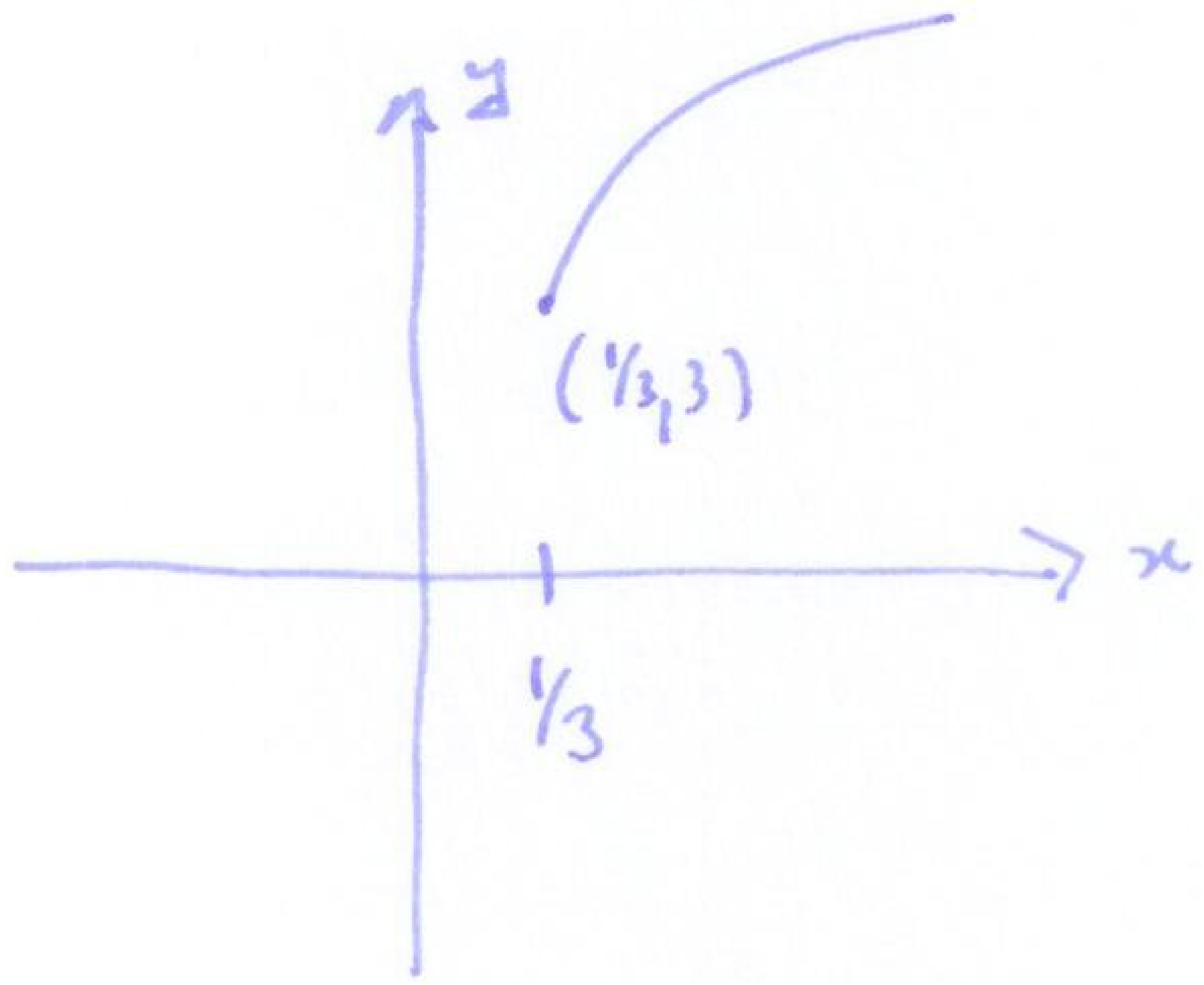
shift up 6 units : $(-x-4)^2+6$

(4) (10 points) Find the domain and the range of the following function and sketch its graph.

$$f(x) = 3 + \sqrt{3x - 1}$$

domain : $x \geq \frac{1}{3}$ or $[1/3, \infty)$

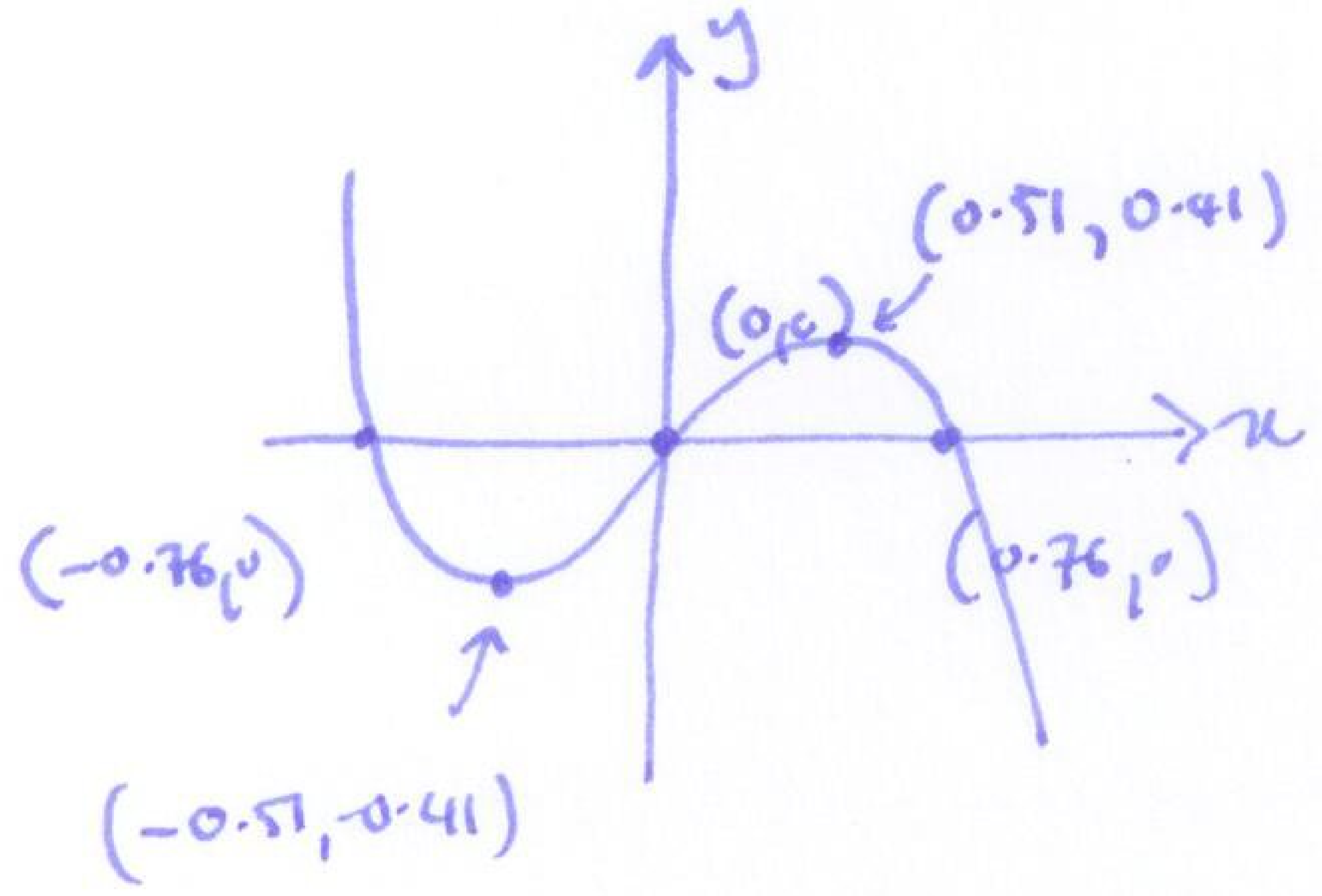
range : $x \geq 3$ or $[3, \infty)$



(5) (10 points) Consider $f(x) = x - 3x^5$. Check f algebraically for symmetries. Graph f using the calculator and find (using the calculator) all zeros, local maxima and local minima, if any.

$f(-x) = (-x) - 3(-x)^5 = -x + 3x^5 \neq f(x)$ not even

$f(-x) = -f(x)$ f is odd



(6) (20 points) Let $p(x) = 3x^3 + 7x^2 - 22x - 8$.

(a) Give a complete list of all possible rational zeros.

(b) Check, using either long division or synthetic division, that $x = 2$ is a rational zero.

(c) Find all remaining zeros.

(d) Write p as a product of linear factors.

(e) Sketch the graph of p.

a) $\pm 1, \pm 2, \pm 4, \pm 8, \pm \frac{1}{3}, \pm \frac{2}{3}, \pm \frac{4}{3}, \pm \frac{8}{3}$.

b)

$$\begin{array}{r|l} & 3x^2 + 13x + 4 \\ \hline x-2 & 3x^3 + 7x^2 - 22x - 8 \\ & 3x^3 - 6x^2 \\ \hline & \quad 13x^2 - 22x - 8 \\ & \quad 13x^2 - 26x \\ \hline & \qquad 4x - 8 \\ & \qquad 4x - 8 \\ \hline & \qquad\quad 0 \end{array}$$

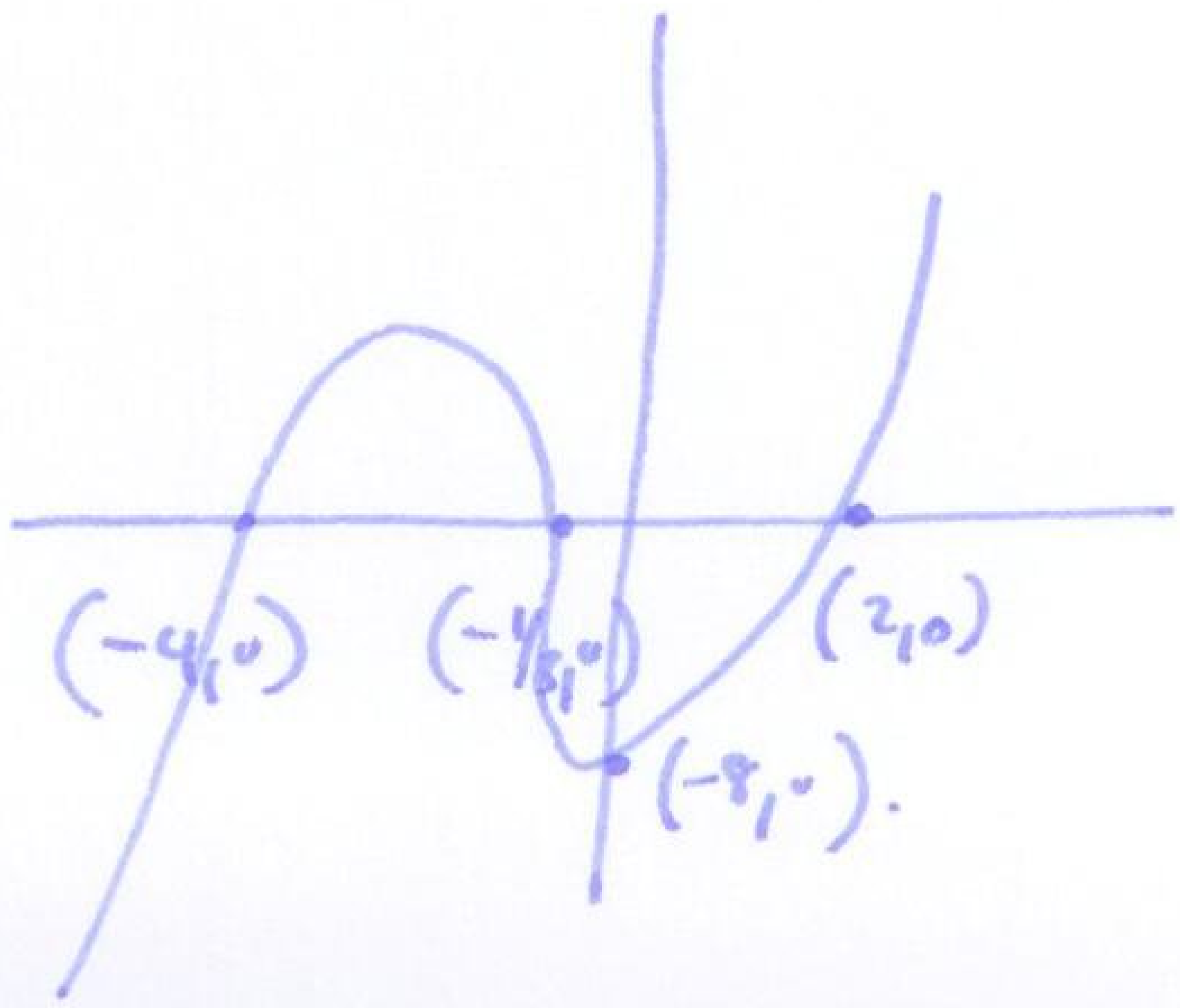
$x = 2$ is a zero.

c) $3x^2 + 13x + 4 = (3x+1)(x+4)$

zeros are $2, -4, -1/3$.

d) $p(x) = (x-2)(3x+1)(x+4)$

e) $p(0) = -8$



(7) (20 points) Consider the function

$$f(x) = \frac{x+2}{3x^2 - 9x + 6}$$

(a) Find the domain of f and the vertical asymptotes.
(b) Find the horizontal asymptote of f, if it has one.
(c) Find the zeros of f and the value of $f(0)$.
(d) Sketch the graph of f.

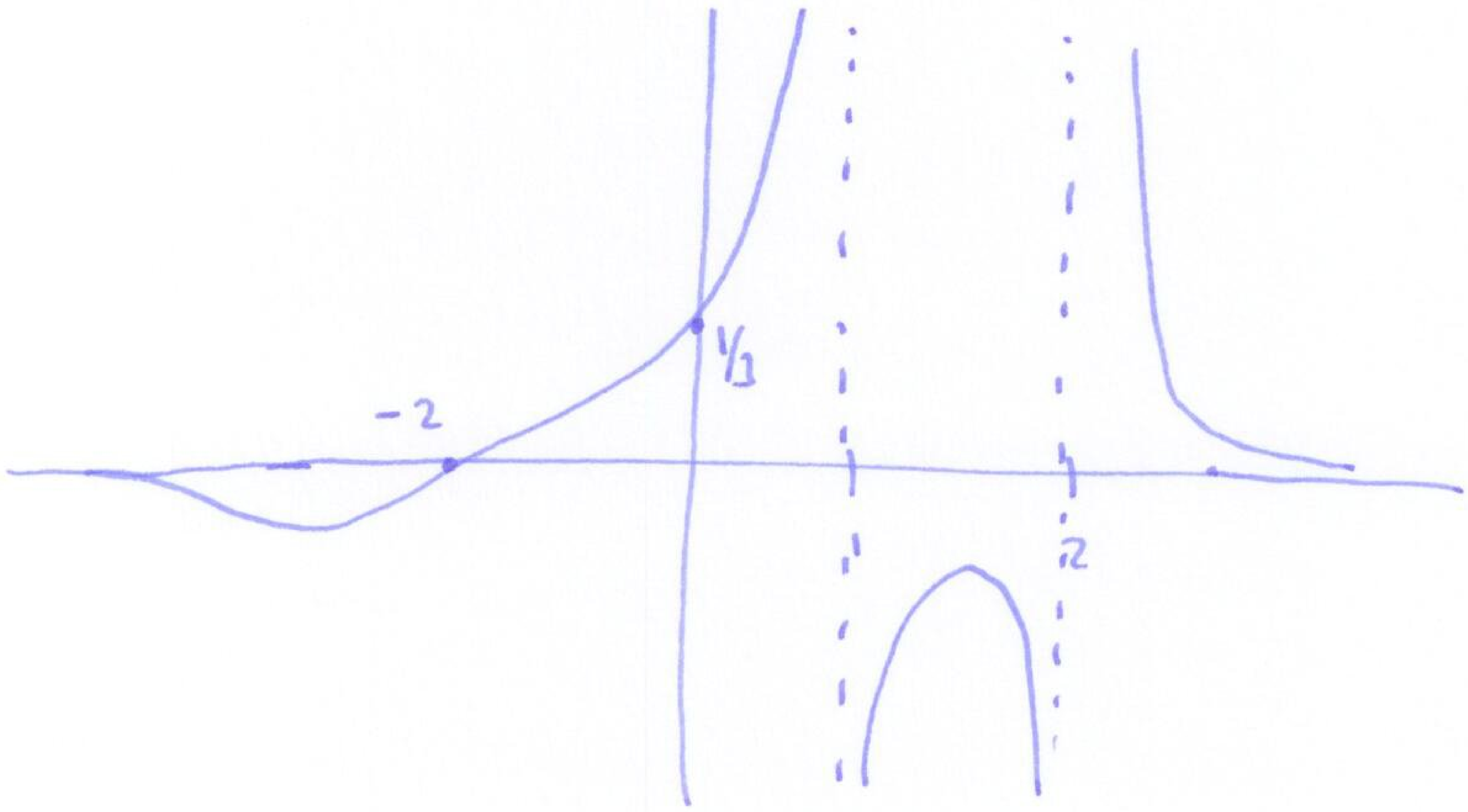
a) $3x^2 - 9x + 6 = 3(x^2 - 3x + 2) = 3(x-2)(x-1)$

domain: $\mathbb{R} \setminus \{1,2\}$ or $(-\infty,1) \cup (1,2) \cup (2,\infty)$

b) $f(x) = \frac{p}{q}$ with $\deg p < \deg q \Rightarrow$ $y=0$ is horizontal asymptote.

c) $f(x) = 0$ at $x = -2$ $\quad f(0) = 1/3$

d)



(8) (15 points) Consider the polynomials p and q given by

$$p(x) = 2x^4 + 3x^3 + 6x^2 + 12x - 8, \qquad q(x) = x^2 + 4$$

(a) Calculate $\frac{p}{q}$ using long division.

(b) Find all real and complex zeros of p.

a)

$$\begin{array}{r|l} & 2x^2 + 3x - 2 \\ \hline x^2+4 & 2x^4 + 3x^3 + 6x^2 + 12x - 8 \\ & 2x^4 \quad\quad + 8x^2 \\ \hline & \quad 3x^3 - 2x^2 + 12x - 8 \\ & \quad 3x^3 \quad\quad + 12x \\ \hline & \quad\quad -2x^2 \quad -8 \\ & \quad\quad -2x^2 \quad -8 \\ \hline & \quad\quad\quad 0 \end{array}$$

$\frac{p}{q} = 2x^2 + 3x - 2$

b) ~~roots~~ : $+2i, -2i, -2, \frac{1}{2}$

$2x^2 + 3x - 2 = (x+2)(2x-1)$