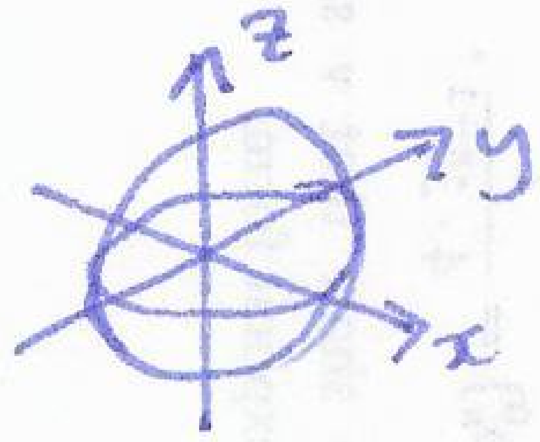
Sample midterm 2 Solutions

Q1 cylindrical : $r^2+z^2=16$ ⊛

rectangular : $r^2=x^2+y^2$ so ⊛ : $x^2+y^2+z^2=16$, sphere of radius 4.



spherical : $\rho=4$.

Q2 $\underline{a}(t)=\langle 6t, 12t^2, -6\rangle$

$\underline{v}(t)=\int \underline{a}(t)dt = \langle 3t^2, 4t^3, -6t\rangle + \underline{c}$

$\underline{v}(0)=\langle 0,0,0\rangle+\underline{c}=\langle 3,-4,5\rangle$

so $\underline{v}(t)=\langle 3t^2+3, 4t^3-4, -6t+5\rangle$.

$\underline{r}(t)=\int \underline{v}(t)dt$

$\underline{r}(t)=\langle t^3+3t, t^4-4t, -3t^2+5t\rangle + \underline{c}$

$\underline{r}(0)=\langle 0,0,0\rangle+\underline{c}=\langle 2,1,-1\rangle$

so $\underline{r}(t)=\langle t^3+3t+2, t^4-4t+1, -3t^2+5t-1\rangle$.

Q3 at $t=0$ $\underline{r}(0)=\langle 0,4\rangle$

$\underline{r}'(0)=\langle 3,0\rangle$

$\underline{r}''(t)=\langle 0,-g\rangle$

$\underline{r}'(t)=\int \underline{r}''(t)\,dt = \langle 0,-gt\rangle+\underline{c}$

$\underline{r}'(0)=\langle 0,0\rangle+\underline{c}$, $\underline{c}=\langle 3,0\rangle$.

$\underline{r}(t) = \int \underline{r}'(t)\,dt$

$= \langle 0, -\frac{1}{2}gt^2 \rangle + \underline{c}t + \underline{d}$

$\underline{r}(0) = \underline{d} = \langle 0, 4 \rangle$

$\underline{r}(t) = \langle 0, -\frac{1}{2}gt^2 \rangle + \langle 3, 0 \rangle t + \langle 0, 4 \rangle$

a) find t when $\underline{r}(t) = \langle *, 0 \rangle$

$-\frac{1}{2}gt^2 + 4 = 0 \quad t^2 = 8g \quad t = \sqrt{8g}$

$\underline{r}(\sqrt{8g}) = \langle 0, -4 \rangle + \langle 3, 0 \rangle \sqrt{8g} + \langle 0, 4 \rangle$

horizontal distance $= 3\sqrt{8g}$

b) speed $= \| \underline{r}'(\sqrt{8g}) \| = \| \langle 0, -g\sqrt{8g} \rangle + \langle 3, 0 \rangle \|$

$= \sqrt{9 + 8g^3}$.

Q4 $\quad \underline{r}(t) = \langle e^t, \sqrt{2}\,t, e^{-t} \rangle$

$\underline{r}'(t) = \langle e^t, \sqrt{2}, -e^{-t} \rangle$

a) speed $= \| \underline{r}'(t) \| = \sqrt{e^{2t} + 2 + e^{-2t}} = \sqrt{(e^t + e^{-t})^2} = e^t + e^{-t}$

b) unit tangent vector $\underline{T}(t) = \frac{\underline{r}'(t)}{\| \underline{r}'(t) \|} = \frac{1}{e^t + e^{-t}} \langle e^t, \sqrt{2}, -e^{-t} \rangle$.

c) length of curve $= \int_1^3 \| \underline{r}'(t) \|\, dt$

$= \int_1^3 e^t + e^{-t}\, dt = \left[e^t - e^{-t} \right]_1^3 = e^3 - e^{-3} - e^1 + e^{-1} = e^3 - e^{-3} + e^{-1} - e.$

Q5 a) $\underline{r}(t) = \langle 4\cos 2\pi t, 4\sin 2\pi t, 5t \rangle \quad 0 \leq t \leq 3$

b) length $= \int_0^3 \|\underline{r}'(t)\|\, dt$

$\underline{r}'(t) = \langle -8\pi \sin 2\pi t, 8\pi \cos 2\pi t, 5 \rangle$

$\|\underline{r}'(t)\| = \sqrt{64\pi^2 + 25}.$

$= \int_0^3 \sqrt{64\pi^2 + 25}\, dt = 3\sqrt{64\pi^2 + 25}.$

Q6 $\|\underline{r}(t)\|^2 = \underline{r}(t) \cdot \underline{r}(t) = c^2$

$\frac{d}{dt}\left(\underline{r}(t) \cdot \underline{r}(t) \right) = \underline{r}'(t) \cdot \underline{r}(t) + \underline{r}(t) \cdot \underline{r}'(t) = 0$

$\Rightarrow \underline{r}'(t) \cdot \underline{r}(t) = 0 \Rightarrow \underline{r}(t), \underline{r}'(t)$ perpendicular.

Q7 straight line parameterized by $\underline{r}(t) = \underline{a} + t\underline{v}$, choose $\|\underline{v}\| = 1$

then $\underline{r}'(t) = \underline{v}$, so unit speed parameterization.

curvature $K = \|T'(t)\| = \|\underline{r}''(t)\|$. but $\underline{r}''(t) = 0$ so $K = 0$.

Q8 $\underline{r}(s)$ parameterized by arc length $\Leftrightarrow \|\underline{r}'(s)\| = 1$.

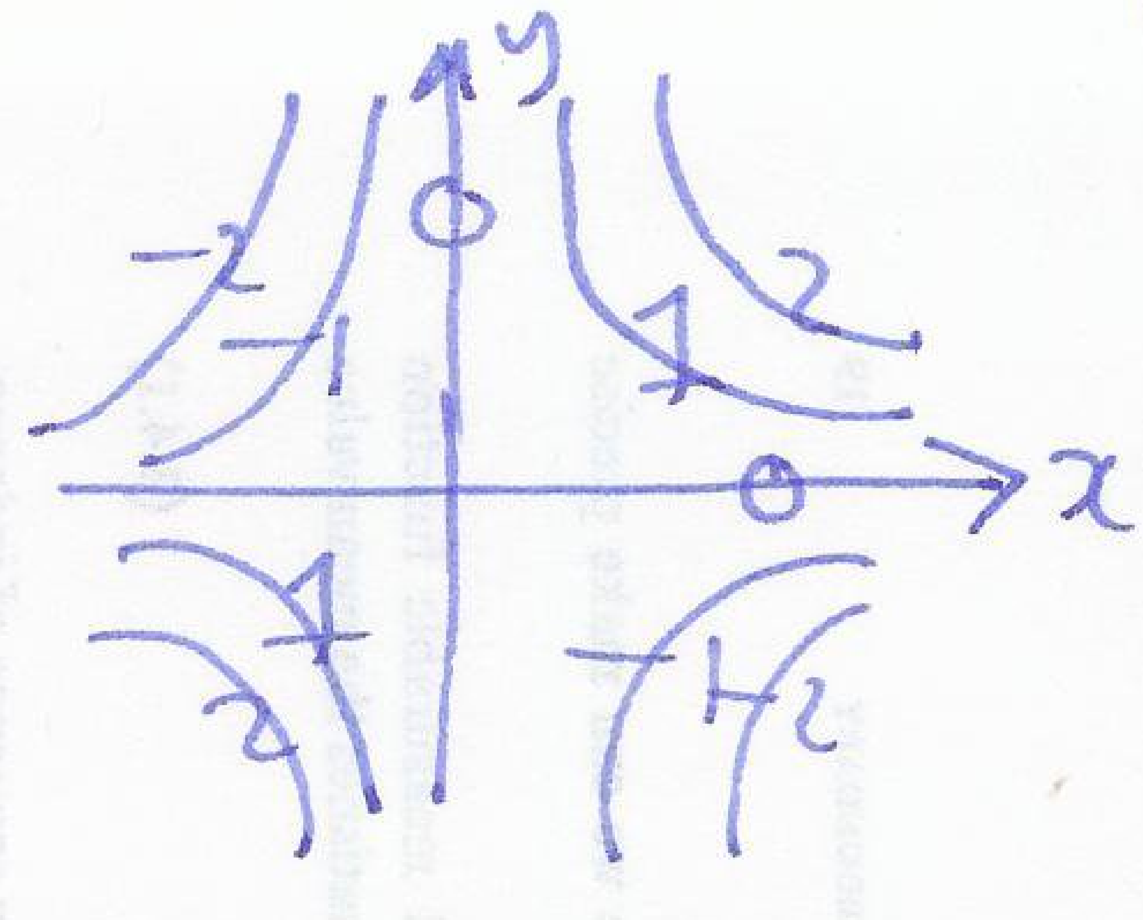
$\underline{r}'(s) = \underline{v}(s)$ $\quad \|\underline{r}'(s)\|^2 = 1 = \underline{r}'(s) \cdot \underline{r}'(s)$

$\underline{r}''(s) = \underline{a}(s).$

so $\frac{d}{dt}\left(\underline{r}'(s) \cdot \underline{r}'(s) \right) = 2\underline{r}''(s) \cdot \underline{r}'(s) = 0$

so $\underline{a}(s), \underline{v}(s)$ perpendicular.

Q9 $f(x,y) = xy$ has level sets $xy = c$



graph of function is a saddle near $(0,0)$.

Q10 $f(x,y,z) = x^2+y^2+z$

has level sets $x^2+y^2+z=c$, $\iff$ $z=-x^2-y^2+c$.