§12.1 Vector valued functions

Real valued functions of 1-variable: \( f: \mathbb{R} \rightarrow \mathbb{R} \),
- \( x \mapsto f(x) \)
- \( e.g. \ x \mapsto x^2 \)

Parameterised curves: \( \mathbb{R} \rightarrow \mathbb{R}^2 \) \( t \mapsto (f(t), g(t)) \)

Alternate notation: \( \boldsymbol{r}(t) = f(t) \mathbf{i} + g(t) \mathbf{j} \) (2d)

Convention: normally don't explicitly describe domain, e.g. \( f(x) = \frac{1}{x} \) not defined for \( x = 0 \).

Examples:
- \( \boldsymbol{r}(t) = (\cos(t), \sin(t)) \)

Observation:
- \( x = \cos t \) \( y = \sin t \) \( \Rightarrow x^2 + y^2 = \cos^2 t + \sin^2 t = 1 \)

Warning: the same curve has many different parameterizations.
- \( \langle \cos(t), \sin(t) \rangle \) \( 0 \leq t < 2\pi \)
- \( \langle \cos(2t), \sin(t) \rangle \) \( 0 \leq t < \pi \)
- \( \langle \cos(t), \sin(t) \rangle \) all values of \( t \).
\[ \Gamma(t) = \langle \cos t, \sin t, t \rangle \]

Finding a parameterization:

E.g. \[ y = x^2 \]

Set \[ x = t \]
\[ y = t^2 \]
\[ \Gamma(t) = \langle t, t^2 \rangle \]

Another example \[ \Gamma(t) = \langle \cos t, \sin 2t \rangle \]

Addition and scalar multiplication

\[ \Gamma_1(t) = \langle f_1(t), g_1(t) \rangle \] then \[ \Gamma_1(t) + \Gamma_2(t) = \langle f_1(t) + f_2(t), g_1(t) + g_2(t) \rangle \]

\[ c \Gamma_1(t) = \langle cf_1(t), cg_1(t) \rangle \]

Limits and continuity

Recall: a function of 1-var may (or may not) have a limit \[ \lim_{t \to a} f(t) \]

\[ \lim_{t \to a} f(t) = L \] if for all \( \epsilon > 0 \) there is a \( \delta > 0 \) st.

\[ |f(t+\delta) - L| < \epsilon \] for all \( |t| \leq \delta \).