(1) (25 points) Compute the derivative $\frac{dy}{dx}$. Do not simplify. Show all your work.

(a) $y = (x^2 + 3x)^9 \sqrt{2x^3 - 5}$

(b) $y = e^{\sin(x^2 - 1)}$

(c) $y = e^{-6x} \cos(2x)$

(d) $y = \frac{x^4}{10x^2 - 8}$

(e) $y = \ln(x^{\frac{3}{2}} + 5)$

a) $\frac{dy}{dx} = 9(x^2+3x)^8(2x+3)\sqrt{2x^3-5} + (x^2+3x)^9 \frac{1}{2}(2x^3-5)^{-1/2} . 6x^2$

b) $\frac{dy}{dx} = e^{\sin(x^2-1)} . \cos(x^2-1) . 2x$

c) $\frac{dy}{dx} = -6e^{-6x}\cos(2x) + e^{-6x} . (-\sin 2x) . 2$

d) $\frac{dy}{dx} = \frac{(10x^2-8)4x^3 - (20x)x^4}{(10x^2-8)^2}$

e) $\frac{dy}{dx} = \frac{1}{(x^{3/2}+5)} . \frac{3}{2}x^{1/2}$

(2) (15 points) Let $f(x) = x^2 - 3x$. Use the *definition of the derivative* to find $f'(x)$.

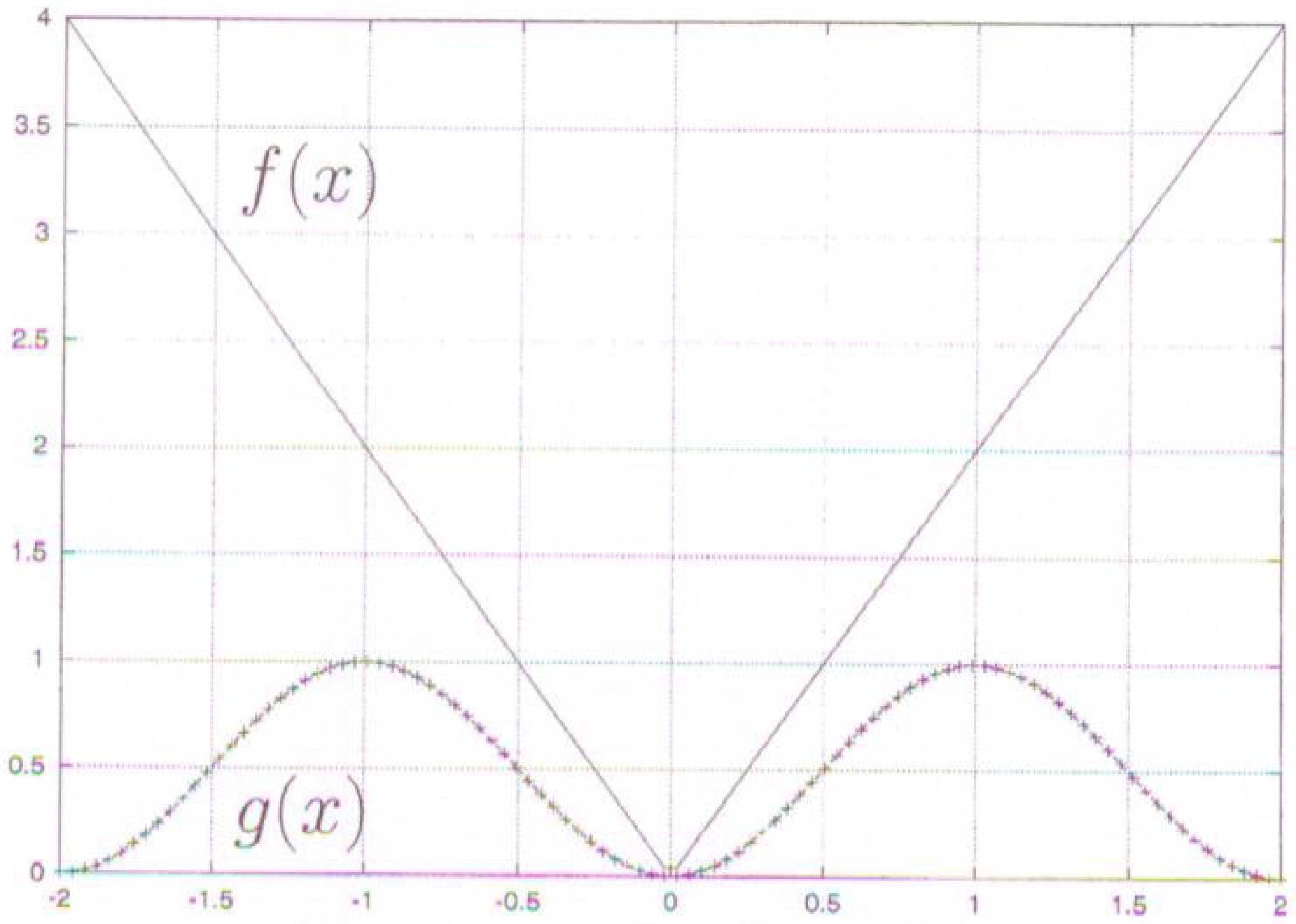
$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{(x+h)^2 - 3(x+h) - x^2 - 3x}{h}$$

$$= \lim_{h \to 0} \frac{x^2 + 2xh + h^2 - 3x - 3h - x^2 - 3x}{h}$$

$$= \lim_{h \to 0} \frac{2xh + h^2 - 3h}{h} = \lim_{h \to 0} 2x + h - 3 = 2x - 3.$$

(3) (30 points) Graphs of $f(x)$ and $g(x)$ are shown below. Show all your work. Evaluate each of the following expressions. If the expression is not defined, state this.



(a) $f(1)$, $f'(-1)$, $f'(0)$, $g'(0.5)$, $g'(1)$

(b) $u(x) = f(x)g(x)$. Find: $u'(1)$.

(c) $v(x) = f(x)/g(x)$. Find: $v'(1)$.

(d) $w(x) = f(g(x))$. Find: $w'(1)$.

(e) Sketch a graph of $f'(x)$.

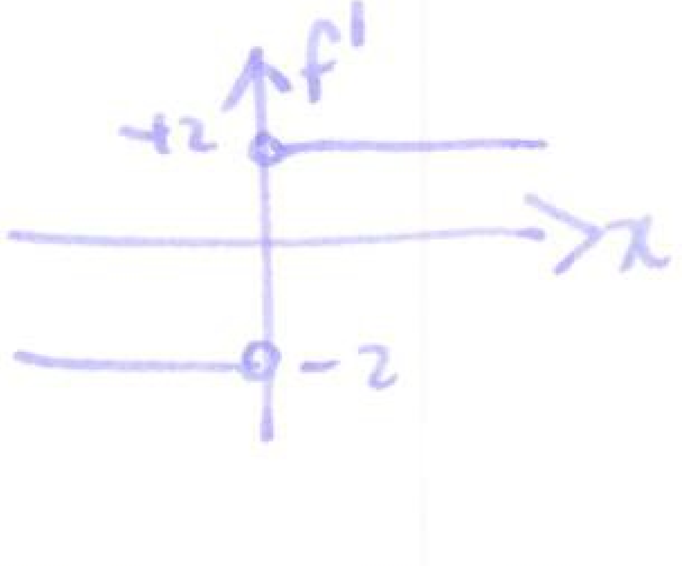
a) $f(1) = 2$, $f'(-1) = -2$, $f'(0)$ undefined, $g'(0.5) = +1$, $g'(1) = 0$

b) $u'(x) = f'(x)g(x) + f(x)g'(x)$, $u'(1) = f'(1)g(1) + f(1)g'(1)$
$2 \cdot 1 + 2 \cdot 0 = 2$

c) $v'(1) = \dfrac{g(1)f'(1) - f(1)g'(1)}{(g(1))^2} = \dfrac{1 \cdot 2 - 2 \cdot 0}{1^2} = 2$

d) $w'(1) = f'(g(1)) \cdot g'(1) = f'(1) \cdot g'(1) = 2 \cdot 0 = 0$.

e)



(4) (20 points) Suppose x and y satisfy $x + x^2y + \cos y = 2$.

(a) Find $\frac{dy}{dx}$ as a function of x and y.

(b) Find the equation of the tangent line to the curve at the point $(1, 0)$.

a) $1 + 2xy + x^2y' + -\sin y . y' = 0$

$y'(x^2 - \sin y) = -1 - 2xy$

$$y' = \frac{1 + 2xy}{\sin y - x^2}$$

b) $x = 1,\ y = 0$ $\quad \frac{dy}{dx} = \frac{1 + 0}{0 - 1^2} = -1$

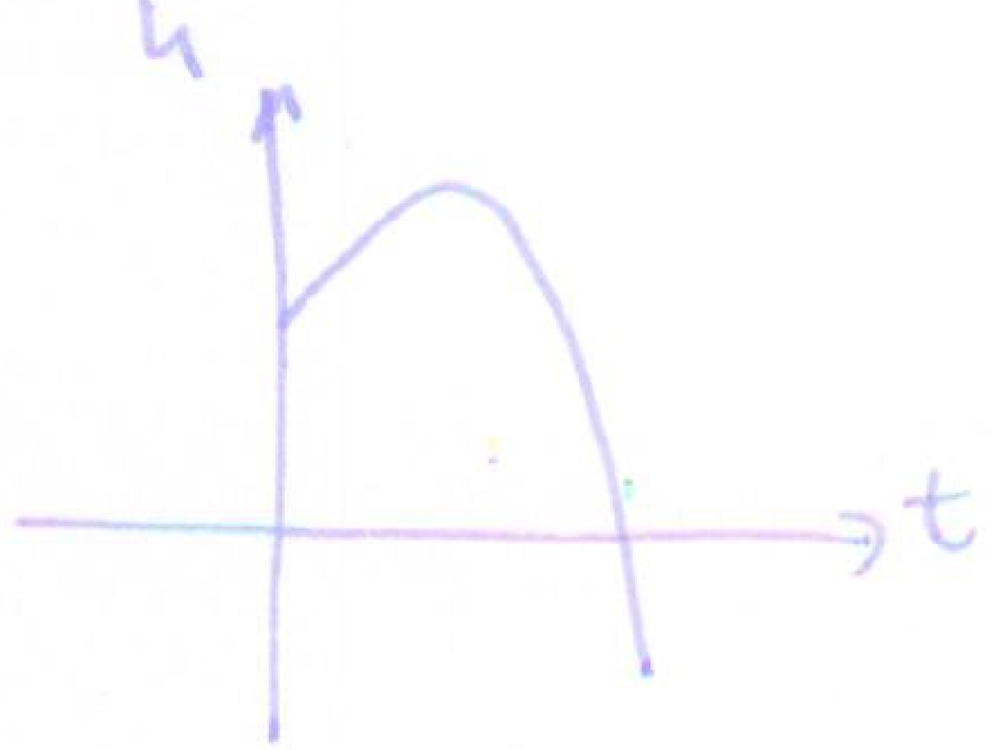
tangent line $\quad y - 0 = (-1)(x - 1)$

$y = -x + 1$

(5) (20 points) A ball is thrown vertically upwards from 64 feet above the ground, with an initial velocity of 48 feet per second.

(a) Find the velocity of the ball when it hits the ground.

(b) Find the maximum height of the ball.



$h(t) = 64 + 48t - \frac{1}{2} 32 t^2$

$h'(t) = 48 - 32t$

a) $h(t) = 0 = 64 + 48t - 16t^2$

$4 + 3t - t^2 = 0$

$-(t^2 - 3t - 4) = 0$

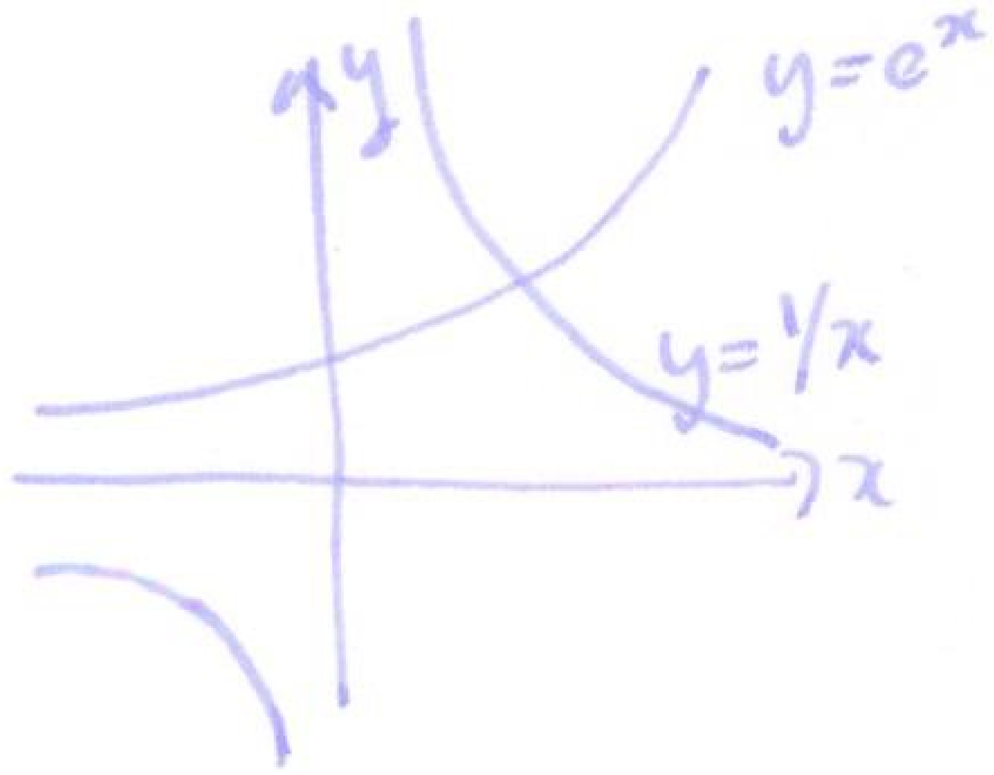
$-(t-4)(t+1)$ $t = 4, -1$

$h'(4) = \cancel{64 + 48 \cdot 4 - 16 \cdot 4^2 =}$
$48 - 32 \cdot 4 = -80$ ft/s.

b) max height when slope = 0 $h'(t) = 0 = 48 - 32t$ $t = \frac{48}{32} = \frac{3}{2}$

$h\left(\frac{3}{2}\right) = 64 + 48 \cdot \frac{3}{2} - 16\left(\frac{3}{2}\right)^2 = 100$ ft

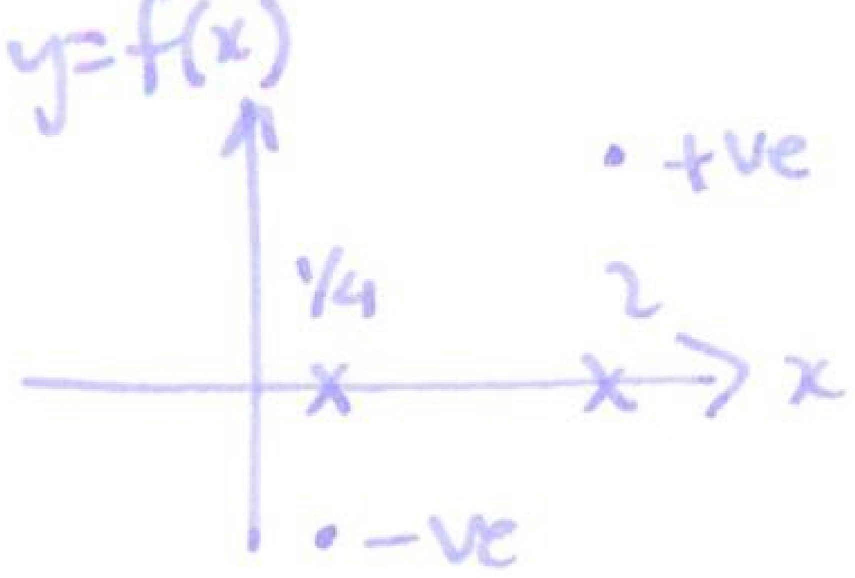
(6) (10 points) Show that $e^x = 1/x$ has a solution for some $x > 0$.



consider $f(x) = e^x - \frac{1}{x}$ continuous as sum of cts functions.

$f(\frac{1}{4}) = e^{1/4} - 1/\frac{1}{4} = \sqrt[4]{e} - 4 < 0$

$f(2) = e^2 - \frac{1}{2} > 0$



so there is an x in $[1/4, 2]$ s.t. $f(x) = 0$, i.e. $e^x = 1/x$ by the intermediate value theorem.