Math 130 Precalculus Common Final Spring '09

Q1 $f(x) = x^3 + 3x^2 + 2x - 6$ possible rational zeros: $\pm 1, \pm 2, \pm 3, \pm 6$

$x=1$ works.

$$\begin{array}{r|l} & x^2 + 4x + 6 \\ \hline x-1 & x^3 + 3x^2 + 2x - 6 \\ & x^3 - x^2 \\ \hline & \quad 4x^2 + 2x - 6 \\ & \quad 4x^2 - 4x \\ \hline & \qquad 6x - 6 \end{array}$$

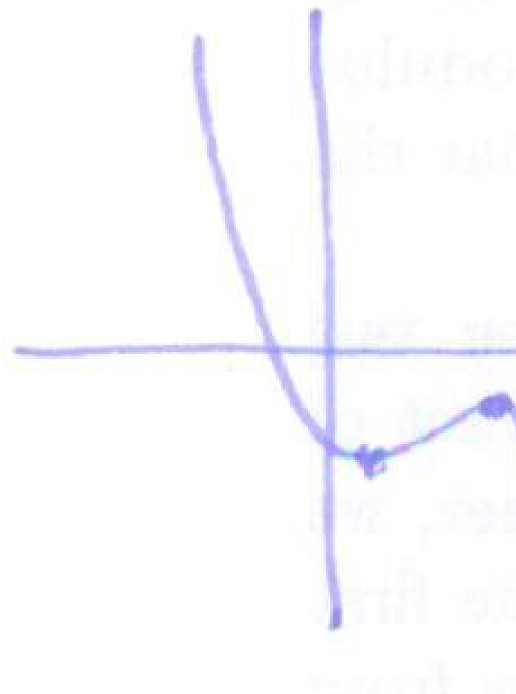
rational zero $x=1$

other zeros: $-2 \pm \sqrt{2}\, i$

other zeros:

$$x = \frac{-4 \pm \sqrt{16-24}}{2}$$

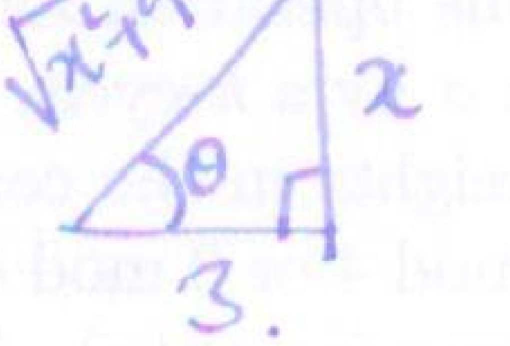
Q2



rel min $x = 1$ increasing on $[1,3]$

rel max $x = 3$. decreasing on $(-\infty, 1], [3, \infty)$.

Q3 $\cos\theta$ $\tan\theta = x/3$

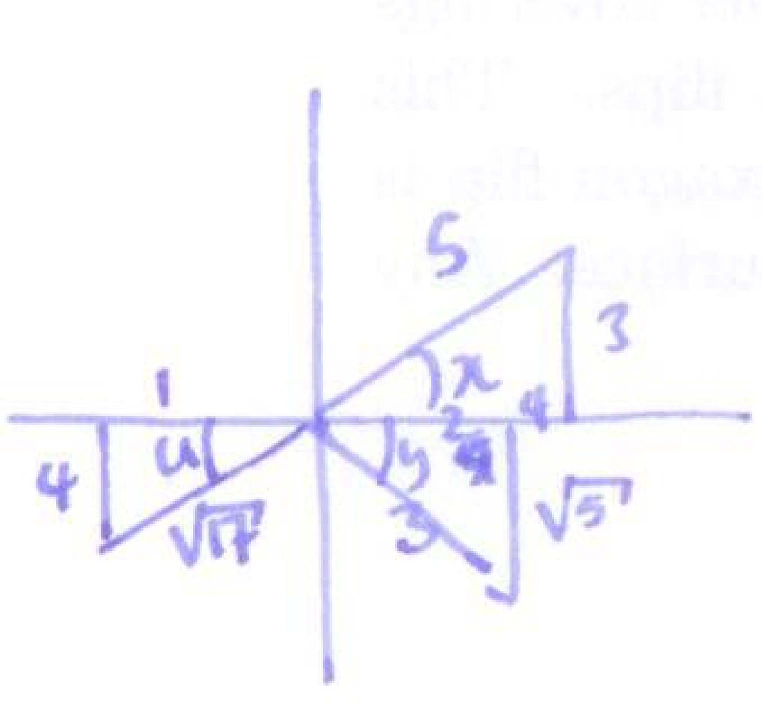


$\cos\theta = \dfrac{3}{\sqrt{x^2+9}}$.

Q4 $z^4 = 2^4\left(\cos(240°) + i\sin(240°)\right) = 16\left(-\frac{1}{2} + i\left(-\frac{\sqrt{3}}{2}\right)\right)$

$= -8 + 8\sqrt{3}\, i$

Q6



a) $\sin 2x = 2\sin x \cos x = 2 \cdot \frac{3}{5} \cdot \frac{4}{5} = \frac{24}{25}$.

b) $\cos(x+y) = \cos x \cos y - \sin x \sin y$

$= \cancel{\frac{2\sqrt{5}}{9}}\ \frac{4}{5} \cdot \frac{2}{3} - \frac{3}{5}\left(\frac{\sqrt{5}}{3}\right) = \frac{8+3\sqrt{5}}{15}$.

c) $\tan\left(\frac{u}{2}\right) = \frac{\sin u}{1+\cos u} = \frac{-4/\sqrt{17}}{1 - 1/\sqrt{17}} = \frac{-4}{\sqrt{17}-1}$.

Q7 $\dfrac{2\tan\theta}{1+\tan^2\theta}$ $\qquad \cos^2\theta + \sin^2\theta = 1$

$\qquad\qquad\qquad\qquad 1 + \tan^2\theta = \sec^2\theta$

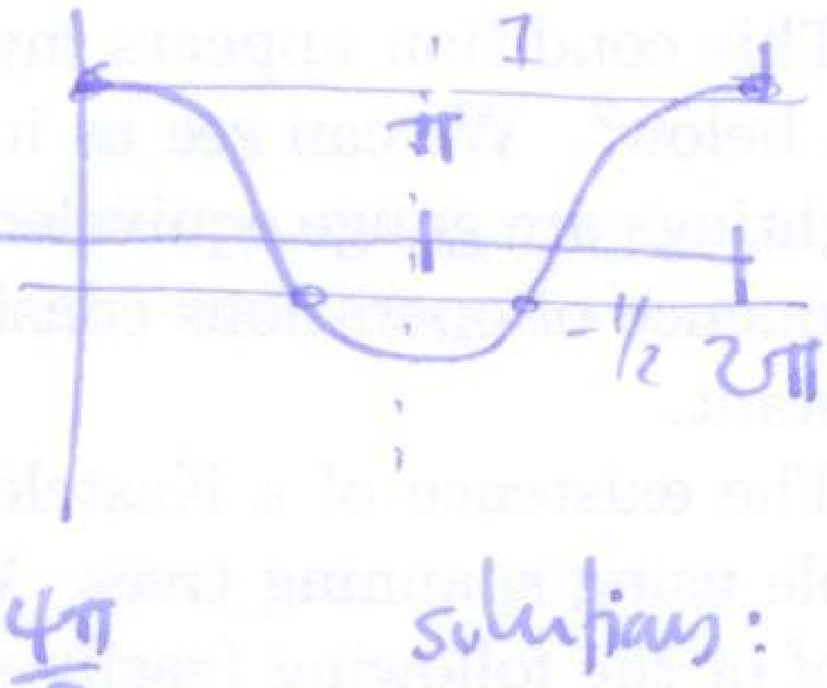
double angle formula.

$$= \frac{2\tan\theta}{\sec^2\theta} = \frac{2\sin\theta}{\cos\theta}\cos^2\theta = 2\sin\theta\cos\theta = \sin 2\theta$$

Q8 $\tan^2 x + \sec x - 1 = 0 \qquad \tan^2\theta = \sec^2\theta - 1$

$\sec^2\theta + \sec x - 2 = 0$

$(\sec\theta + 2)(\sec\theta - 1) = 0$



$\sec\theta = 1 \iff \cos\theta = 1 \quad \theta = 0$

$\sec\theta = -2 \iff \cos\theta = -1/2 \quad \theta = \frac{2\pi}{3},\ 2\pi - \frac{2\pi}{3} = \frac{4\pi}{3}.$ $\quad$ solutions: $0, \frac{2\pi}{3}, \frac{4\pi}{3}$.

Q9 $f(x) = \sqrt{x}$

$f(x) = \sqrt{x+3}$ horizontal shift left 3 units

$f(x) = -5\sqrt{x+3}$ reflect in x-axis and scale by factor of +5 (expansion) in vertical direction

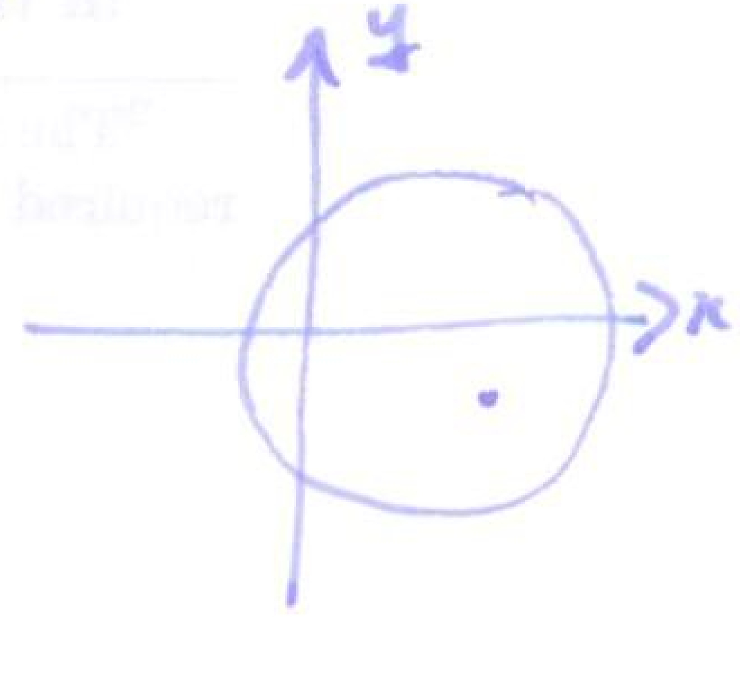
$f(x) = -5\sqrt{x+3} + 7$ shift up 7 units.

Q10 $x^2 - 6x + y^2 + 2y = 6.$

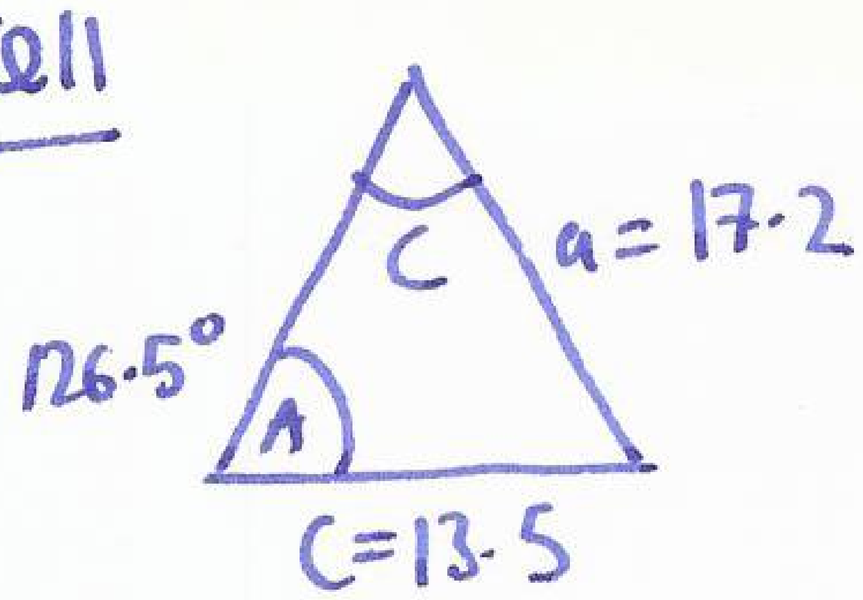
$(x-3)^2 - 9 + (y+1)^2 - 4 = 6.$

$(x-3)^2 + (y+1)^2 = 19$ $\quad$ center $(3,-1)$

radius $\sqrt{19}$.



Q11



$$\frac{\sin 126.5}{17.2} = \frac{\sin C}{13.5}$$

$$C = \sin^{-1}\left(\frac{13.5}{17.2}\sin 126.5\right)$$

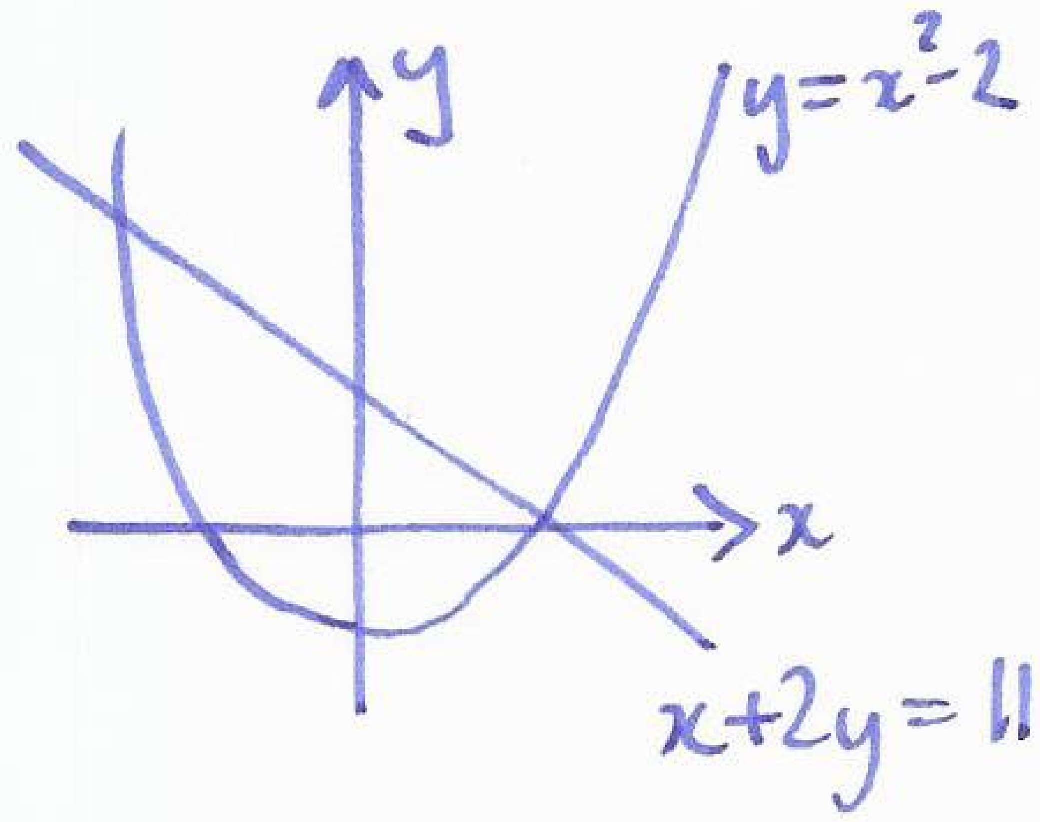
$$C \approx 39.1^\circ$$

(other soln too big).

$$B = 180 - A - C \approx 14.4$$

$$\frac{\sin 126.5}{17.2} = \frac{\sin 14.4}{b} \qquad b = 17.2\,\frac{\sin 14.4}{\sin 126.5} \approx 5.3.$$

Q12



$$y = (11 - 2y)^2 - 2$$

$$y = 4y^2 - 44y + 121 - 2$$

$$4y^2 - 45y + 119 = 0$$

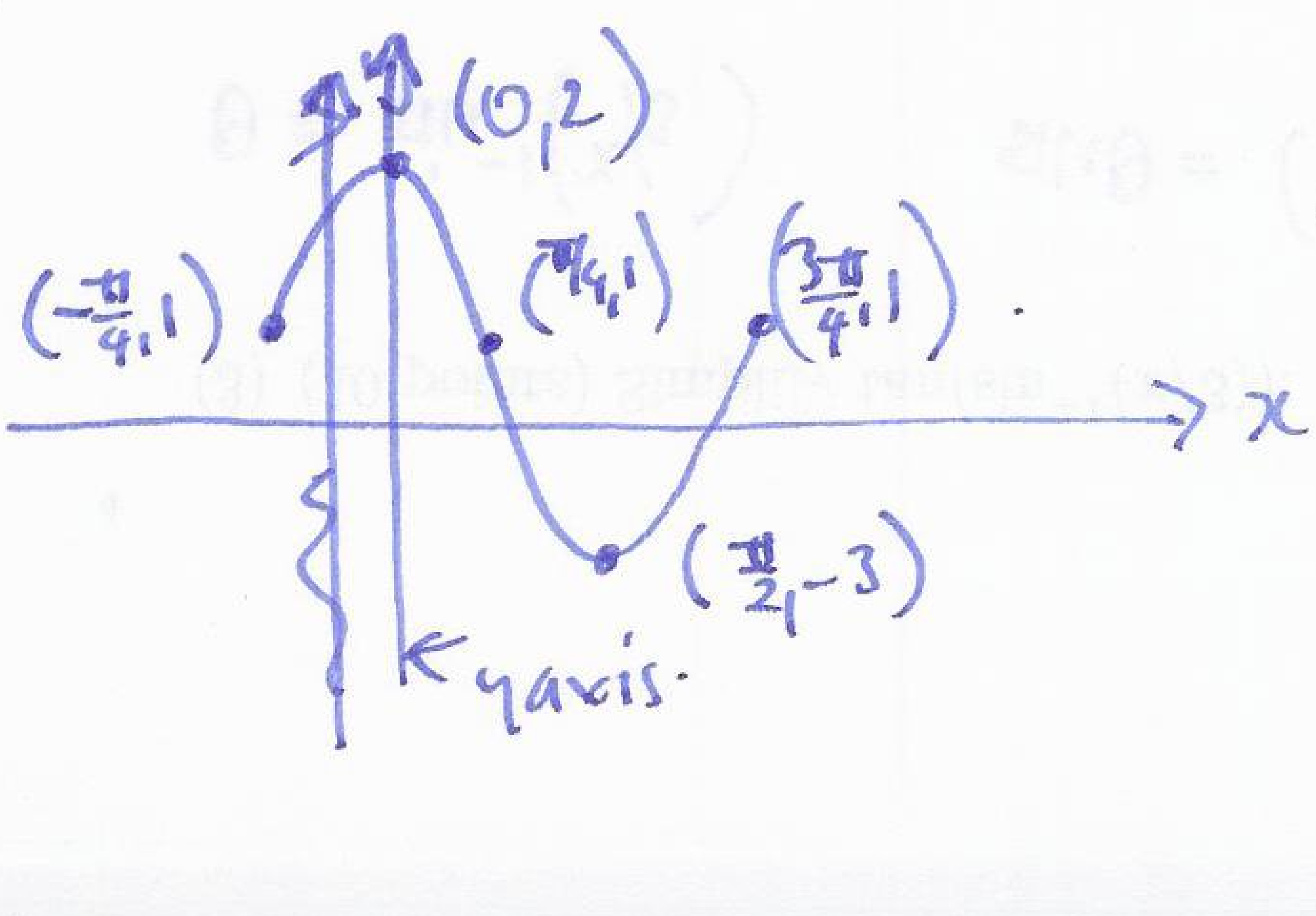
$$y = \frac{45 \pm \sqrt{45^2 - 4.4.119}}{8} = \frac{45 \pm 11}{8} = 7, \frac{17}{4}$$

$y = 7$, $x = 11 - 2y = -3$

$y = 17/4$, $x = -5/2$.

Q13 $y = 3\sin\left(2x + \frac{\pi}{2}\right) + 1 = 3\sin\left(2\left(x + \frac{\pi}{4}\right)\right) + 1$

amplitude = 3, period = π, phase shift = $-\pi/4$.



Q14 $f(x) = \dfrac{x-5}{x-1} + 3$

a) $f(0) = \dfrac{-5}{-1} + 3 = 8.$

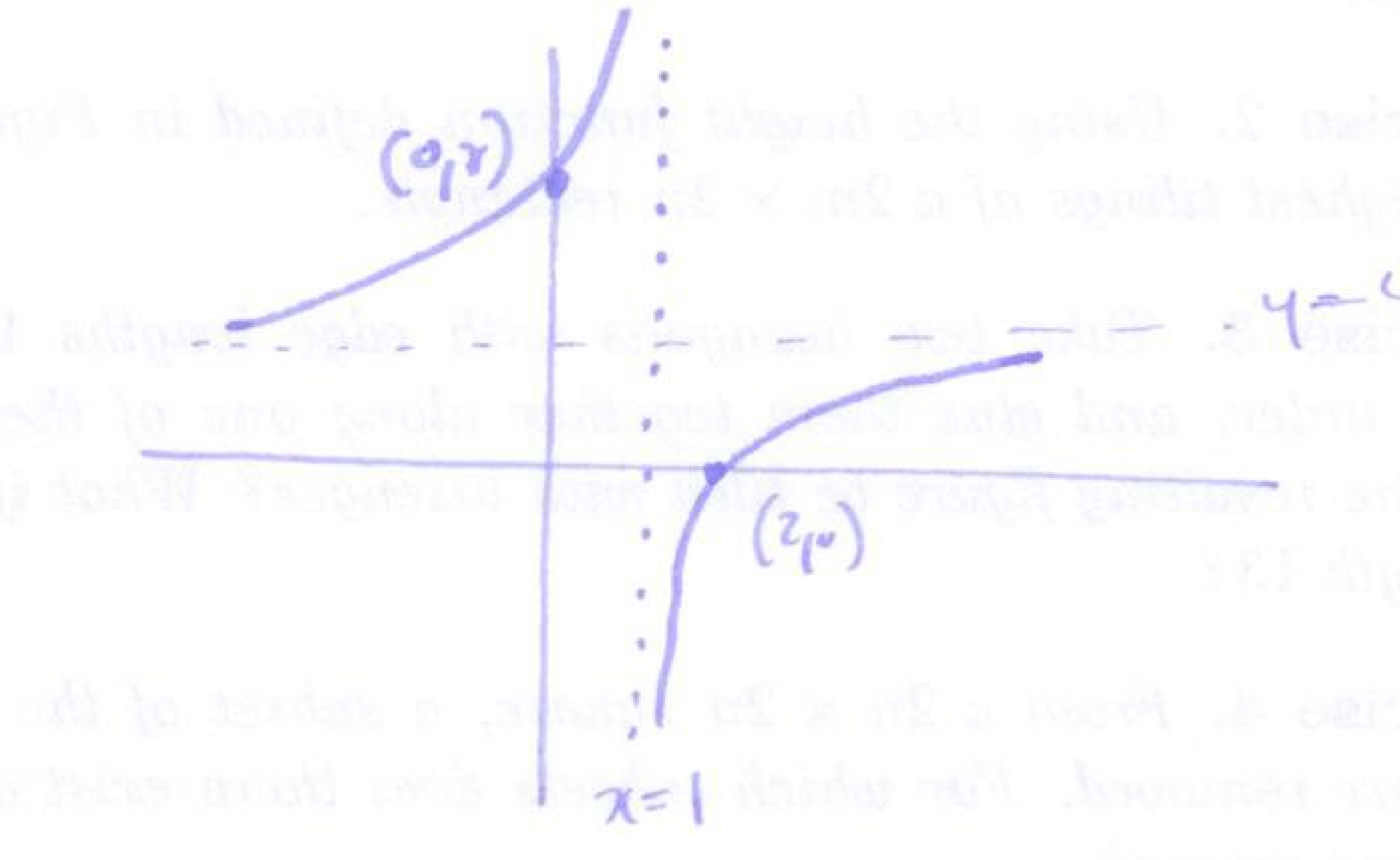
b) $\dfrac{x-5}{x-1} + 3 = 0$ $\qquad x-5 = -3(x-1) = -3x+3.$

$4x = 8 \quad x = 2.$

c) $x = 1$

d) $y = 4$

e)



f) $(1,2]$

Q15 center $(1,0)$ $\qquad \dfrac{(x-h)^2}{a^2} + \dfrac{(y-k)^2}{b^2} = 1 \qquad c^2 = a^2 - b^2 = 16.$

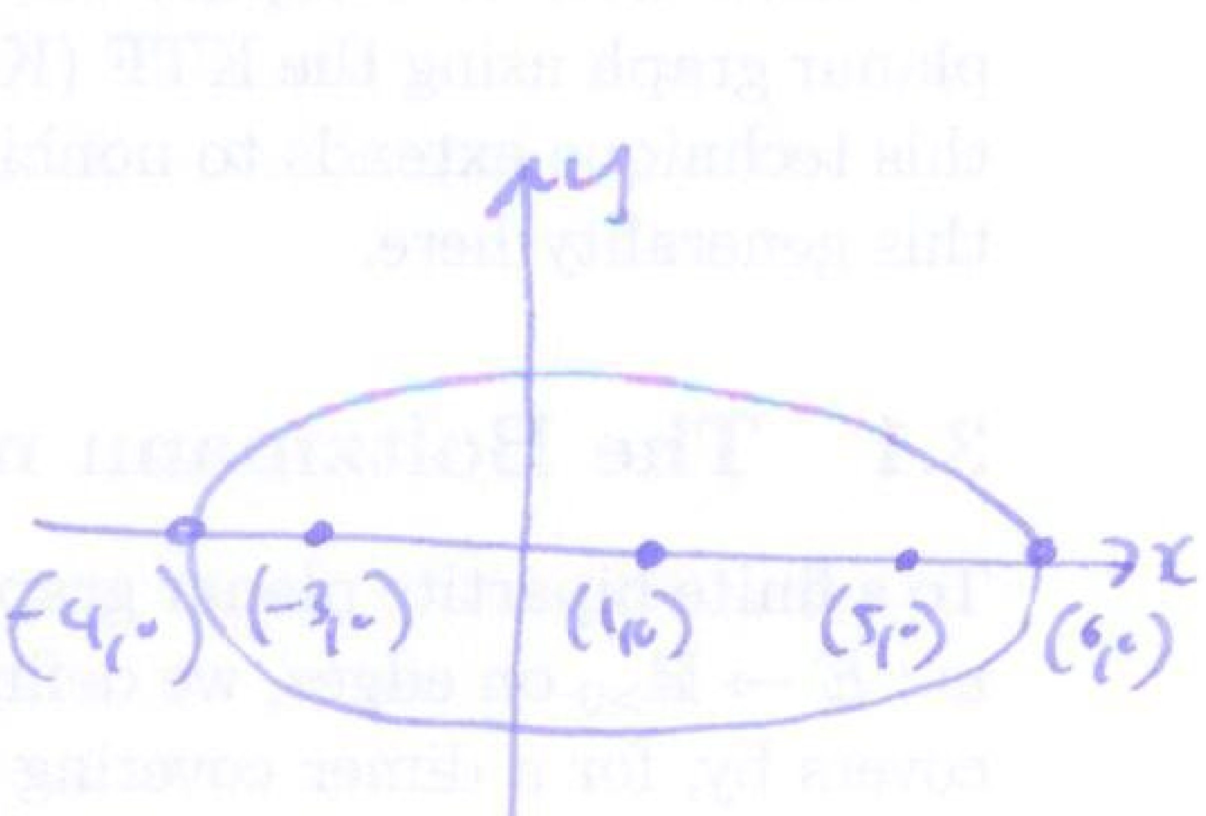
$a > b > 0.$

foci $(h \pm c, k)$ $\qquad \dfrac{(x-1)^2}{5^2} + \dfrac{y^2}{3^2} = 1$ $\qquad$ 5 3

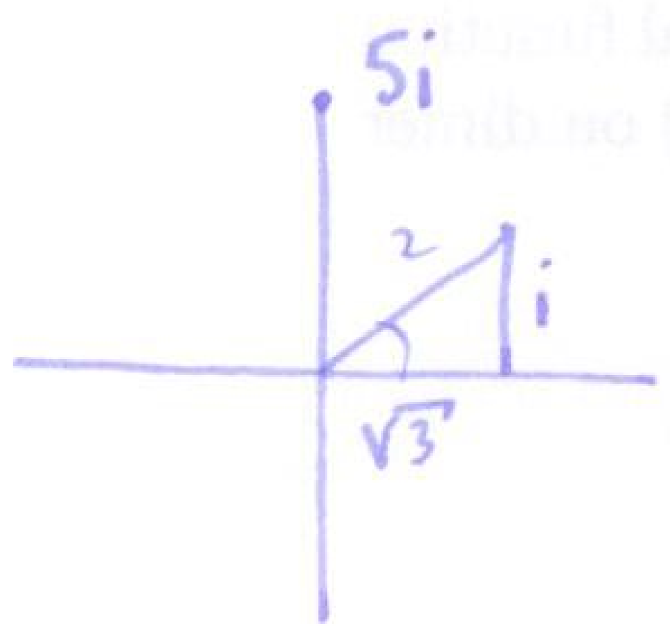
horizontal axis large.

i.e. $(5,0), (-3,0)$

vertices: $(h \pm a, k)$ i.e. $(6,0), (-4,0)$



Q16



$z_1 = 5i = 5\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right).$

$z_2 = \sqrt{3} + i = 2\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$

$z_1 z_2 = 10\left(\cos\left(\frac{\pi}{2} + \frac{\pi}{6}\right) + i\sin\left(\frac{\pi}{2} + \frac{\pi}{6}\right)\right) = 10\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right).$

$z_1 / z_2 = 5/2\left(\cos\left(\frac{\pi}{2} - \frac{\pi}{6}\right) + i\sin\left(\frac{\pi}{2} - \frac{\pi}{6}\right)\right) = 10\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$