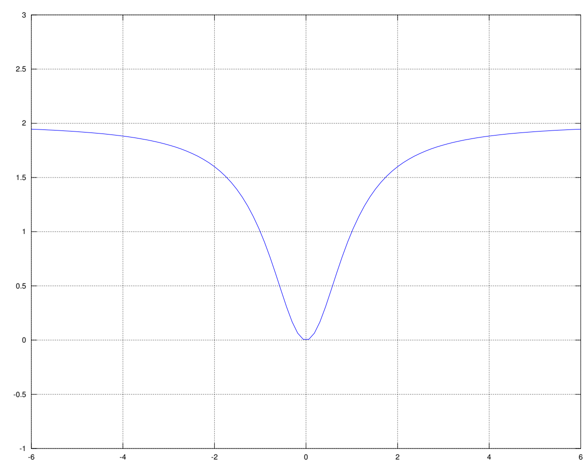
Math 231 Calculus 1 Fall 10 Sample midterm 3

(1) Consider the function $f(x)$ defined by the following graph.



(a) Label all regions where $f'(x) < 0$.
(b) Label all regions where $f'(x) > 0$.
(c) What is $\lim_{x\to\infty} f'(x)$?
(d) What is $\lim_{x\to-\infty} f''(x)$?
(e) Sketch a graph of $f'(x)$ on the figure.
(f) Label the approximate locations of all points of inflection.

(2) True or False. Indicate whether the following statements are True or False.
(a) Assume $f(x)$ is differentiable for all x. If $f'(c) = 0$ for some c, then $f(x)$ has a local maximum or minimum at $x = c$.
(b) If the relative maxima of some differentiable function $f(x)$ are $f(1) = 4$ and $f(5) = 16$ then $f'(c) = 3$ for some c in the interval $(1, 4)$.
(c) If $f'(x) > 0$ for all values of x, then $\lim_{x\to\infty} f(x)$ does not exist.
(d) If $f'(c) = 0$ and $f''(c) > 0$, the function $f(x)$ has a local maximum at $x = c$.

(3) Sketch a graph of a differentiable function f that satisfies the following conditions and has $x = -1$ as its only critical point.

$$\begin{aligned} f(-1) &= 10 \\ f'(-1) &= 0 \\ f'(x) &> 0 \text{ for } x < -1 \\ f'(x) &< 0 \text{ for } x > -1 \\ \lim_{x\to\infty} f(x) &= \lim_{x\to-\infty} f(x) = 4 \end{aligned}$$

(4) Consider the following function:

$$g(x) = \tfrac{1}{4}(x^4 - 4x^3)$$

(a) Find, if they exist, the coordinates of all relative maxima and minima.
(b) Determine the interval(s) where g is increasing and those where g is decreasing.
(c) Find, if they exist, the coordinates of all points of inflection.
(d) Determine the intervals where g is concave up and those where g is concave down.
(e) Sketch the curve as accurately as possible.

(5) Consider the function

$$f(x) = \frac{1}{x^2 - 4}$$

(a) Find all vertical and horizontal asymptotes of the function.
(b) Find all critical points of the function.
(c) Determine the intervals where $f(x)$ is increasing and decreasing.
(d) Use the 2nd derivative test to attempt to identify all local maxima and minima.
(e) Sketch the function and label all relative maxima and minima.

(6) We all know that the cube root of 8 is 2. In other words, $\sqrt[3]{8} = 8^{1/3} = 2$. Use calculus to approximate $\sqrt[3]{9}$. Use your calculator to find $\sqrt[3]{9}$ and state whether or not your approximation is "good".

(7) The equation for a circle of unit radius is $x^2 + y^2 = 1$. In the first quadrant, this implies that $y = \sqrt{1 - x^2}$. Find the dimensions of the rectangle of largest area, with sides parallel to the coordinate axes, that one can inscribe between

the x-axis and the circle in the first quadrant. What is the maximum area? What is the shape of the rectangle with largest area inscribed in a circle?

(8) Compute the following limits. Show all work.
   (a) $\lim_{x\to\infty} \frac{6x^5-12x^4-24}{2x^5+30}$
   (b) $\lim_{x\to-\infty} \frac{x^2}{1-x^2}$
   (c) $\lim_{x\to\infty} \frac{3x}{\sqrt{x^2+5}}$
   (d) $\lim_{x\to 0} \frac{e^{2x^2}-1}{1-\cos(3x)}$

(9) Abner goes fishing with a fishing rod, and catches a fish. The fish swims away from Abner at the surface of the water at a speed of 2 ft/s. The fishing line is taut (it is straight line) and the end of Abner's fishing pole is 4 ft above the water surface. Draw and label a picture of the situation. What is the rate at which Abner must let out line when the length of line between the tip of the fishing pole and the fish is 5 feet? (Assume the line stays straight!)