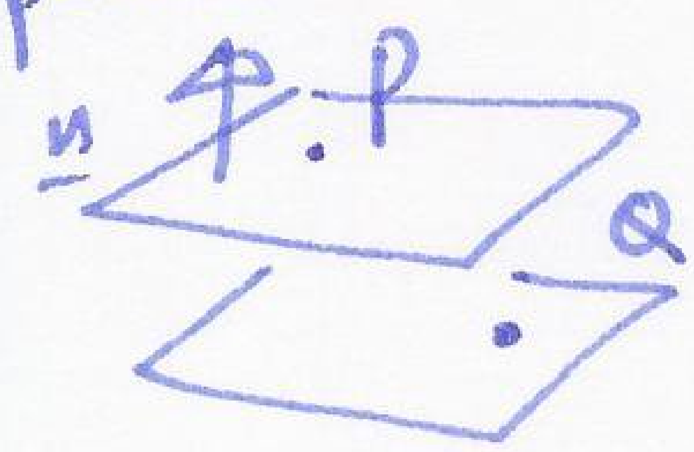
Math 233 Calculus 3 Fall 09 Midterm 1

Name: Solutions

Problem 1. Show that the two planes $x+y-z=4$ and $2x+2y-2z=0$ are parallel, and find the distance between them.

normal vectors are $\langle 1,1,-1\rangle$, $\langle 2,2,-2\rangle$, parallel, so planes are parallel.



find points P, Q on each plane, e.g.

$P=(4,0,0) \quad Q=(0,0,0)$

distance between planes = distance from point on one plane to the other, i.e. $\dfrac{\overrightarrow{PQ}\cdot \underline{n}}{\|\underline{n}\|}$

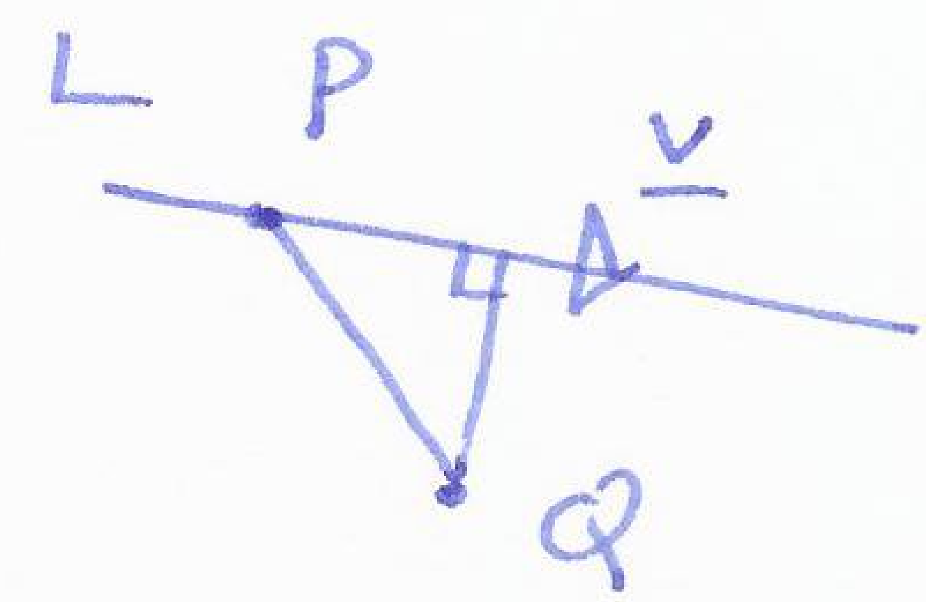
$$= \frac{\langle 4,0,0\rangle\cdot\langle 1,1,-1\rangle}{\|\langle 1,1,-1\rangle\|} = \frac{4}{\sqrt{3}}$$

Problem 2. Let L be the line through the point $P = (1, 1, 1)$ in the direction $\langle 1, -1, 1 \rangle$, and let $Q = (4, 1, 4)$.

(a) Find a unit vector in the direction of L.

(b) Write the vector $\vec{\mathbf{PQ}}$ as a sum of two vectors, one parallel to L, and one perpendicular to L.

(c) Find the distance from the line L to the point Q.



a) $\frac{1}{\sqrt{3}} \langle 1, -1, 1 \rangle = \hat{\underline{v}}$

b) projection vector : $(\overrightarrow{PQ} \cdot \hat{\underline{v}})\, \hat{\underline{v}}$

$= \left(\langle 3, 0, 3 \rangle \cdot \frac{1}{\sqrt{3}} \langle 1, -1, 1 \rangle\right) \frac{1}{\sqrt{3}} \langle 1, -1, 1 \rangle$

$= \frac{1}{3} \cdot 6 \langle 1, -1, 1 \rangle = \langle 2, -2, 2 \rangle$

so $\overrightarrow{PQ} = \underbrace{\langle 2, -2, 2 \rangle}_{\text{parallel to } \underline{v}} + \underbrace{\langle 1, +2, 1 \rangle}_{\text{perpendicular to } \underline{v}}$

c) distance from Q to L = length of perpendicular

$\| \langle 1, 2, 1 \rangle \| = \sqrt{6}$

Problem 3. Consider three points $A(-1,-1,-1)$, $B(2,2,3)$, $C(4,4,5)$.

(a) Find the area of the triangle formed by A, B, C.

(b) Find the equation of the plane that contains A, B, C.

a) area of triangle $= \frac{1}{2} \| \vec{AB} \times \vec{AC} \|$

$\vec{AB} = \langle 3, 3, 4 \rangle \qquad \vec{AC} = \langle 5, 5, 6 \rangle$

$$\vec{AB} \times \vec{AC} = \begin{vmatrix} i & j & k \\ 3 & 3 & 4 \\ 5 & 5 & 6 \end{vmatrix} = i \begin{vmatrix} 3 & 4 \\ 5 & 6 \end{vmatrix} - j \begin{vmatrix} 3 & 4 \\ 5 & 6 \end{vmatrix} + k \begin{vmatrix} 3 & 3 \\ 5 & 5 \end{vmatrix}$$

$$= \langle -2, 2, 0 \rangle$$

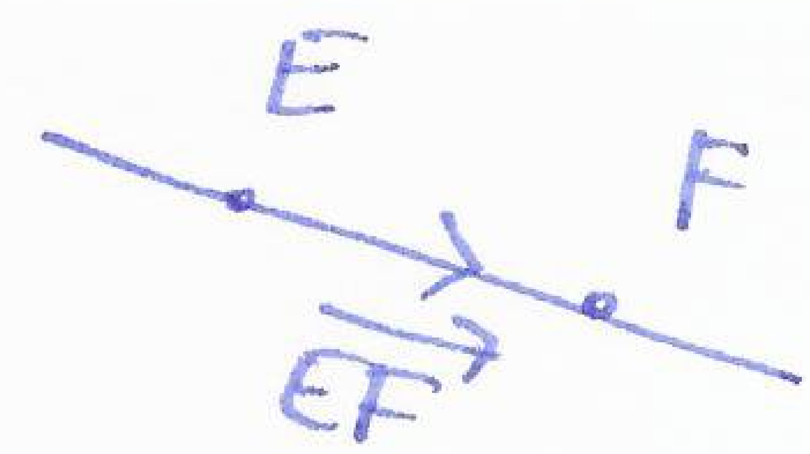
$$\frac{1}{2} \| \vec{AB} \times \vec{AC} \| = \frac{1}{2} \sqrt{4+4} = \sqrt{2}$$

b) normal vector parallel to $\vec{AB} \times \vec{AC} = \langle -2, 2, 0 \rangle$

$(-1,-1,-1)$ is a point on the plane

$$-2(x+1) + 2(y+1) + 0(z+1) = 0$$

$$-2x + 2y = 0$$



Problem 4. Consider two points $E(0,1,1)$, $F(4,-3,-3)$.

(a) Find a parametric equation of the line through E and F.

(b) Find the symmetric equation of the line through E and F.

a) $\overrightarrow{EF} = \langle 4,-4,-4 \rangle$

$\underline{x} = \langle 0,1,1 \rangle + t\langle 4,-4,-4 \rangle$

$\iff \quad x = 4t,\ y = 1-4t,\ z = 1-4t$

b)

$$\frac{x}{4} = \frac{y-1}{-4} = \frac{z-1}{-4}$$

Problem 5. For each equation below, sketch the surface in $\mathbb{R}^3$ that it describes.

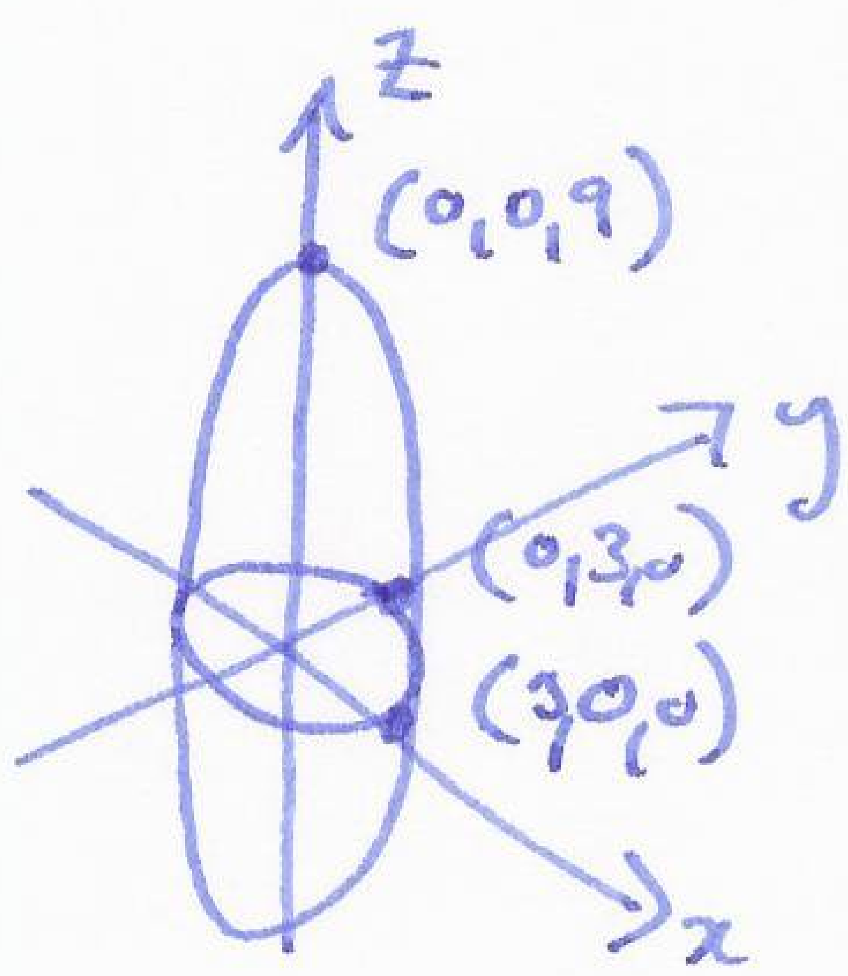
(a) $9x^2 + 9y^2 + z^2 = 81$

(b) $z = 4x^2 - y^2$

(c) $4x^2 + 4y^2 = 4z^2 + 16$

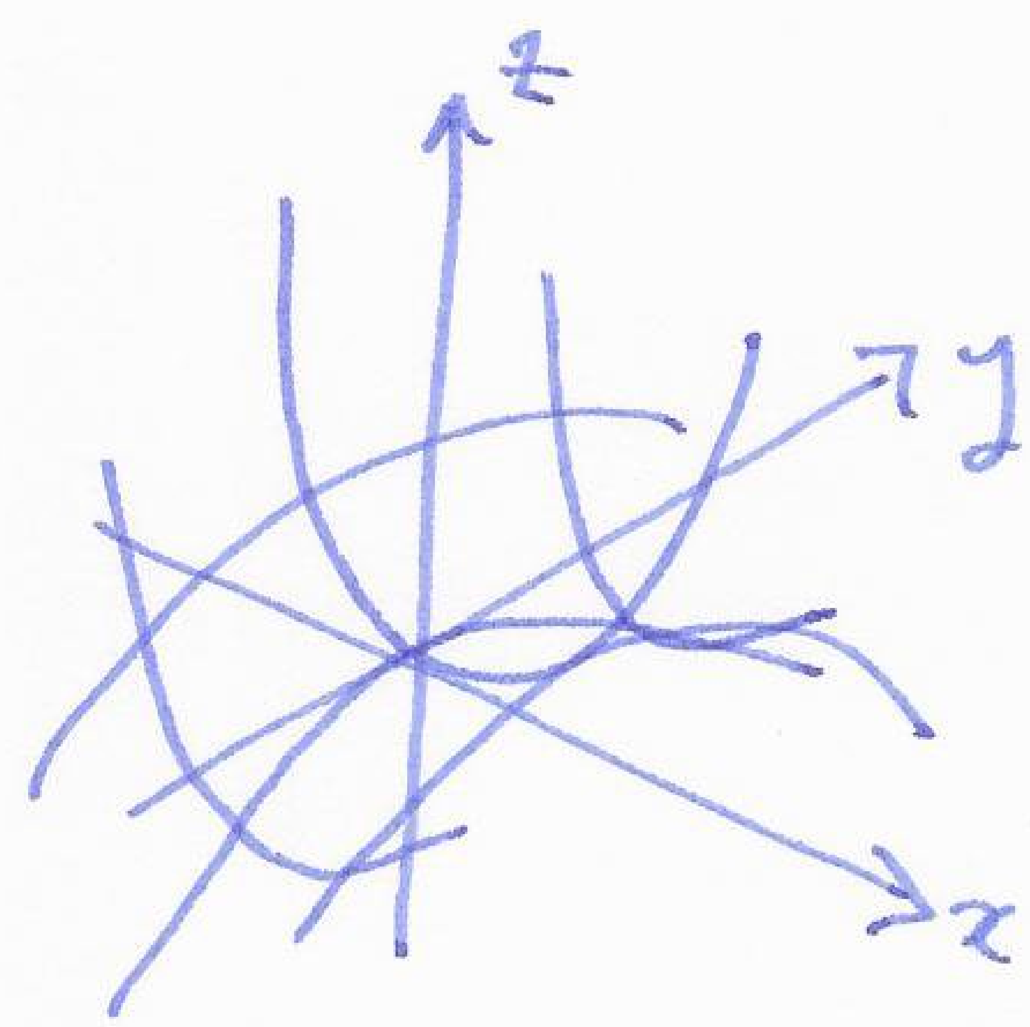
(d) $x^2 + 2x + y^2 + z^2 = 1$

a)



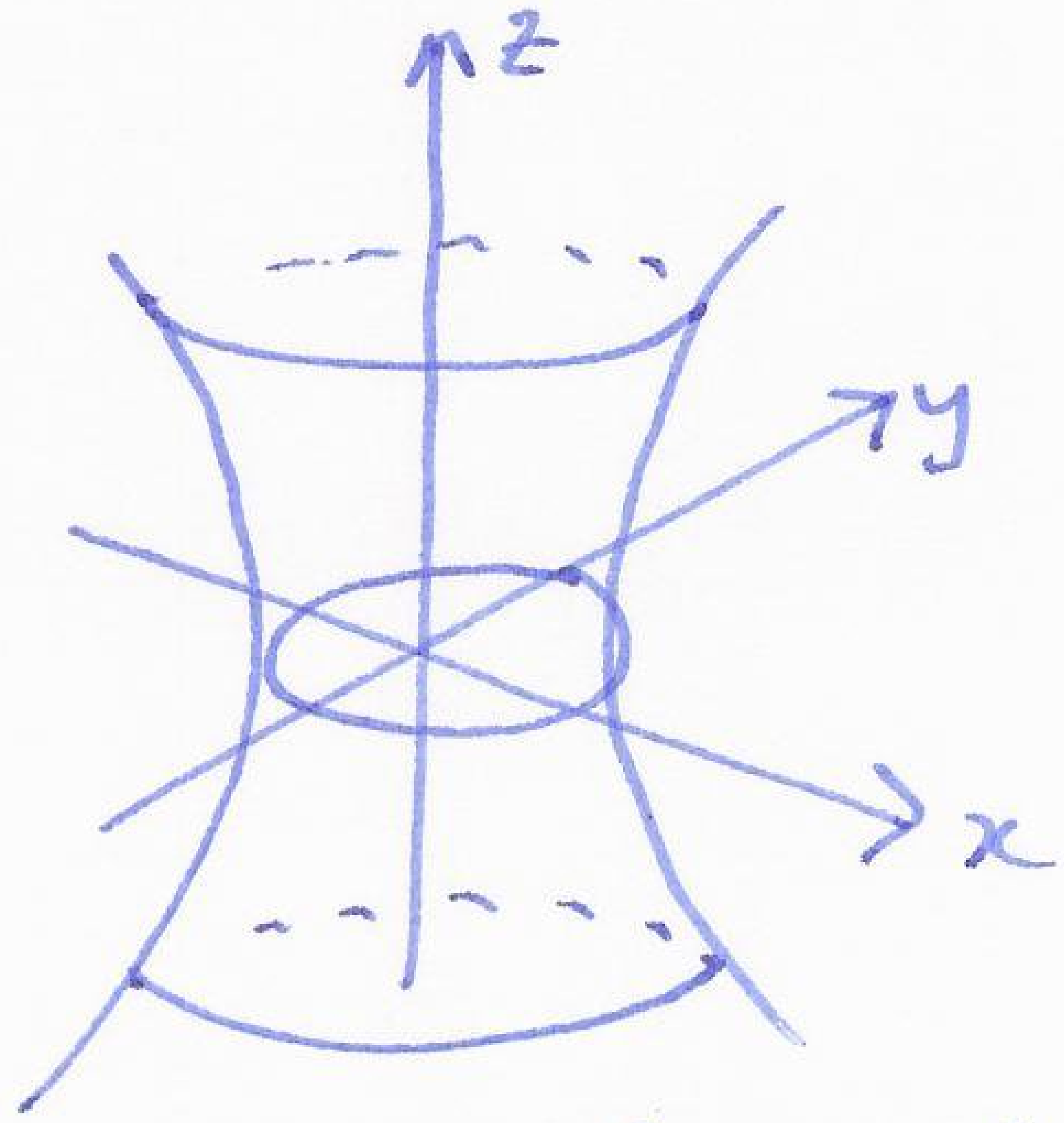
ellipsoid

b)



saddle surface

c)

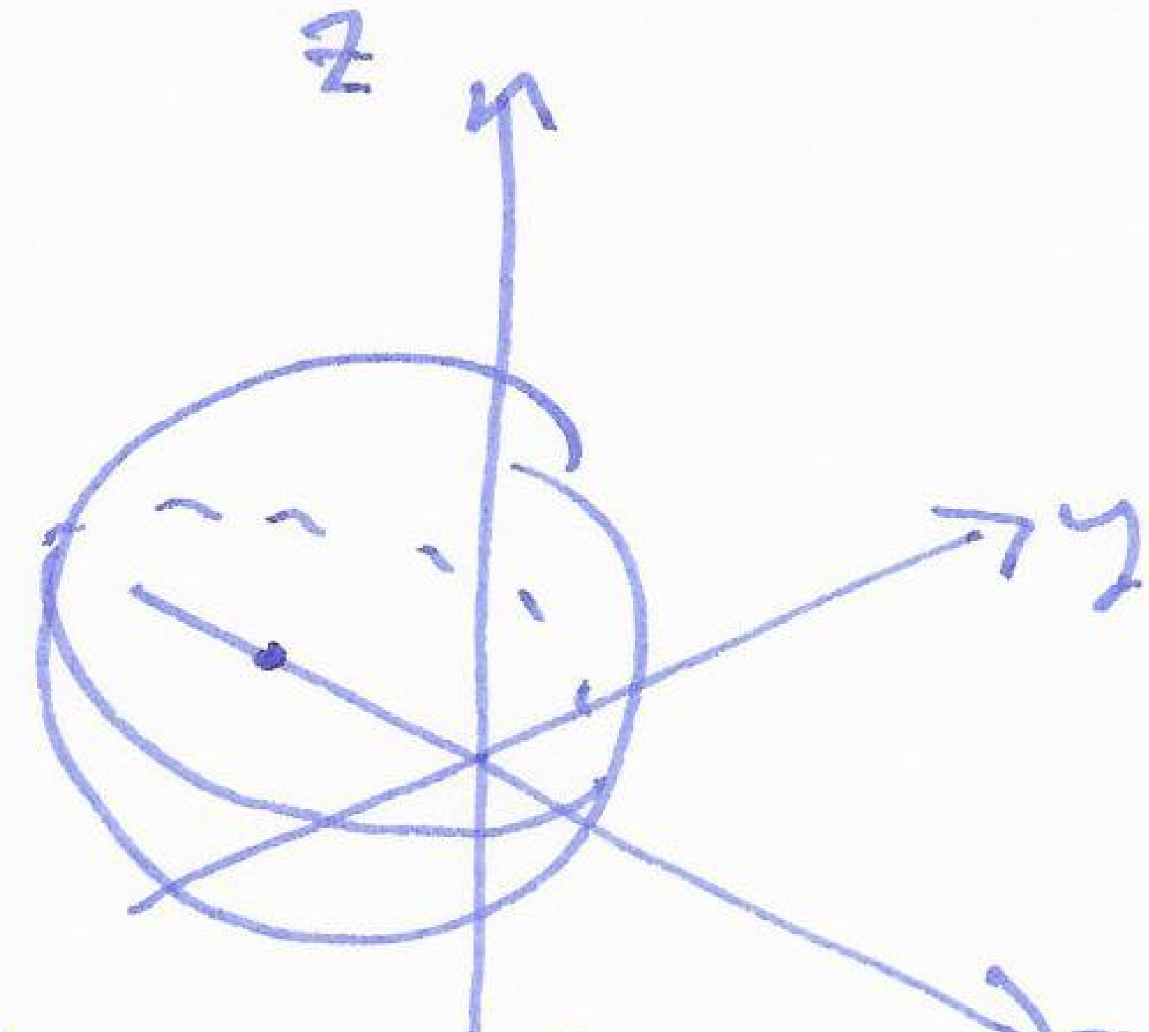


hyperboloid of 1 sheet.

complete the square:

d) $(x+1)^2 - 1 + y^2 + z^2 = 1$

$(x+1)^2 + y^2 + z^2 = 2$



center $(-1,0,0)$
radius $\sqrt{2}$.