

Math 231 Calculus 1 Sample midterm 2 solutions

①

Q1 a)
$$\frac{dy}{dx} = \frac{(7 + \cos(3x)) 5e^{5x} - (-3\sin(3x))e^{5x}}{(7 + \cos(3x))^2}$$

b)
$$\frac{dy}{dx} = 14 \left((7x)^{1/3} + (x^2+4)^{1/2} \right)^{13} \cdot \left(\frac{1}{3} (7x)^{-2/3} \cdot 7 + \frac{1}{2} (x^2+4)^{-1/2} \cdot 2x \right)$$

c)
$$\frac{dy}{dx} = \frac{\sec^2(3x+4) \cdot 3}{2 + \tan(3x+4)}$$

d)
$$e^y + x e^y \cdot \frac{dy}{dx} = \frac{dy}{dx} - 1$$

$$\frac{dy}{dx} (x e^y - 1) = -1 - e^y$$

$$\frac{dy}{dx} = \frac{1 + e^y}{1 - x e^y}$$

Q2 $f(x) = \frac{1}{2x+3}$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{\frac{1}{2(x+h)+3} - \frac{1}{2x+3}}{h}$$

$$= \lim_{h \rightarrow 0} \frac{(2x+3) - (2x+2h+3)}{h(2x+2h+3)(2x+3)} = \lim_{h \rightarrow 0} \frac{-2h}{h(2x+2h+3)(2x+3)}$$

$$= \lim_{h \rightarrow 0} \frac{-2}{(2x+2h+3)(2x+3)} = \lim_{h \rightarrow 0} \frac{-2}{(2x+3)^2}$$

②

Q3 a) $A'(x) = f'(x)g(x) + f(x)g'(x)$

$$A'(2) = f'(2)g(2) + f(2) \cdot g'(2)$$

$$= (-1) \cdot 4 + 1 \cdot 2 = -4 + 2 = -2$$

b) $B'(x) = f'(g(x)) \cdot g'(x)$

$$B'(2) = f'(g(2)) \cdot g'(2) = f'(4) \cdot 2 = 3 \cdot 2 = 6$$

c) $c(x) = g^{-1}(x) \quad 0 \leq x \leq 3$

$$c(2) = 1$$

$$c'(2) = \frac{1}{g'(c(2))} = \frac{1}{g'(1)} = \frac{1}{2}$$

Q4 slope: implicit differentiation $2x + 4y \frac{dy}{dx} = 0$

slope = 1 : $\left. \begin{array}{l} 2x + 4y = 0 \\ x^2 + 2y^2 = 1 \end{array} \right\} \begin{array}{l} x = -2y \\ (-2y)^2 + 2y^2 = 1 \end{array}$

$$6y^2 = 1$$

$$y = \pm \frac{1}{\sqrt{6}}$$

$$x = \mp \frac{2}{\sqrt{6}}, \text{ solutions } \left(\frac{2}{\sqrt{6}}, -\frac{1}{\sqrt{6}} \right), \left(-\frac{2}{\sqrt{6}}, \frac{1}{\sqrt{6}} \right)$$

Q5 $h(t) = h_0 + v_0 t - \frac{1}{2} g t^2$

$h(t) = 0 + 25t - 5t^2$

$h'(t) = 25 - 10t$

a) max height when slope is zero. $h'(t) = 25 - 10t = 0$

so $t = 2.5$

$h(2.5) = 25 \cdot 2.5 - 5(2.5)^2 = 31.25$

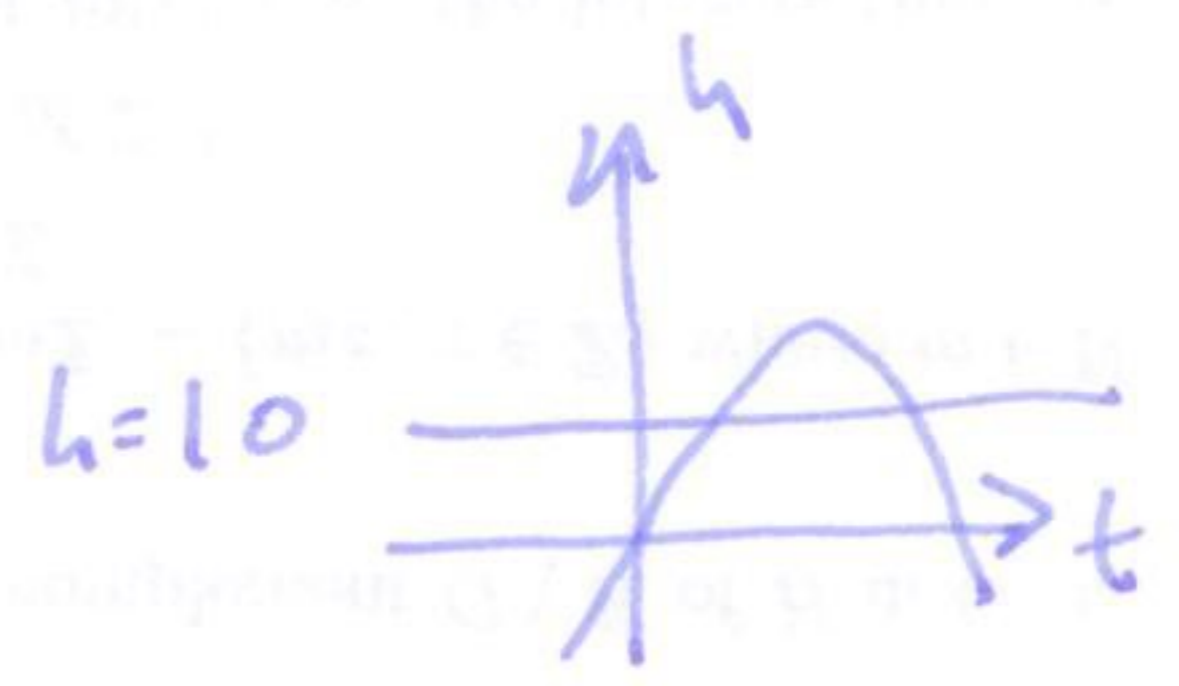
b) $h(t) = 10 = 25t - 5t^2$

$5t^2 - 25t + 10 = 0$

$t^2 - 5t + 2 = 0$

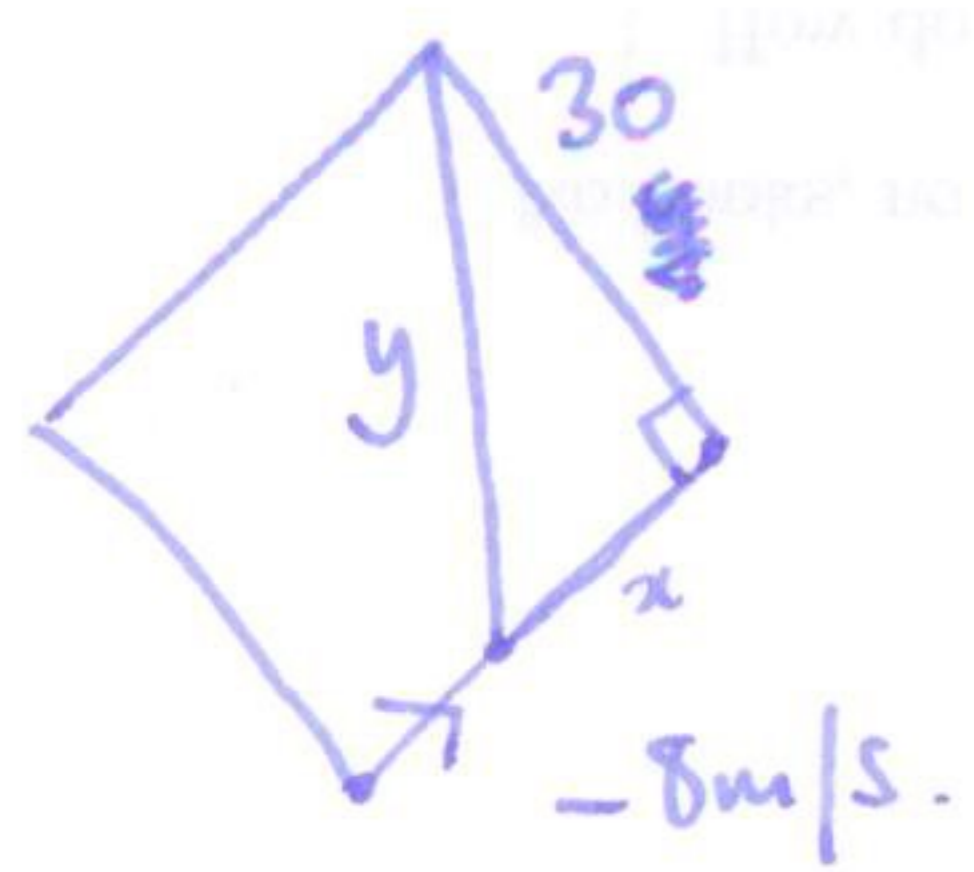
$t = \frac{5 \pm \sqrt{25 - 8}}{2} = \frac{5 \pm \sqrt{17}}{2}$

choose smallest answer $\frac{5 - \sqrt{17}}{2}$



velocity $h'(\frac{5 - \sqrt{17}}{2}) = 25 - 10(\frac{5 - \sqrt{17}}{2}) = 5\sqrt{17}$

Q6



$x^2 + 30^2 = y^2$

$2x = 2y \frac{dy}{dx}$

$\frac{dy}{dx} = \frac{x}{y}$

$x = 15 \Rightarrow y = \sqrt{675} \approx 26$

$\frac{dy}{dx} \approx \frac{15}{26} \approx 0.58 \text{ m/s}$