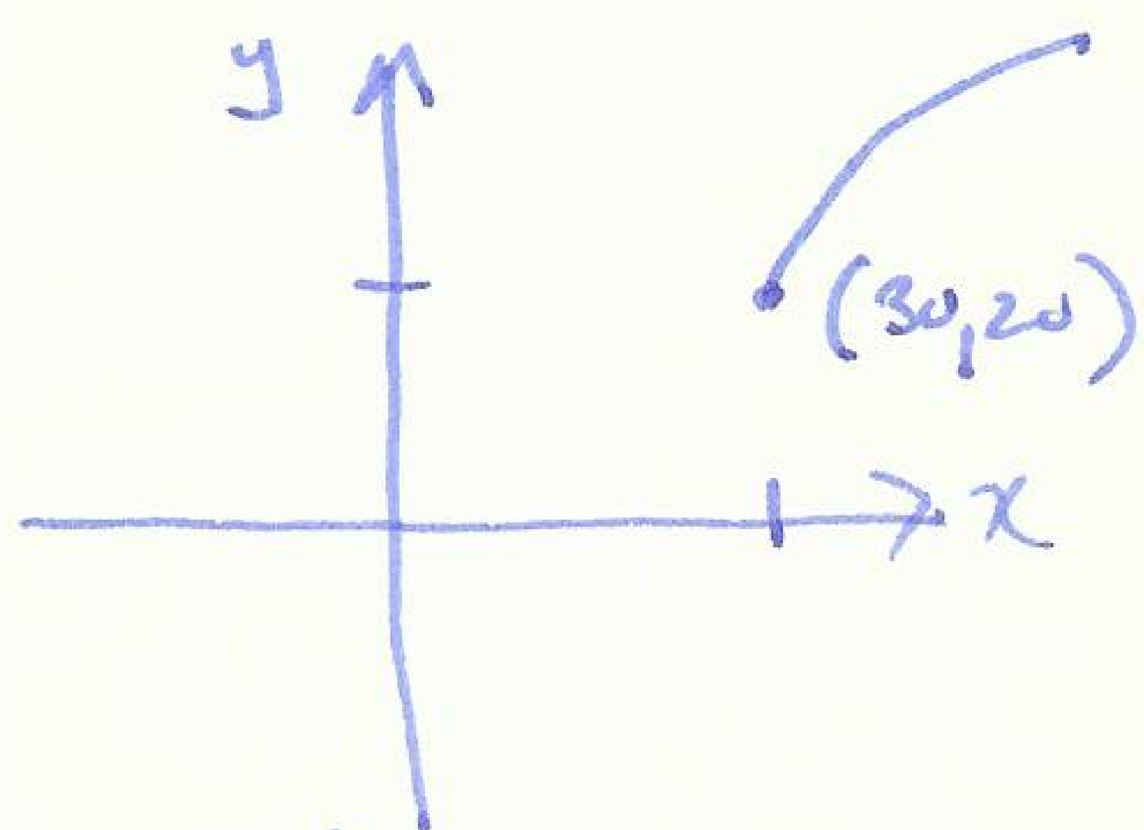
Math 130 Precalculus Sample midterm 1 solutions

Q1 a) They are the same functions: for each input you get the same output.

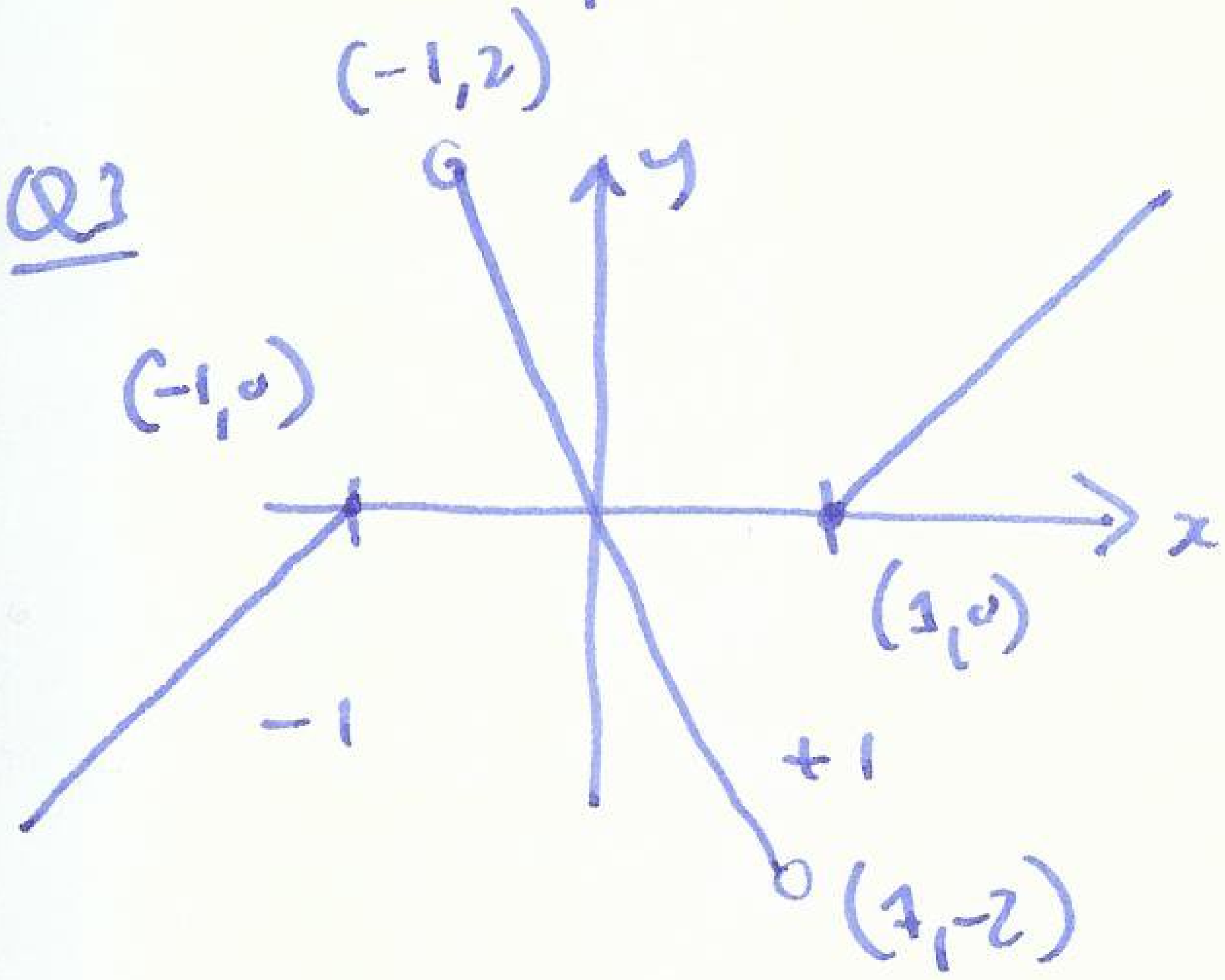
Q2



domain: $[30, \infty)$

range: $[20, \infty)$

Q3



Q4 $f(x) = x^2$ reflect across x-axis: $-x^2$

shift 10 units to right: $-(x-10)^2$

shift 2 units up: $-(x-10)^2+2$

Q5 $u = 3-2i$ $v = 2+3i$

$u+v = 5+i$ $u-v = 1-5i$

$uv = (3-2i)(2+3i) = 6+9i-4i-6i^2 = 12+5i$

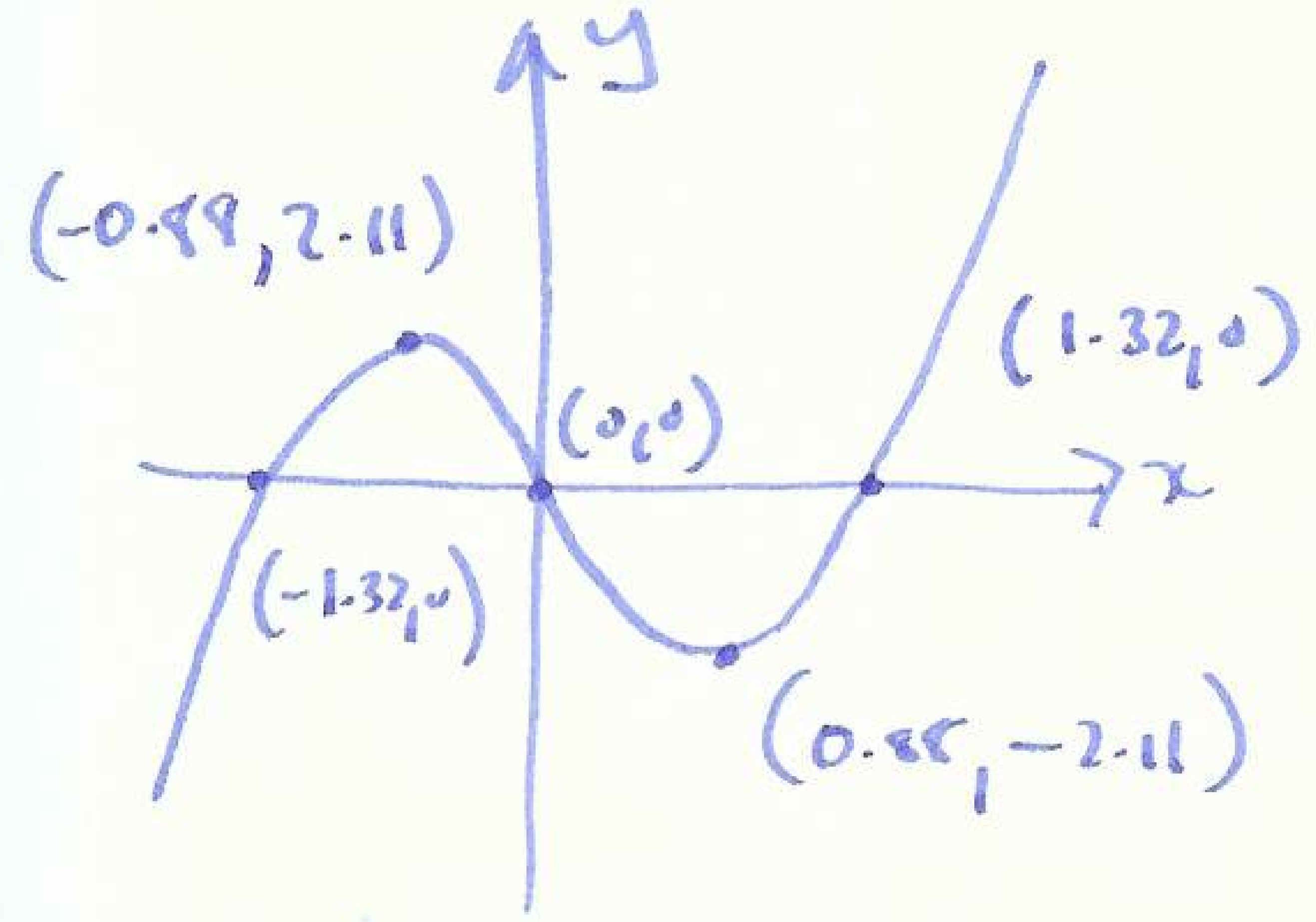
$\frac{u}{v} = \frac{3-2i}{2+3i}\,\frac{2-3i}{2-3i} = \frac{6-9i-4i+6i^2}{4+9} = \frac{-13i}{13} = -i$

$$\frac{v}{u} = \frac{2+3i}{3-2i}\frac{3+2i}{3+2i} = \frac{6+4i+9i+6i^2}{9+4} = \frac{13i}{13} = i$$

Q6 $f(x) = x^5 - 3x$

even? $f(-x) = (-x)^5 - 3(-x) = -x^5 + 3x \neq f(x)$ NO.

odd? $f(-x) = -x^5 + 3x = -f(x)$ YES.



Q7 $p(x) = 3x^3 - 7x^2 - 22x + 8$

a) possible rational roots: $\pm 1 \quad \pm 2 \quad \pm 4 \quad \pm 8$

$\pm 1/3 \quad \pm 2/3 \quad \pm 4/3 \quad \pm 8/3$

b)

$$\begin{array}{r|l} & 3x^2 - x - 24 \\ x-2 & 3x^3 - 7x^2 - 22x + 8 \\ & 3x^3 - 6x^2 \\ \hline & -x^2 - 22x + 8 \\ & -x^2 + 2x \\ \hline & -24x + 8 \\ & -24x + 48 \\ \hline & -40 \end{array}$$

$x=2$ not a root.

$$\begin{array}{r|l} & x^2 - 2x - 8 \\ 3x-1 & 3x^3 - 7x^2 - 22x + 8 \\ & 3x^3 - x^2 \\ \hline & -6x^2 - 22x + 8 \\ & -6x^2 + 2x \\ \hline & -24x + 8 \\ & -24x + 8 \\ \hline & 0 \end{array}$$

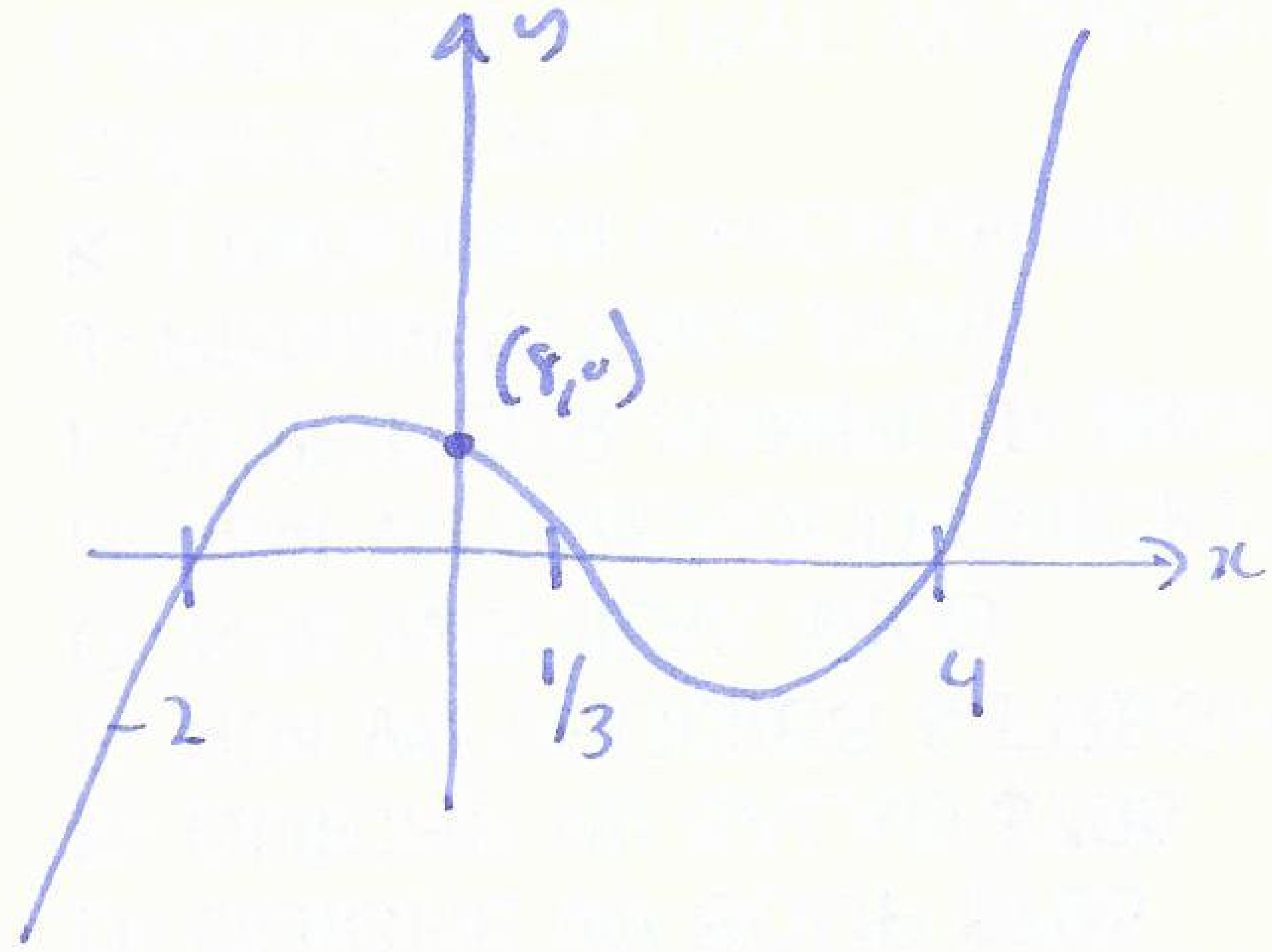
$x = 1/3$ is a root.

c) $x^2 - 2x - 8 = (x-4)(x+2)$

so roots are $\frac{1}{3}, 4, -2$.

d) $p(x) = (3x-1)(x-4)(x+2)$

e)



Q8 $f(x) = \dfrac{2x^2-2}{3x^2-3x-18}$

a) $3x^2 - 3x - 18 = 3(x-3)(x+2)$ vertical asymptotes at $x = 3, -2$.

" $3(x^2 - x - 6)$

domain $\mathbb{R} \setminus \{-2, 3\} = (-\infty, -2) \cup (2, 3) \cup (3, \infty)$.

b) horizontal asymptote: $y = \frac{2}{3}$.

c) $2x^2 - 2 = 2(x^2-1) = 2(x-1)(x+1)$ $f(x) = 0$ when $x = \pm 1$.

$f(0) = \dfrac{-2}{-18} = \dfrac{1}{9}$

d) solve $$\frac{2x^2-2}{3x^2-3x-18} = \frac{2}{3}.$$

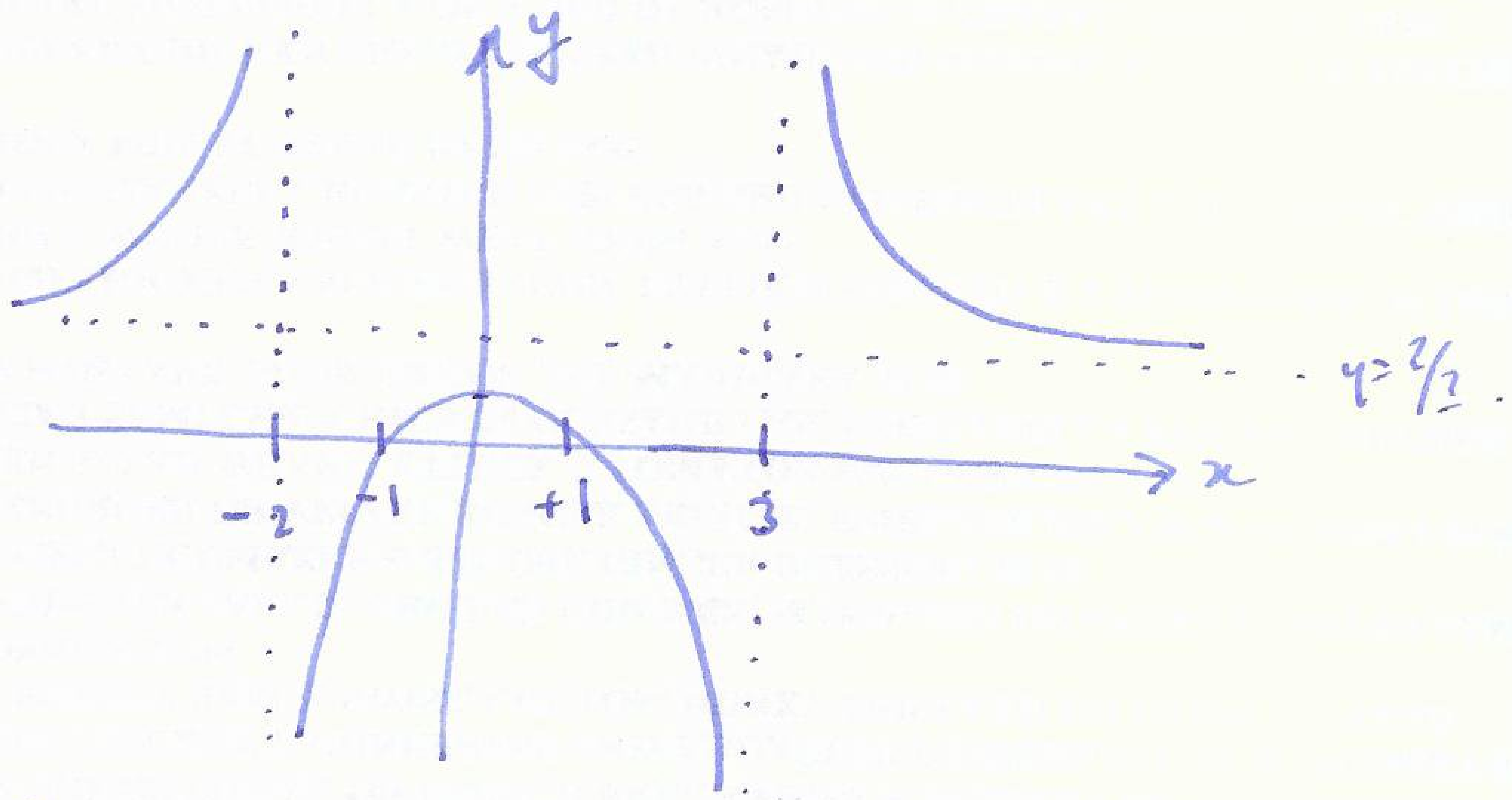
$$6x^2 - 6 = 6x^2 - 6x - 36$$

$$6x = -30$$

$$x = -5$$

answer: yes at $x=-5$.

e)



$$f(x) = \frac{2(x+1)(x-1)}{3(x-3)(x+2)}$$

check sign:

$x > 3$	$\frac{++}{++}$	+
$1 < x < 3$	$++/-+$	−
$-1 < x < 1$	$+-/-+$	+
$-2 < x < 1$	$--/-+$	−
$-2 < x$	$--/--$	+

Q9 $\frac{x+3}{x-4} - \frac{x+1}{x+2} \geqslant 0$

$$\frac{(x+3)(x+2)-(x+1)(x-4)}{(x-4)(x+2)} = \frac{x^2+5x+6-(x^2-3x-4)}{(x-4)(x+2)}$$

$$= \frac{8x+10}{(x-4)(x+2)} = \frac{2(4x+5)}{(x-4)(x+2)} \geqslant 0 .$$

$f(x)=0$ at $x=-\frac{5}{4}$ vertical asymptotes at $x=4$, $x=-2$.

check signs

$4<x$	$+/++$	$+$
$-\frac{5}{4} \leqslant x < 4$	$+/-+$	$-$
$-2 < x \leq \frac{5}{4}$	$-/-+$	$+$
$x<-2$	$-/--$	$-$

solution : $x > 4$, $-2 < x \leq -5/4$

or $(4,\infty) \cup (-2,-\frac{5}{4}]$

Q10

Q10

$$
\begin{array}{r|l}
 & x^2 + 3x - 10 \\
\hline
x^2+4 & x^4 + 3x^3 - 6x^2 + 12x - 40 \\
 & x^4 \qquad\quad + 4x^2 \\
\hline
 & \qquad 3x^3 - 10x^2 + 12x - 40 \\
 & \qquad 3x^3 \;\cancel{-12x}\; -12x \\
\hline
 & \qquad\qquad -10x^2 \quad - 40 \\
 & \qquad\qquad -10x^2 \quad - 40 \\
\hline
 & \qquad\qquad\qquad\qquad 0
\end{array}
$$

so $p(x) = (x^2+4)(x^2+3x-10)$

$\quad = (x+2i)(x-2i)(x+5)(x-2)$

roots: $\pm 2i, -2, 5$.