## Math 231 Calculus 1 Fall 10 Midterm 1b

**Name:** Solutions

- You may use a calculator, but no notes.

| | | |
|---|---|---|
| 1 | 20 | |
| 2 | 20 | |
| 3 | 20 | |
| 4 | 20 | |
| 5 | 20 | |
| 6 | 20 | |
| | 120 | |

| | |
|---|---|
| Midterm 1 | |
| Overall | |

(1) (20 points) The graph of $y = f(x)$ is shown below. Evaluate each limit, or write DNE if the limit does not exist. No justifications are necessary.

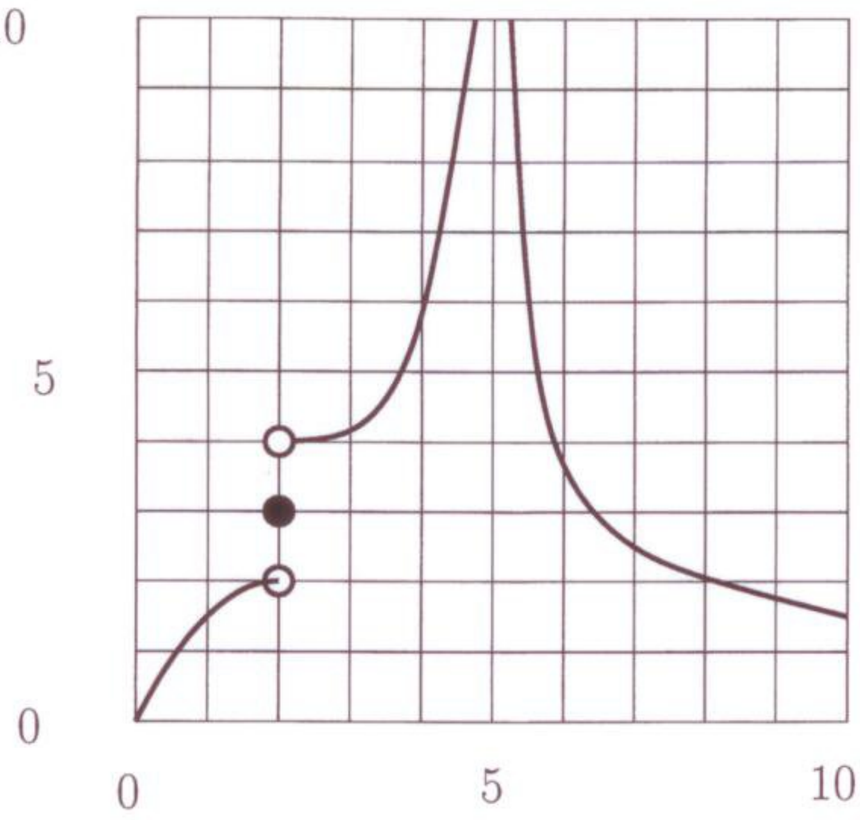



FIGURE 1. $f(x)$

(a) $\lim_{x\to 2+} f(x)$ 4

(b) $\lim_{x\to 2} f(x)$ DNE

(c) $\lim_{x\to 5} f(x)$ $+\infty$

(d) $\lim_{x\to 8-} f(x)$ 2

(2) (20 points) For what value of $c$ (if any) is the function $f(x)$ continuous at $x = 2$? Justify your answer.

$$f(x) = \begin{cases} x + \frac{1}{x-1} & x < 2 \\ c & x = 2 \\ \frac{6\cos(\pi x)}{x} & x > 2 \end{cases}$$

$\lim_{x \to 2-} f(x) = \lim_{x \to 2-} x + \frac{1}{x-1} = 2 + \frac{1}{1} = 3.$

$\lim_{x \to 2+} f(x) = \lim_{x \to 2+} \frac{6\cos(\pi x)}{x} = \frac{6}{2} = 3$

so right limit = left limit, so $f(x)$ is cts if we choose $c = 3$.

(3) (20 points) Evaluate these limits. For an infinite limit, write $+\infty$ or $-\infty$. If a limit does not exist (DNE), you must justify why this is the case.

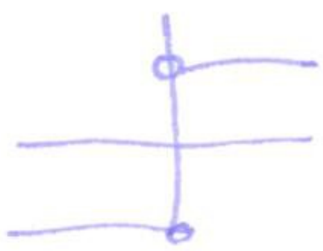
(a) $\lim_{x\to 2} \frac{x-2}{|x-2|}$

(b) $\lim_{x\to 0} \frac{\sin 2x \sin 3x}{x^2}$

(c) $\lim_{x\to 8} \frac{2-\sqrt{x-4}}{x-8}$

(d) $\lim_{h\to 0} \frac{4x^2-(2x+h)^2}{h}$

a) $\lim_{x\to 2} \frac{x-2}{|x-2|} = \lim_{x\to 0} \frac{x}{|x|} = \begin{cases} +1 & x>0 \\ -1 & x<0 \end{cases}$



different right/left limits so DNE.

b) $\lim_{x\to 0} \frac{\sin 2x \sin 3x}{x^2} = \lim_{x\to 0} \frac{\sin 2x}{x} \frac{\sin 3x}{x}$

$\lim_{x\to 0} \frac{\sin 2x}{x} = \lim_{\substack{\theta\to 0 \\ 2x=\theta}} \frac{\sin\theta}{\theta/2} = 2\lim_{\theta\to 0} \frac{\sin\theta}{\theta} = 2$

$\lim_{x\to 0} \frac{\sin 3x}{x} = \lim_{\substack{\phi\to 0 \\ \phi=3x}} \frac{\sin\phi}{\phi/3} = 3\lim_{\phi\to 0} \frac{\sin\phi}{\phi} = 3$

both limits exist so product limit exists and is the product, so $\lim_{x\to 0} \frac{\sin 2x \sin 3x}{x^2} = 6$

c) $\lim_{x\to 8} \frac{(2-\sqrt{x-4})}{(x-8)} \frac{(2+\sqrt{x-4})}{(2+\sqrt{x-4})} = \lim_{x\to 8} \frac{4-x+4}{(x-8)(2+\sqrt{x-4})} = \lim_{x\to 8} \frac{-1}{2+\sqrt{x-4}} = -\frac{1}{4}$

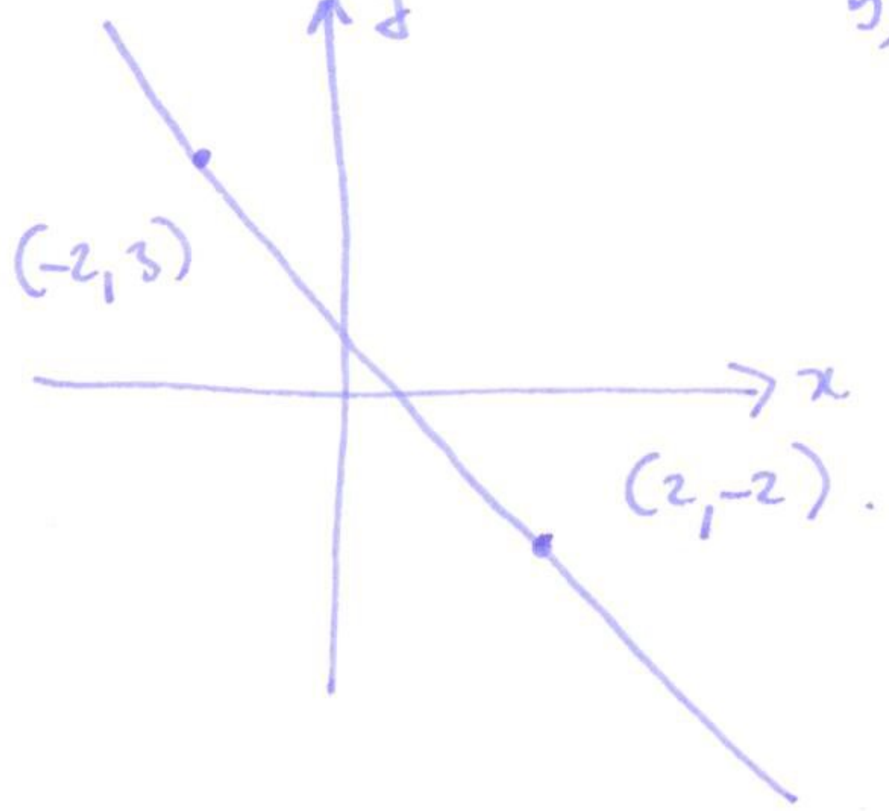
d) $\lim_{h\to 0} \frac{4x^2-(2x+h)^2}{h} = \lim_{h\to 0} \frac{4x^2-4x^2-4xh-h^2}{h} = \lim_{h\to 0} -4x-h = -4x.$

(4) (20 points)

(a) Plot the two points $(-2, 3)$ and $(2, -2)$ in the $xy$-plane, and draw the straight line that runs through both of them.

(b) Write down the equation of the line.

a)




b) $\text{slope} = \frac{\Delta y}{\Delta x} = \frac{3-(-2)}{-2-2} = -\frac{5}{4}$

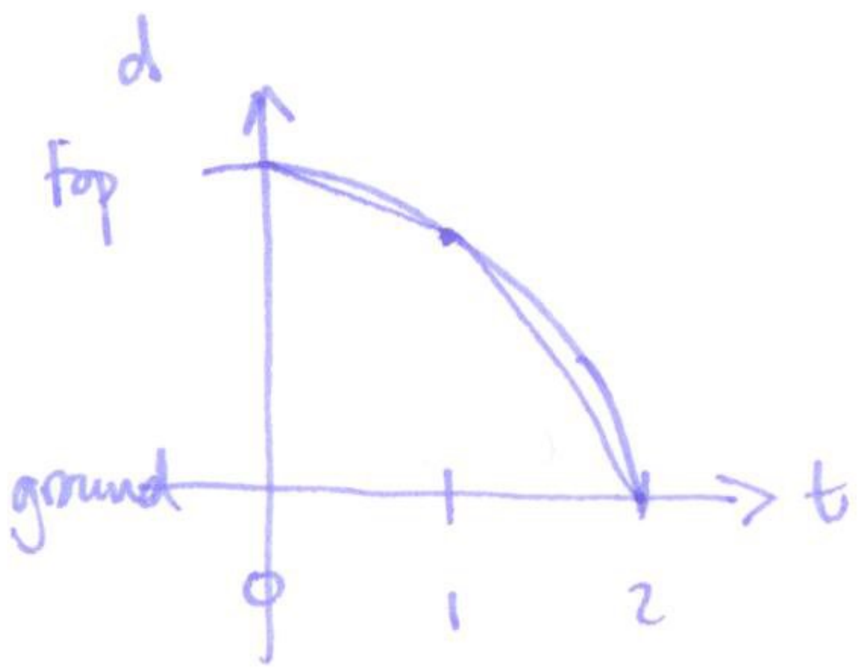
$y - y_0 = m(x - x_0)$

$y - 3 = -\frac{5}{4}(x+2)$

$y = -\frac{5}{4}x - \frac{5}{2} + 3$

$y = -\frac{5}{4}x + \frac{1}{2}$

(5) (20 points) You drop a stone off the top of an apartment building, and it takes roughly two seconds to hit the ground.
   (a) Draw a rough sketch of the graph of distance against time for the stone.
   (b) Looking at your graph, how would you compare the average rate of change between times 0 and 1 second, and between times 1 and 2 seconds.




both rates of change negative,

and

average ROC between 0,1 > average rate of change between 1,2 .

(6) (20 points) A population of bacteria doubles in size every minute. If there are 100 bacteria at time 0, what is the average rate of change in population between 2 and 4 minutes?

| time | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| # bacteria | 100 | 200 | 400 | 800 | 1600 |

$\#\text{bacteria} = 100 \cdot 2^t$ ($t$ in minutes)

average rate of change between $t=2, 4$ is $\dfrac{1600-400}{2} = 600$ bacteria/minute.