Math 231 Calculus 1 Fall 10 Midterm 1

Name: Solutions

- You may use a calculator, but no notes.

1	20	
2	20	
3	20	
4	20	
5	20	
6	20	
	120	

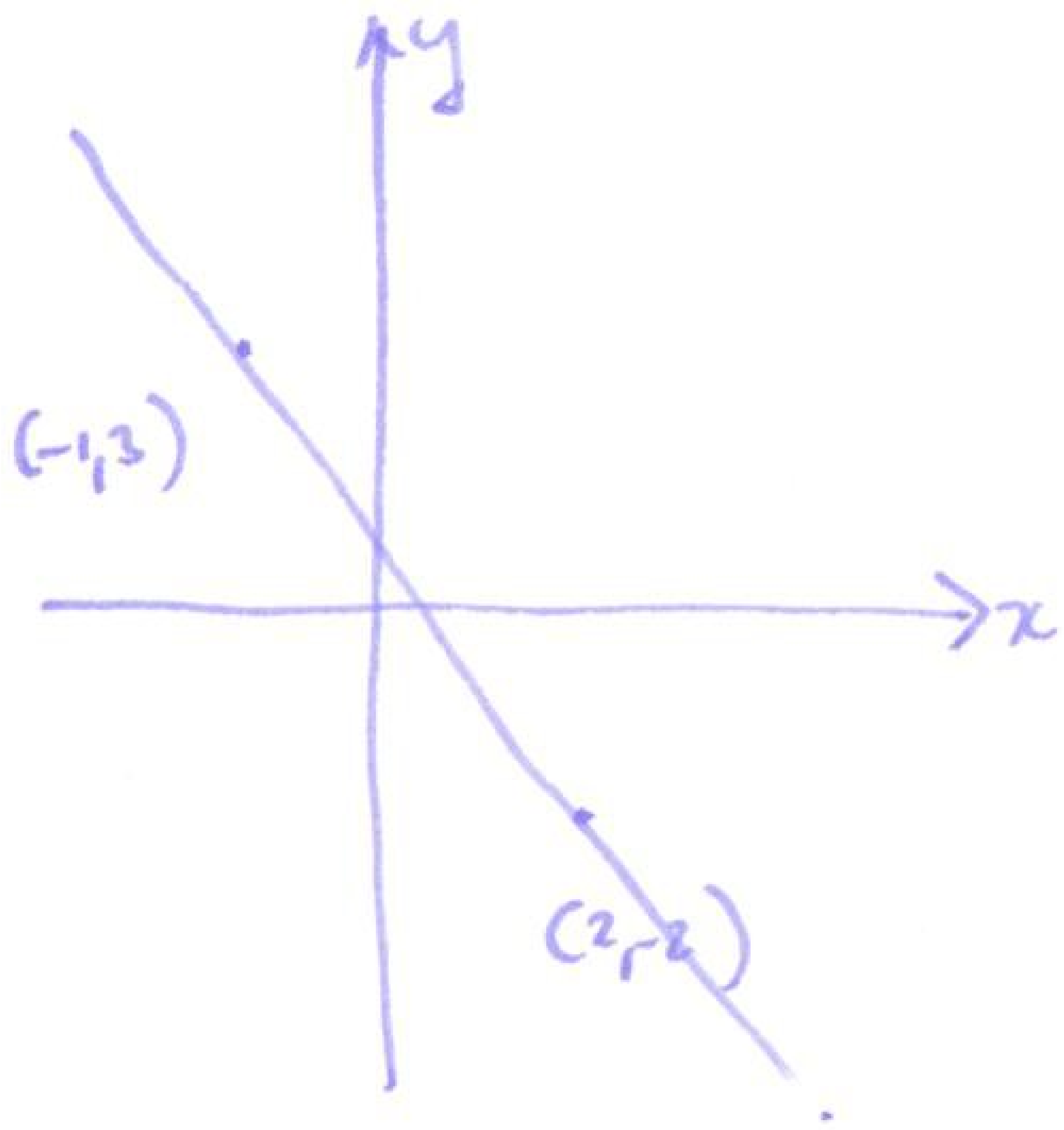
Midterm 1	
Overall	

(1) (20 points)

(a) Plot the two points $(-1, 3)$ and $(2, -2)$ in the xy-plane, and draw the straight line that runs through both of them.

(b) Write down the equation of the line.

a)



b) $\text{slope} = \frac{\Delta y}{\Delta x} = \frac{3-(-2)}{-1-2} = -\frac{5}{3}$

$$y - y_0 = m(x - x_0)$$

$$y - 3 = -\frac{5}{3}(x - (-1))$$

$$y = -\frac{5}{3}x - \frac{5}{3} + 3$$

$$y = -\frac{5}{3}x + \frac{4}{3}$$

(2) (20 points) The graph of $y = f(x)$ is shown below. Evaluate each limit, or write DNE if the limit does not exist. No justifications are necessary.

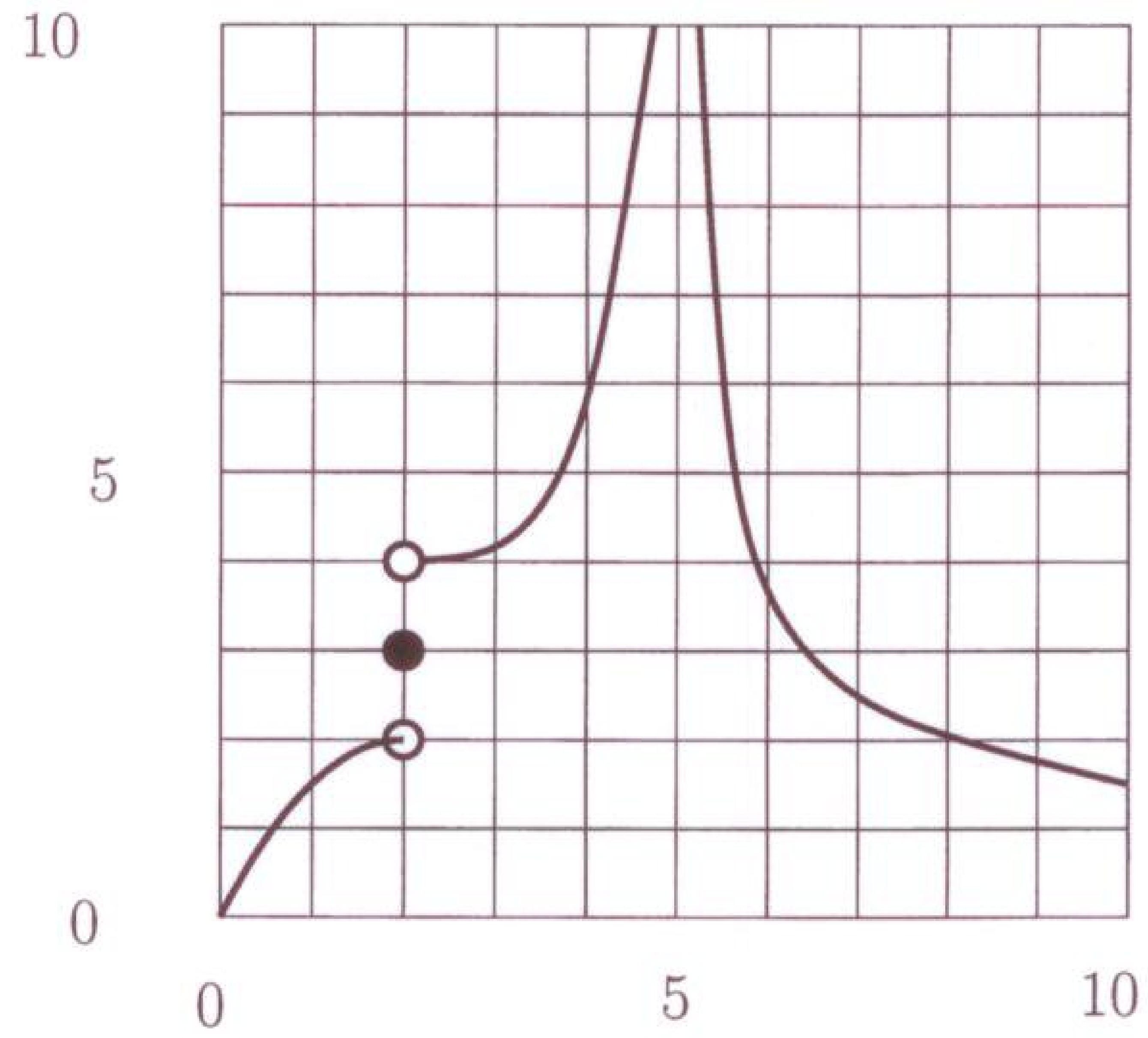


FIGURE 1. $f(x)$

(a) $\lim_{x \to 2-} f(x)$ 2

(b) $\lim_{x \to 2} f(x)$ DNE

(c) $\lim_{x \to 5} f(x)$ $+\infty$

(d) $\lim_{x \to 8+} f(x)$ 2

(3) (20 points) Evaluate these limits. For an infinite limit, write $+\infty$ or $-\infty$. If a limit does not exist (DNE), you must justify why this is the case.

(a) $\lim_{x\to 1} \frac{x-1}{|x-1|}$

(b) $\lim_{x\to 0} \frac{\sin 2x \sin 3x}{x^2}$

(c) $\lim_{x\to 10} \frac{2-\sqrt{x-6}}{x-10}$

(d) $\lim_{h\to 0} \frac{(3x+h)^2-9x^2}{h}$

a) $\lim_{x\to 1} \frac{x-1}{|x-1|} = \lim_{x\to 0} \frac{x}{|x|} = \begin{cases} +1 & x>0 \\ -1 & x<0 \end{cases}$ different left/right limits so DNE.

b) $\lim_{x\to 0} \frac{\sin 2x \sin 3x}{x^2} = \lim_{x\to 0} \frac{\sin 2x}{x} \frac{\sin 3x}{x}$

$\lim_{x\to 0} \frac{\sin 2x}{x} \underset{\theta=2x}{=} \lim_{\theta\to 0} \frac{\sin\theta}{\theta/2} = \lim_{\theta\to 0} 2\frac{\sin\theta}{\theta} = 2$

$\lim_{x\to 0} \frac{\sin 3x}{x} \underset{\theta=3x}{=} \lim_{\theta\to 0} \frac{\sin\theta}{\theta/3} = \lim_{\theta\to 0} 3\frac{\sin\theta}{\theta} = 3.$

both limits exist so $\lim_{x\to 0} \frac{\sin 2x}{x}\frac{\sin 3x}{x} = \lim_{x\to 0} \frac{\sin 2x}{x} \lim_{x\to 0} \frac{\sin 3x}{x} = 6.$

c) $\lim_{x\to 10} \frac{2-\sqrt{x-6}}{x-10} \frac{2+\sqrt{x-6}}{2+\sqrt{x-6}} = \frac{4-x+6}{(x-10)(2+\sqrt{x-6})} = \frac{-1}{2+\sqrt{x-6}} = -\frac{1}{4}$

d) $\lim_{h\to 0} \frac{(3x+h)^2-9x^2}{h} = \lim_{h\to 0} \frac{9x^2+6xh+h^2-9x^2}{h} = \lim_{h\to 0} 6x+h = 6x.$

(4) (20 points) For what value of c (if any) is the function $f(x)$ continuous at $x = 2$? Justify your answer.

$$f(x) = \begin{cases} x + \frac{1}{x-1} & x < 2 \\ c & x = 2 \\ \frac{6\cos(\pi x)}{x} & x > 2 \end{cases}$$

$$\lim_{x \to 2-} f(x) = \lim_{x \to 2-} x + \frac{1}{x-1} = 2 + \frac{1}{2-1} = 3.$$

$$\lim_{x \to 2+} f(x) = \lim_{x \to 2+} \frac{6\cos(\pi x)}{x} = \frac{6}{2} = 3$$

both left and right limits exist and are the same.

so function continuous if $c = 3$.

(5) (20 points) A population of bacteria doubles in size every minute. If there are 100 bacteria at time 0, what is the average rate of change in population between 2 and 4 minutes?

time	0	1	2	3	4
# bacteria	100	200	400	800	1600

bacteria = $100 \cdot 2^t$ (t in minutes)

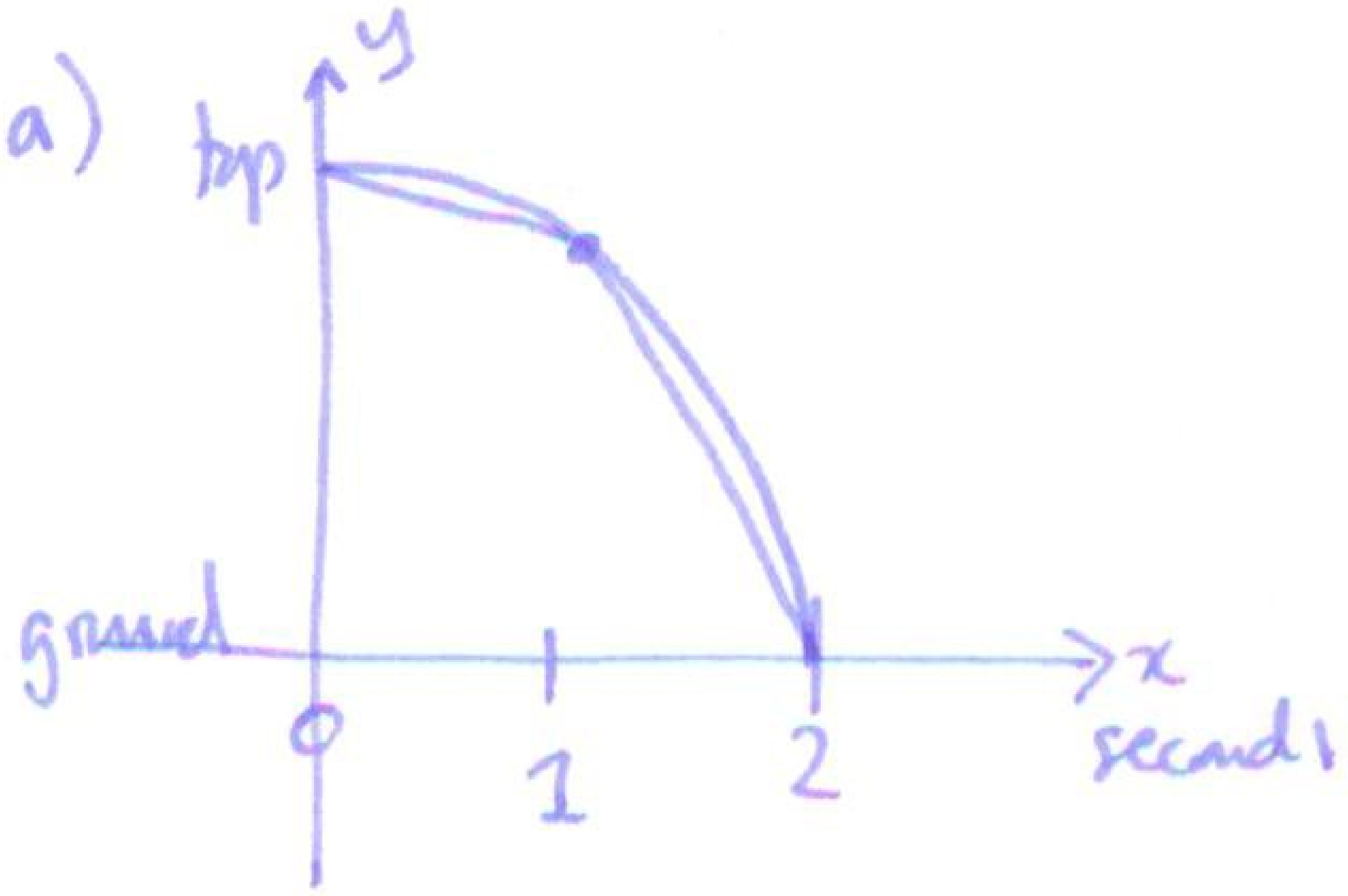
average rate of change between $t=2$ and 4 is $\frac{\Delta y}{\Delta x} = \frac{1600-400}{2} = 600$ bacteria/minutes.

(6) (20 points) You drop a stone off the top of an apartment building, and it takes roughly two seconds to hit the ground.

(a) Draw a rough sketch of the graph of distance against time for the stone.

(b) Looking at your graph, how would you compare the average rate of change between times 0 and 1 second, and between times 1 and 2 seconds.

a)



both rates of change negative,

average RoC between 0,1 > average RoC between 1,2

i.e. stone goes downwards faster between $t=1,2$ than between $t=0,1$.