THE COLLEGE OF STATEN ISLAND
DEPARTMENT OF MATHEMATICS
Math 230/231 Final Exam
Fall 2010

FORM A

Name:______________________________

PART I – Answer all questions in this part.

a) Evaluate $\int \frac{3+x^2-5x^3}{x}dx$

(4%)

$\int \frac{3}{x} + x - 5x^2 dx$

a) $3\ln|x| + \frac{1}{2}x^2 - \frac{5}{3}x^3 + C$

b) Evaluate $\int e^{4x-3}dx$

(4%)

b) $\frac{1}{4}e^{4x-3} + C$

c) Evaluate $\int (\cos 7x - \sin 3x)\, dx$

(4%)

c) $\frac{1}{7}\sin 7x + \frac{1}{3}\cos 3x + C$

d) If $y=\frac{74}{x^2}-\sqrt[5]{x^3}+e^4$ find $\frac{dy}{dx}$ d)________________

(4%) $y = 74x^{-2} - x^{3/5} + e^4$

$\frac{dy}{dx} = -2\cdot 74x^{-3} - \frac{3}{5}x^{-2/5}$

$= -148x^{-3} - \frac{3}{5}x^{-2/5}$

e) If $f(x)=\frac{3x}{7x^3+4}-x^3\left(\sqrt{3x-7}\right)$ find $f'(x)$

(4%) e)________________

$f'(x) = \frac{(7x^3+4)3-(3x)(21x^2)}{(7x^3+4)^2} - 3x^2(3x-7)^{1/2} - x^3\frac{1}{2}(3x-7)^{-1/2}\cdot 3$

f) If $f(x)=\ln(7x^3+3)+e^{5x}$ find $f''(x)$ f)________________

(4%) $f'(x) = \frac{1}{7x^3+3}\cdot 21x^2 + 5e^{5x}$

$f''(x) = \frac{(7x^3+3)42x-(21x^2)\cdot 21x^2}{(7x^3+3)^2} + 25e^{5x}$

Find each of the following limits: Your answer may be a real number, $-\infty$, $+\infty$ or DNE (does not exist). Justify your answer.

g) $\lim_{x \to 5}\left(\frac{x^2-25}{5-x}\right) = \lim_{x \to 5} \frac{(x+5)(x-5)}{-(x-5)}$ g) -10

(4%)

$= \lim_{x \to 5} -(x+5)$

h) $\lim_{x \to 6^-}\left(\frac{7}{6-x}\right)$ h) $+\infty$

(4%)

i) $\lim_{x \to \infty}\left(\frac{x^4-3x^2+\sqrt{x}}{4x^5-7\sqrt{x}}\right)$ i) 0

(4%)

Consider the function $f(x) = \dfrac{7x^2}{x^2+4}$

j. Find the equation of each vertical asymptote in the xy plane (if any). Justify your answer.

j)_______________

(4%)

$x^2+4=0$ no real solutions

as $x^2 \geq 0$ so $x^2+4 \geq 4 > 0$.

k) Find the equation of each horizontal asymptote in the xy plane (if any). Justify your answer.

k)_______________

(4%) $\dfrac{7x^2}{x^2+4} = \dfrac{7}{1+4/x^2}$

$\lim_{x \to +\infty} \dfrac{7}{1+4/x^2} = 7$ so horizontal asymptotes are both $y=7$

$\lim_{x \to -\infty} \dfrac{7}{1+4/x^2} = 7$

Consider the function $f(x) = \frac{1}{3}x^3 + \frac{3}{2}x^2 + 2x + 4$

l) Determine the interval(s) on which f(x) is increasing or decreasing.

(4%)

Increasing $(-\infty, -2) \cup (-1, \infty)$

$f'(x) = x^2 + 3x + 2$

$= (x+2)(x+1)$

Decreasing $(-2, -1)$

$+ \quad | \quad - \quad | \quad +$

$-2 \quad -1$

m) Find the x-values of all relative maxima or minima (if any).

(4%) $f'(x) = 0$ when $x = -2, -1$

Rel maxima $x = -2$

Rel minima $x = -1$

$f''(x) = 2x+3$

$f''(-2) = -1 \Rightarrow$ local max

$f''(-1) = +1 \Rightarrow$ local min

n) Determine the interval(s) where f(x) is concave up or concave down.

(4%) $f''(x)$ $- \quad | \quad +$

$-3/2$

f concave up on $(-3/2, \infty)$

f concave down on $(-\infty, -3/2)$

$f''(x) > 0$ concave up

$f''(x) < 0$ concave down

o) Find the x-values of the point(s) of inflection (if any).

PT of inflection $-3/2$

(3%) $f''(x) = 0$ at $x = -3/2$

Part II – Answer any TWO (2) of the following (10% each)

1. An oil tanker in the Atlantic Ocean has just sprung a leak. The radius of the oil slick formed is increasing at the rate of thirty centimeters per minute. Find the rate at which the area of the oil slick is increasing when the radius is 27 cm? (Remember: Area of Circle = πr^2)

$A = \pi r^2$ $\quad \frac{dr}{dt} = 30 \text{ cm/min}$

1 $1620\pi \text{ cm}^2/\text{min}$

$\frac{dA}{dt} = \pi 2r \frac{dr}{dt}$ $\quad$ when $r = 27$ $\quad \frac{dA}{dt} = \pi . 2 . 27 . 30$

2. A curve is defined by the equation $3x^4 + 7x^3 = 7xy - 4$

a) Find a formula for the derivative $\frac{dy}{dx}$ in terms of x and y.

$12x^3 + 21x^2 = 7x\frac{dy}{dx} + 7y$

2a)________________

$\frac{dy}{dx} = \frac{12x^3 + 21x^2 - 7y}{7x}$

b) Find the slope of the tangent line at the point (1,2).

$\frac{dy}{dx} = \frac{12 + 21 - 14}{7} = \frac{19}{7}$

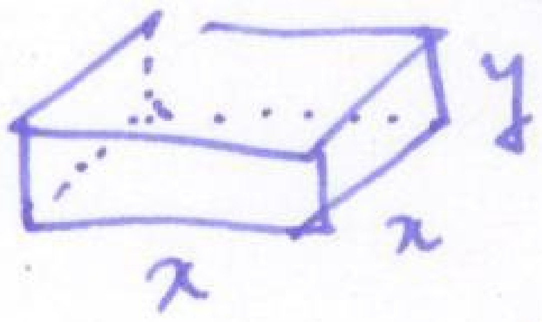
2b)________________

c) Find the equation of the tangent line at this point.

2c)________________

$y - 2 = \frac{19}{7}(x - 1)$

3. A box is constructed of two different types of metal. The metal for the top and bottom, which are both square, costs \$1/ft^2 and the metal for the sides costs \$2/ft^2. Find the dimensions that minimize cost if the box has a volume of 20 ft^3.



$V = 20 = x^2y \qquad y = \frac{20}{x^2}.$

$C = 2x^2 + 2.4xy$

Height____________

Length____________

Width____________

$C = 2x^2 + 8x . \frac{20}{x^2} = 2x^2 + \frac{160}{x}$

$\frac{dC}{dx} = 4x - \frac{160}{x^2} = 0 \qquad x^3 = 40 \qquad x = \sqrt[3]{40} \qquad y = 20/\sqrt[3]{1600}$

Part III – Answer any two (2) of the following: (10% each)

1a) Using the definition of the derivative as a limit, find $f'(x)$ for the function $f(x) = 3x^2 - 2x$

1a)____________

$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$

$f'(x) = \lim_{h\to 0} \frac{3(x+h)^2 - 2(x+h) - 3x^2 + 2x}{h} = \lim_{h\to 0} \frac{3x^2 + 6xh + 3h^2 - 2x - 2h - 3x^2 + 2x}{h}$

$f'(x) = \lim_{h\to 0} 6x + 3h - 2 = 6x - 2$

1b) Check your answer by using the appropriate derivative rules.

1b)____________

$f'(x) = 6x - 2$

2) Using Newton's method find the zero(s) of $f(x) = \cos x - x$, (complete three iterations with an initial guess of x=0.8) $\left[\text{Hint: } x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}\right]$

$$x_{n+1} = x_n - \frac{\cos(x_n) - x_n}{-\sin(x_n) - 1}$$

2)________________

$x_0 = 0.8$

$x_1 = 0.73985$

$x_2 = 0.73909$

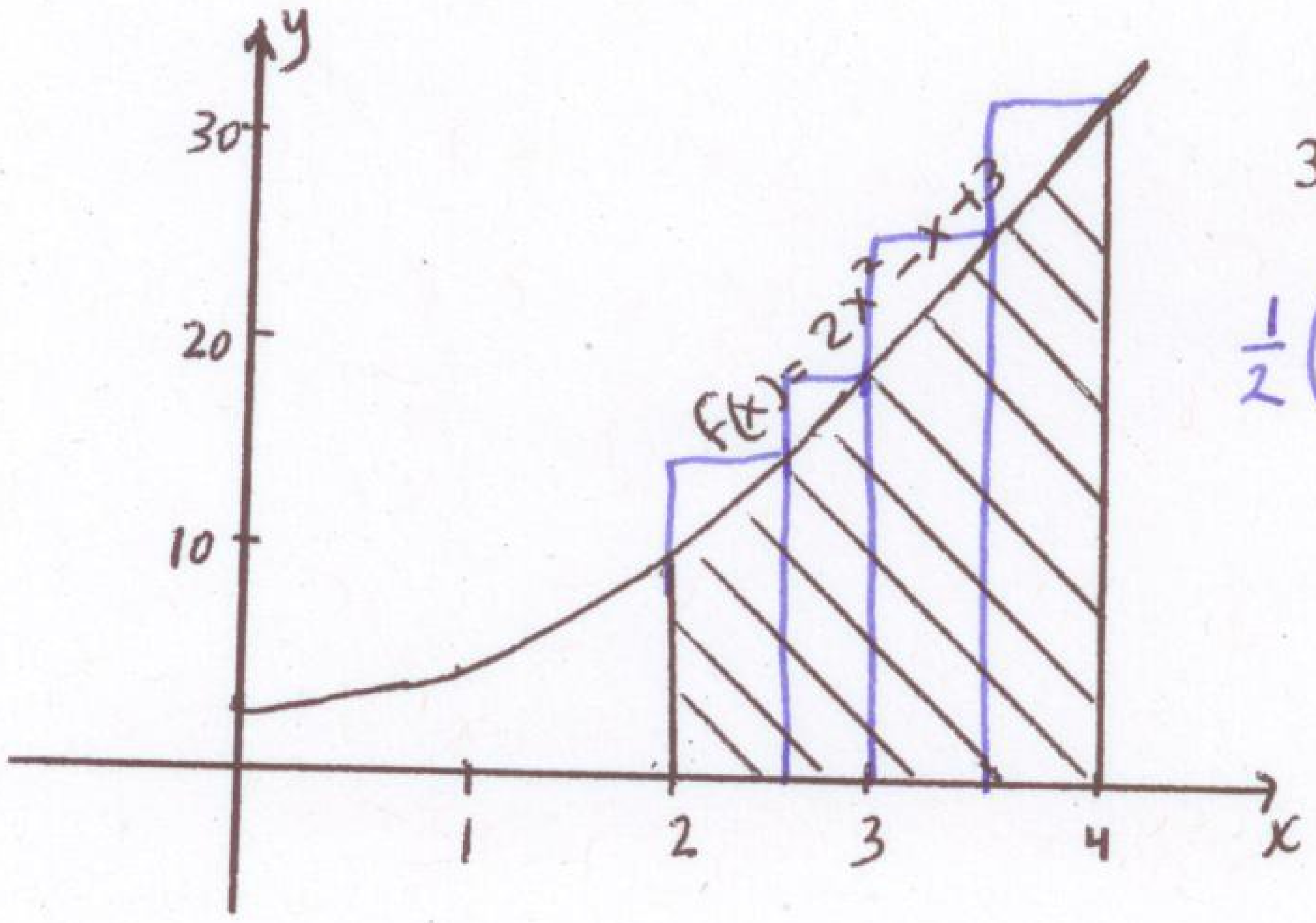
$x_3 = 0.73909$

2b) Consider the function $f(x) = (x-1)(x-3)$ $= x^2 - 4x + 3$. Find a point C in the interval [1,3] satisfying the MVT (mean value theorem).

$f(1) = 0$ $f(3) = 0$ so average RoC $= 0$

$f'(x) = 2x - 4$ so $f'(x) = 0$ has solution $x = 2$

3a) Find an estimate of the area shown by using a Riemann Sum with four rectangles of equal width and by using the right end points of each rectangle for the height. The region is bounded by $f(x) = 2x^2 - x + 3$, the x-axis, x=2 and x=4.



3a 43

$$\frac{1}{2}\left(f(2\tfrac{1}{2})+f(3)+f(3\tfrac{1}{2})+f(4)\right)$$

3b) Find the exact area by setting up and evaluating an appropriate integral.

$$\int_2^4 2x^2-x+3\,dx = \left[\frac{2}{3}x^3-\frac{1}{2}x^2+3x\right]_2^4$$

$$= \left(\frac{2}{3}\cdot 64-\frac{1}{2}\cdot 16+12\right)-\left(\frac{16}{3}-2+6\right)$$

3b) $37\frac{1}{3} = \frac{112}{3}$