Math 130 Precalculus Spring 10 Midterm 1a

Name: Solutions

- You may use a graphing calculator.
- You may use a 3×5 index card of notes.

1	10	
2	10	
3	10	
4	10	
5	10	
6	20	
7	20	
8	15	
	105	

(1) (10 points)

(a) What is the maximum number of distinct roots a cubic polynomial can have? Write down a cubic polynomial with this number of roots.

(b) What is the smallest number of distinct roots a cubic polynomial can have? Write down a cubic polynomial with this number of roots.

a) 3 $(x-1)(x-2)(x-3)$

b) 1 x^3

(2) (10 points) Given $u = 2 - 5i$ and $v = 1 + 4i$, compute $u + v$, uv and u/v.

$$u + v = 3 - i$$

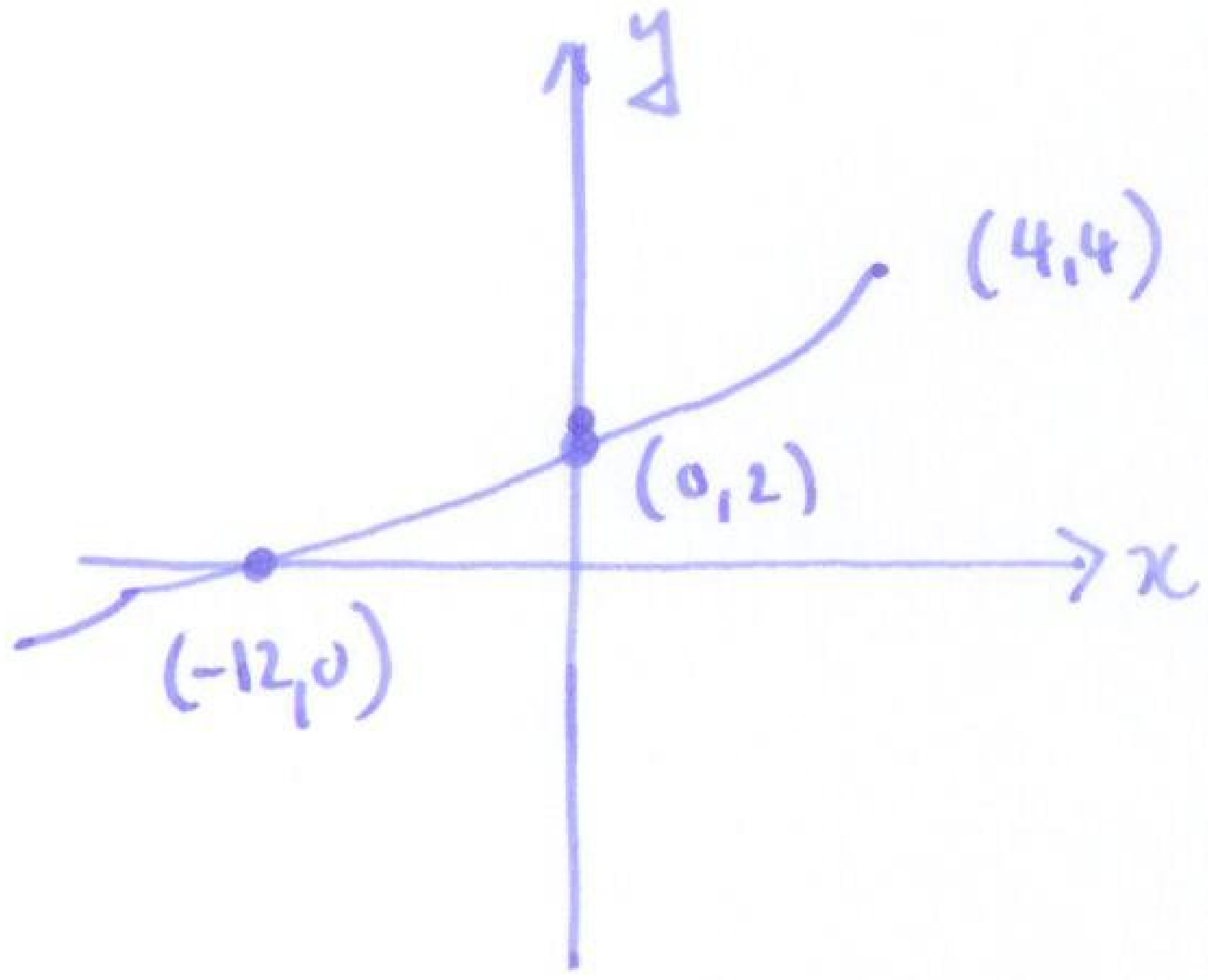
$$uv = (2-5i)(1+4i) = 2 + 8i - 5i - 20i^2$$

$$= 22 + 3i$$

$$\frac{u}{v} = \frac{(2-5i)(1-4i)}{(1+4i)(1-4i)} = \frac{2 - 8i - 5i + 20i^2}{1 + 16} = \frac{-18}{17} - \frac{13}{17}i$$

(3) (10 points) Find the domain and the range of the following function and sketch its graph.

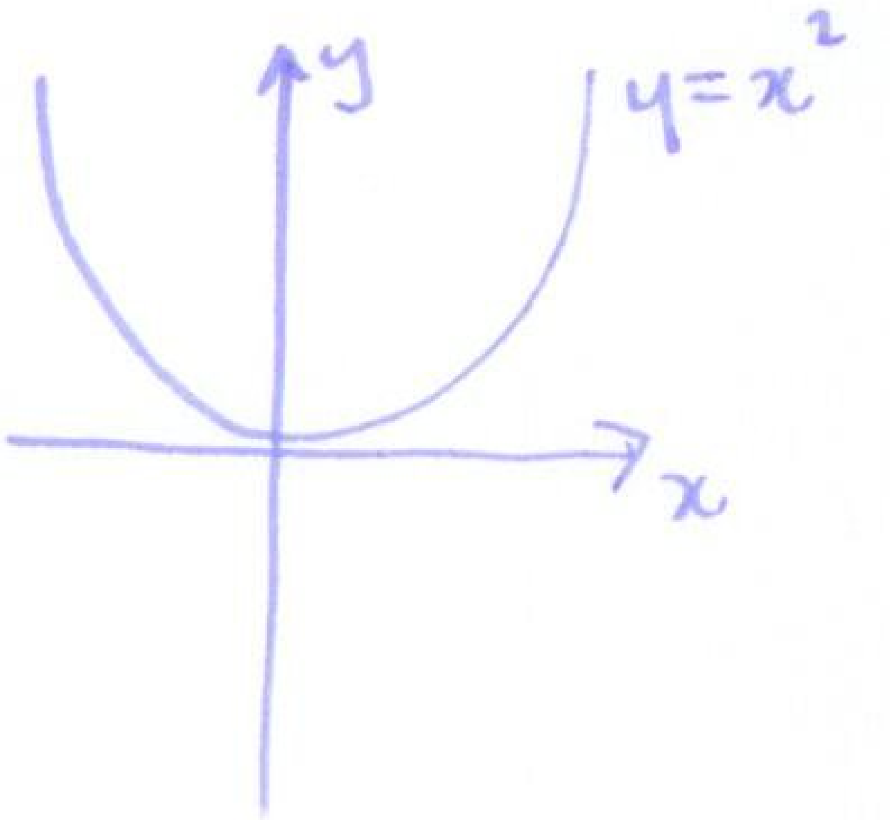
$$f(x) = 4 - \sqrt{4 - x}$$



domain : $x \leq 4$ or $(-\infty, 4]$

range : $x \leq 4$ or $(-\infty, 4]$

(4) (10 points) Find the equation of the graph obtained by taking the graph of $f(x) = x^2$ and shifting it 7 units to the left, then reflecting it across the y-axis, and finally shifting it down 3 units.



shift to left : $(x+7)^2$

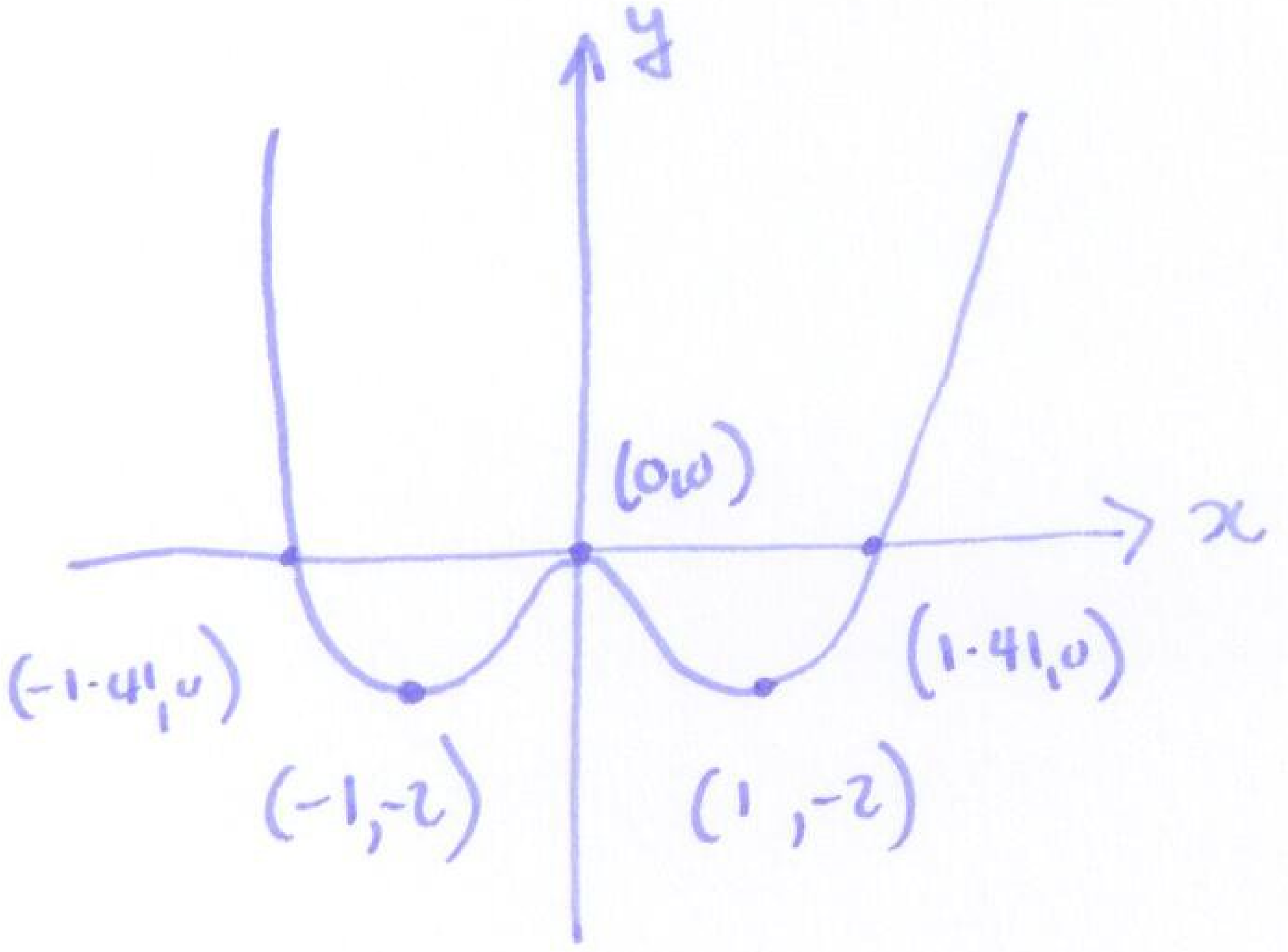
reflect across y-axis : $(-x+7)^2$

shift down three units : $(-x+7)^2 - 3$

(5) (10 points) Consider $f(x) = 2x^4 - 4x^2$. Check f algebraically for symmetries. Graph f using the calculator and find (using the calculator) all zeros, local maxima and local minima, if any.

$f(-x) = 2(-x)^4 - 4(-x)^2 = 2x^4 - 4x^2 = f(x)$ f is even ✓.

$f(-x) = f(x) \neq -f(x)$ f is not odd.



local max: $(0,0)$

local min: $(1,-2)$

$(-1,-2)$.

(6) (20 points) Let $p(x) = 2x^3 - x^2 - 7x + 6$.

(a) Give a complete list of all possible rational zeros.

(b) Check, using either long division or synthetic division, that $x = 1$ is a rational zero.

(c) Find all remaining zeros.

(d) Write p as a product of linear factors.

(e) Sketch the graph of p.

a) $\pm 1, \pm 2, \pm 3, \pm 6, \quad \pm\frac{1}{2}, \pm\frac{3}{2}$

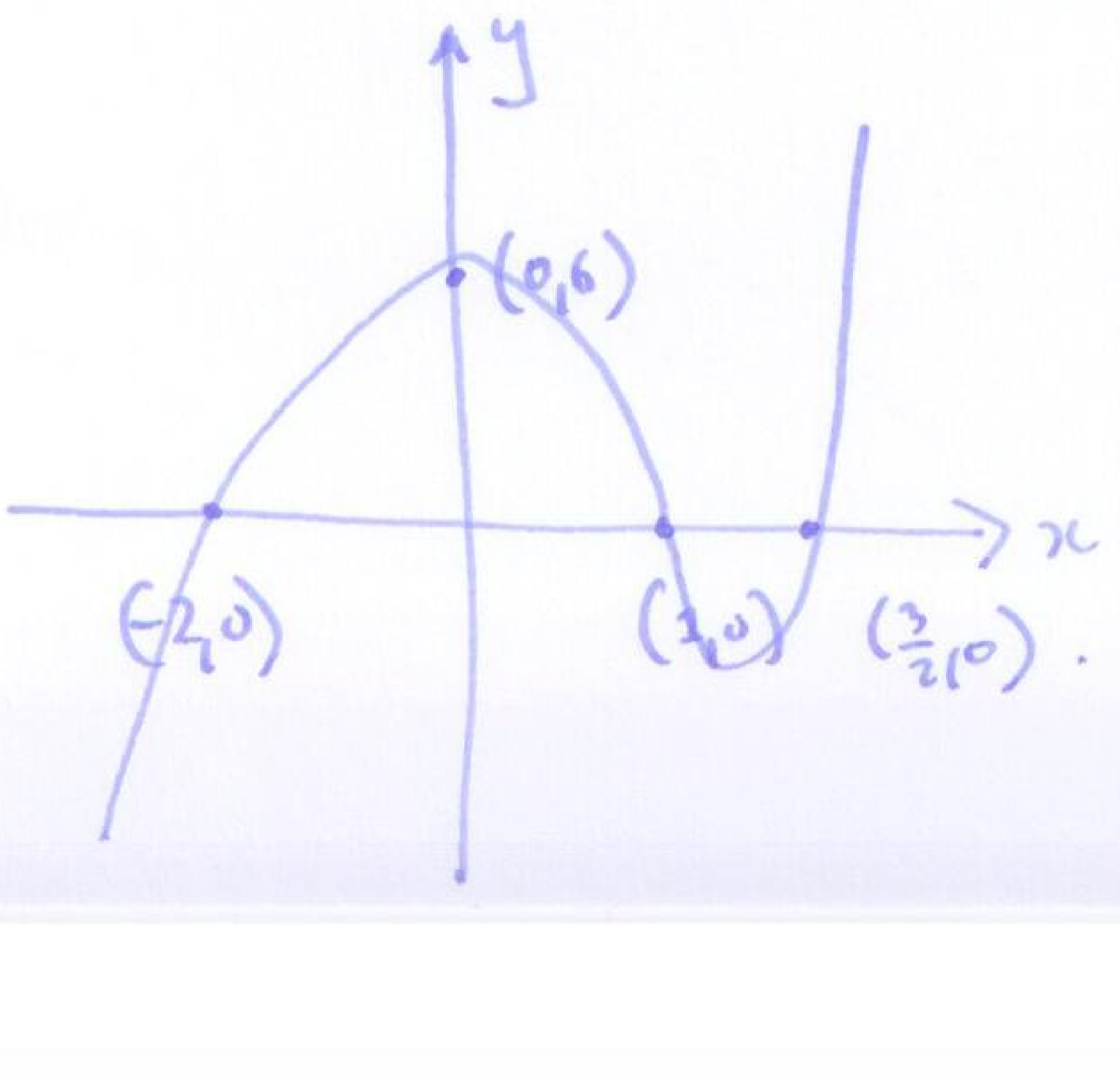
b)

$$
\begin{array}{r|l}
 & 2x^2 + x - 6 \\
x-1 & 2x^3 - x^2 - 7x + 6 \\
 & 2x^3 - 2x^2 \\
\hline
 & x^2 - 7x + 6 \\
 & x^2 - x \\
\hline
 & -6x + 6 \\
 & -6x + 6 \\
\hline
 & 0
\end{array}
$$

c) $2x^2 + x - 6 = (2x-3)(x+2) \qquad x = -2, \frac{3}{2}$

d) $p(x) = (x-1)(x+2)(2x-3)$

e)



(7) (20 points) Consider the function

$$f(x) = \frac{x+3}{2x^2-2x-12}$$

(a) Find the domain of f and the vertical asymptotes.
(b) Find the horizontal asymptote of f, if it has one.
(c) Find the zeros of f and the value of $f(0)$.
(d) Sketch the graph of f.

a) $2x^2-2x-12 = 2(\cancel{4}x-3)(x+2)$

$= 2(x^2-x-6)$

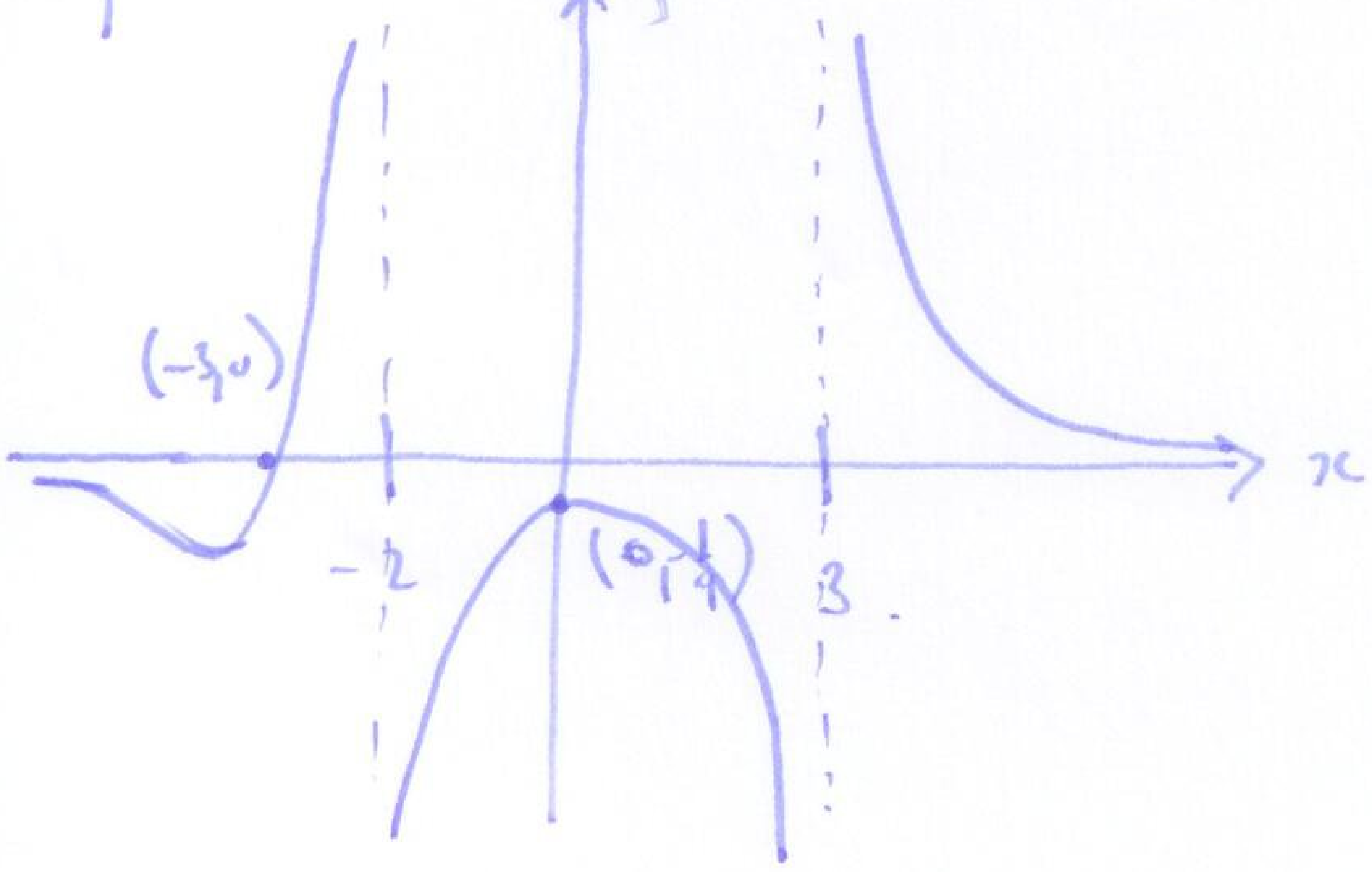
domain : $\mathbb{R}\setminus\{-2,3\}$ or $(-\infty,-2)\cup(-2,3)\cup(3,\infty)$.

b) $f(x) = \frac{p(x)}{q(x)}$ where $\deg p(x) < \deg(q(x))$ so $y=0$ is horizontal asymptote.

c) $f(x)=0$ when $x+3=0$ i.e. $x=-3$.

$f(0) = \frac{3}{-12} = -\frac{1}{4}$.

d)



signs

$x>3$	+ /	+	+	+
$-2<x<3$	+ /	−	+	−
$-3<x<-2$	+ /	−	−	+
$-3<x$	− /	−	−	−

(8) (15 points) Consider the polynomials p and q given by

$$p(x) = x^4 + x^3 + 3x^2 + 9x - 54, \qquad q(x) = x^2 + 9$$

(a) Calculate $\frac{p}{q}$ using long division.

(b) Find all real and complex zeros of p.

a)

$$\begin{array}{r|l} & x^2 + x - 6 \\ \hline x^2+9 & x^4 + x^3 + 3x^2 + 9x - 54 \\ & x^4 + 9x^2 \\ \hline & x^3 - 6x^2 + 9x - 54 \\ & x^3 + 9x \\ \hline & -6x^2 - 54 \\ & -6x^2 - 54 \\ \hline & 0 \end{array}$$

$$\frac{p}{q} = x^2 + x - 6$$

b) $x^2 + x - 6 = (x+3)(x-2)$

roots are : $3i, -3i, -3, 2$.