Sample midterm 1 Solutions

Q1 a) $\|\underline{v}\| = \sqrt{4+1+1} = \sqrt{6}$

unit vector : $\frac{1}{\sqrt{6}}\langle 2,-1,1\rangle = \langle \frac{2}{\sqrt{6}}, \frac{-1}{\sqrt{6}}, \frac{1}{\sqrt{6}}\rangle = \hat{\underline{v}}$

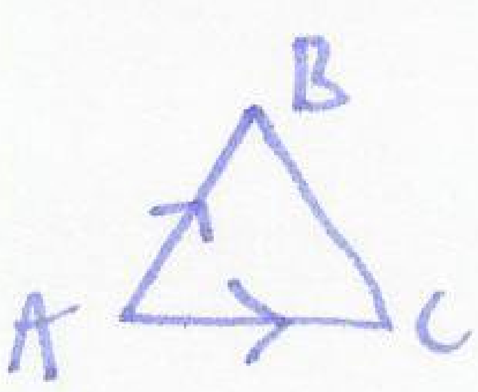
b) $\|\text{proj}_{\underline{v}}\, \underline{u}\| = (\underline{u}\cdot\hat{\underline{v}})\,\hat{\underline{v}} = \left(\langle 4,4,5\rangle \cdot \frac{1}{\sqrt{6}}\langle 2,-1,1\rangle\right)\cdot\langle \frac{2}{\sqrt{6}}, \frac{-1}{\sqrt{6}}, \frac{1}{\sqrt{6}}\rangle$

$= \frac{9}{\sqrt{6}}\langle \frac{2}{\sqrt{6}}, \frac{-1}{\sqrt{6}}, \frac{1}{\sqrt{6}}\rangle = \langle 3, -\frac{3}{2}, \frac{3}{2}\rangle$

c) $\underline{u} = \underbrace{\langle 3, \frac{-3}{2}, \frac{3}{2}\rangle}_{\text{parallel to } \underline{v}} + \underbrace{\left(\underline{u} - \langle 3, \frac{-3}{2}, \frac{3}{2}\rangle\right)}_{\text{perpendicular to } \underline{v}}$

$= \langle 3, \frac{-3}{2}, \frac{3}{2}\rangle + \langle 1, \frac{11}{2}, \frac{7}{2}\rangle$

Q2

a) 

$\overrightarrow{AB} = \langle 3, 1, 3\rangle$

$\overrightarrow{AC} = \langle 3, 0, 6\rangle$

$$\overrightarrow{AB}\times\overrightarrow{AC} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 3 & 1 & 3 \\ 3 & 0 & 6 \end{vmatrix} = \underline{i}\begin{vmatrix} 1 & 3 \\ 0 & 6 \end{vmatrix} - \underline{j}\begin{vmatrix} 3 & 3 \\ 3 & 6 \end{vmatrix} + \underline{k}\begin{vmatrix} 3 & 1 \\ 3 & 0 \end{vmatrix}$$

$= \langle 6, -9, -3\rangle \neq \underline{0}$

if A, B, C were colinear, $\overrightarrow{AB}$ would be parallel to $\overrightarrow{AC}$, so $\overrightarrow{AB}\times\overrightarrow{AC} = \underline{0}$.

so $\overrightarrow{AB}\times\overrightarrow{AC} \neq \underline{0} \Rightarrow A, B, C$ not colinear.

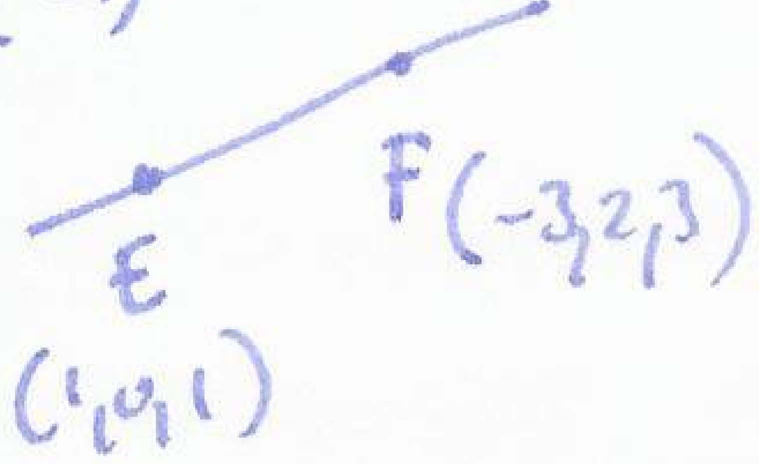
b) area triangle $ABC = \frac{1}{2}\|\overrightarrow{AB}\times\overrightarrow{AC}\| = \frac{1}{2}\sqrt{36+81+9}$ $\frac{\sqrt{126}}{2}$.

c) normal vector $\underline{n} = \langle 6, -9, -3 \rangle$

contains $A = (-2, 1, -1)$

plane : $6(x+2) - 9(y-1) - 3(z+1) = 0$.

Q3 a)



$\underline{v} = \overrightarrow{EF} = \langle -4, +2, 2 \rangle$

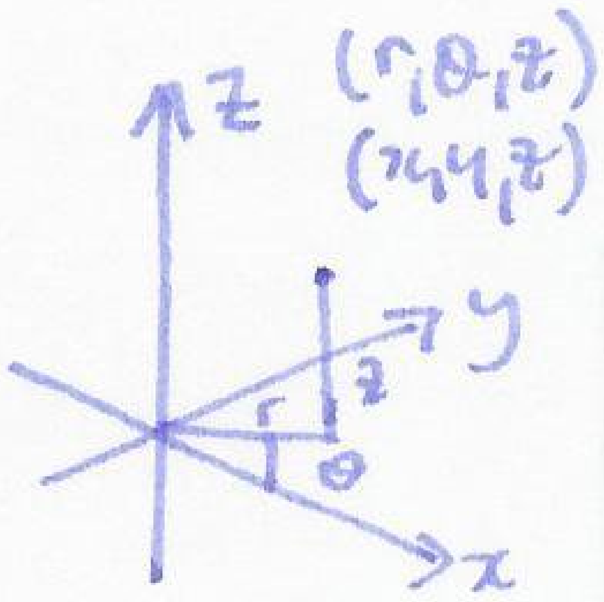
$x = -4t + 1$

$y = 2t$

$z = 2t + 1$.

b) $\dfrac{x-1}{-4} = \dfrac{y}{2} = \dfrac{z-1}{2}$.

c)

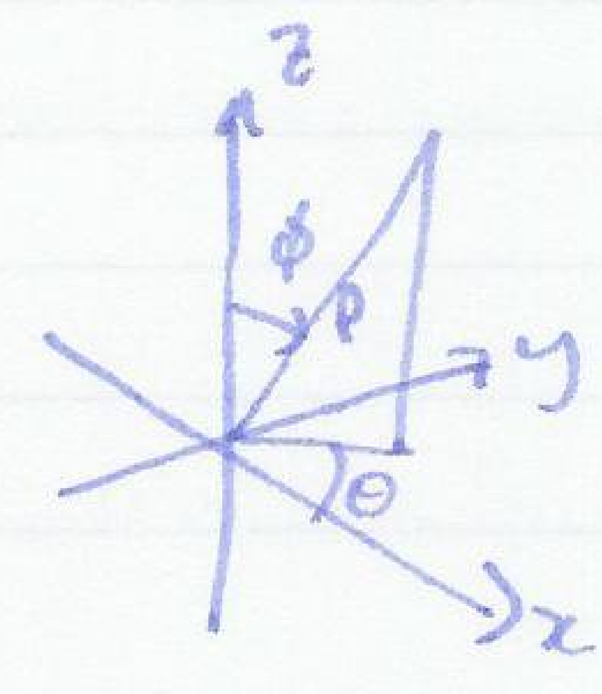


$x = 1$ $\quad r = \sqrt{x^2+y^2} = 1$

$y = 0$ $\quad \theta = \tan^{-1}(y/x) = 0$

$z = 1$ $\quad z = 1$

d) $\rho = \sqrt{x^2+y^2+z^2} = \sqrt{1+1} = \sqrt{2}$

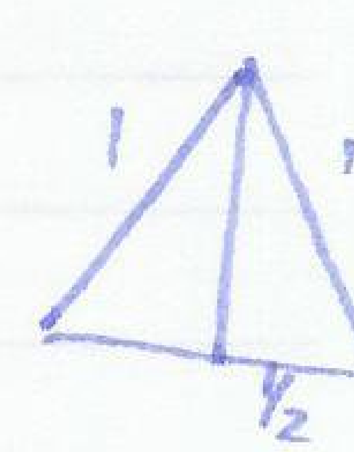


$\theta = \theta$ (cylindricals) $= 0$.

$\phi = \cos^{-1}\left(\dfrac{z}{\rho}\right) = \cos^{-1}\left(\dfrac{1}{\sqrt{2}}\right) = \dfrac{\pi}{4}$.

Q4 a) $L_1 : x - y = 3$ $\quad \underline{n}_1 = \langle 1, -1, 0 \rangle$

$L_2 : -y + z = 1$ $\quad \underline{n}_2 = \langle 0, -1, 1 \rangle$



angle : $\cos\theta = \dfrac{\underline{n}_1 \cdot \underline{n}_2}{\|\underline{n}_1\| \, \|\underline{n}_2\|} = \dfrac{1}{\sqrt{2}\sqrt{2}} = \dfrac{1}{2} \Rightarrow \theta = \dfrac{\pi}{3}$

line of intersection parallel to $\underline{n}_1 \times \underline{n}_2 = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 1 & -1 & 0 \\ 0 & -1 & 1 \end{vmatrix}$

$= \underline{i}\begin{vmatrix} -1 & 0 \\ -1 & 1 \end{vmatrix} - \underline{j}\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} + \underline{k}\begin{vmatrix} 1 & -1 \\ 0 & -1 \end{vmatrix} = \langle -1, -1, -1 \rangle$.

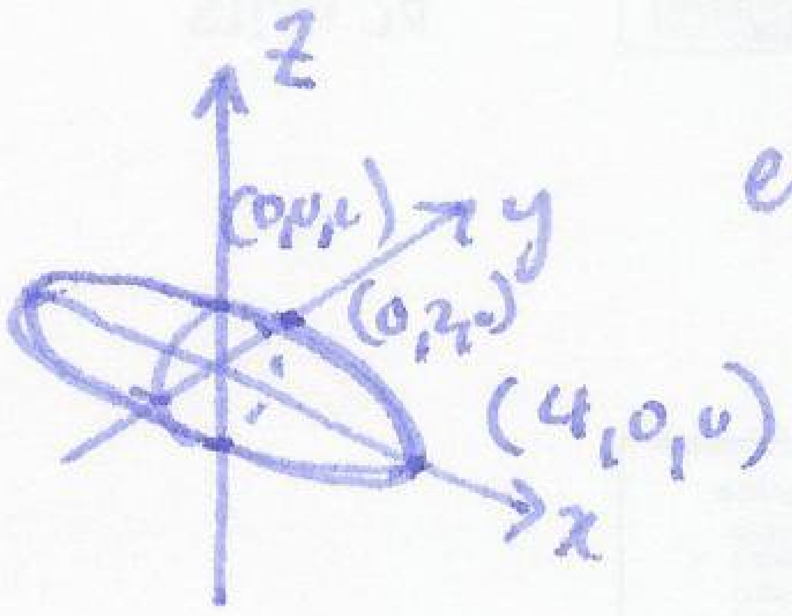
find common point, set $y = 0$, $x = 3$, $z = 1$, i.e. $(3, 0, 1)$

line is $\langle 3, 0, 1 \rangle + t\langle 1, 1, 1 \rangle$.

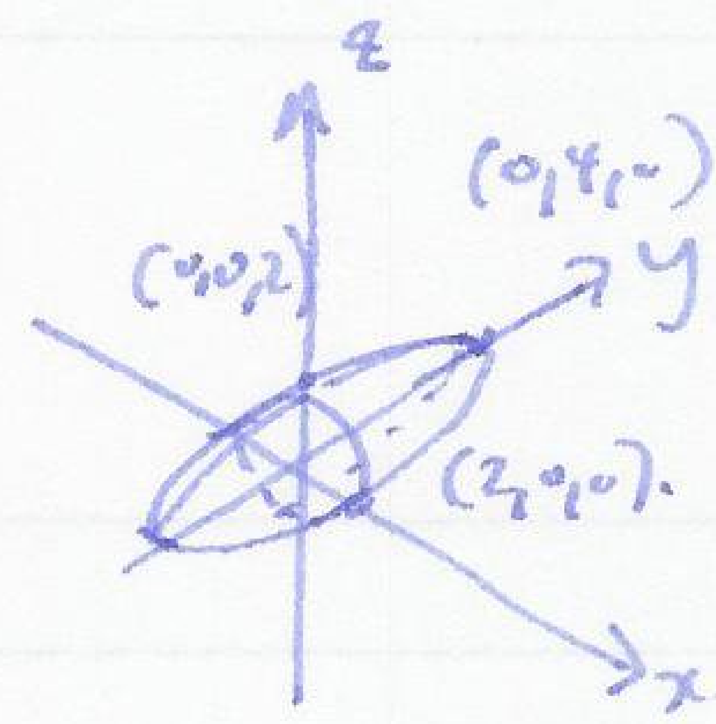
b) line parallel to $\langle 1, 2, 4 \rangle = \underline{n}$ normal vector for plane.

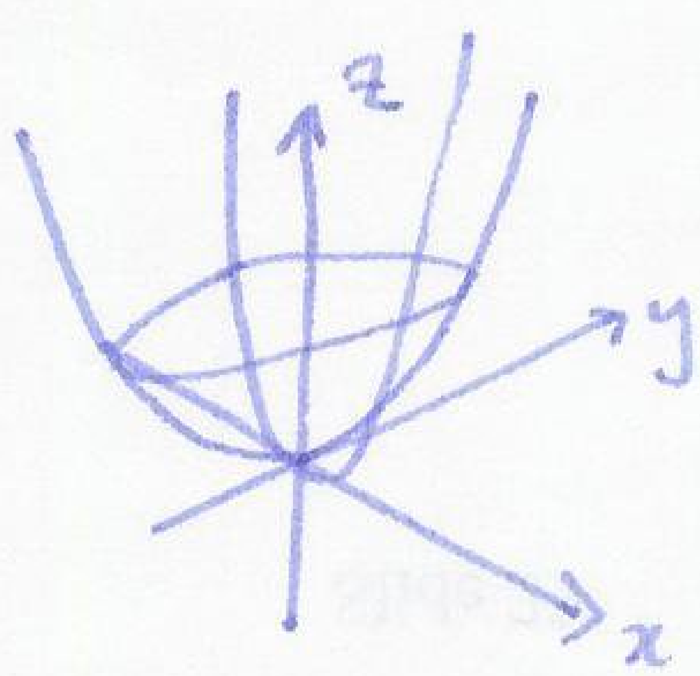
plane : $(x-1) + 2(y-2) + 4(z+1) = 0$.

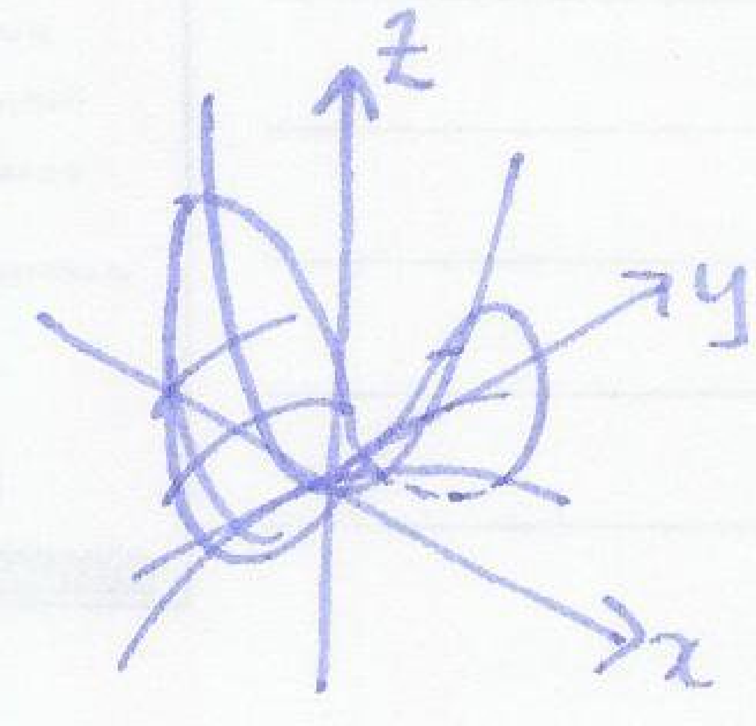
Q5

a) 

ellipse.

b) 

c) 

d) 

e) $9x^2 + 4y^2 - 2z^2 = 72$

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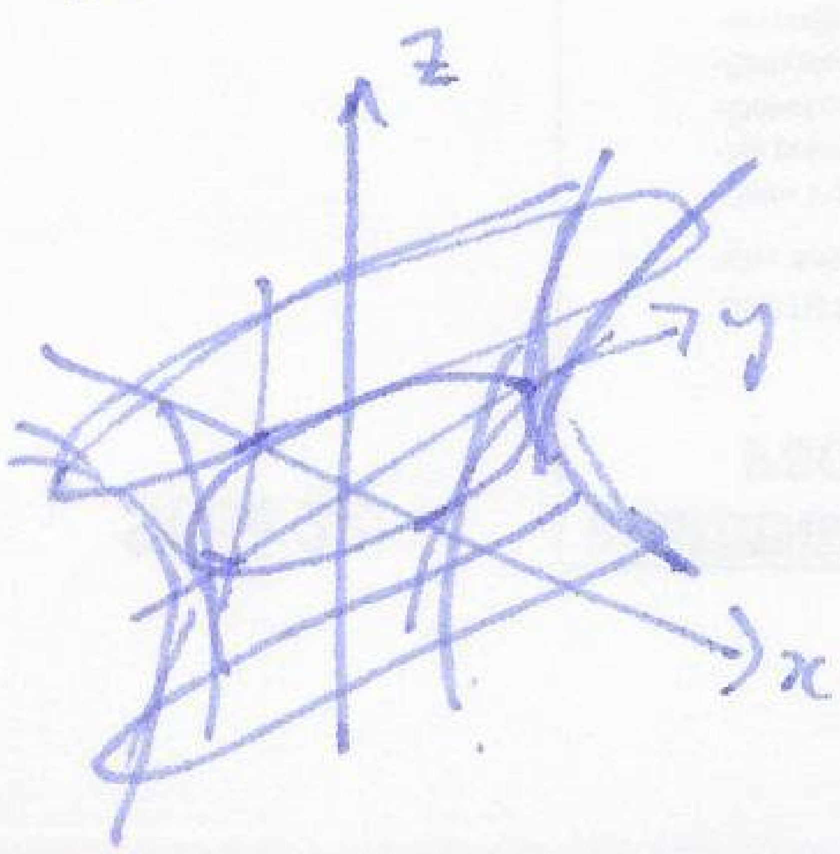
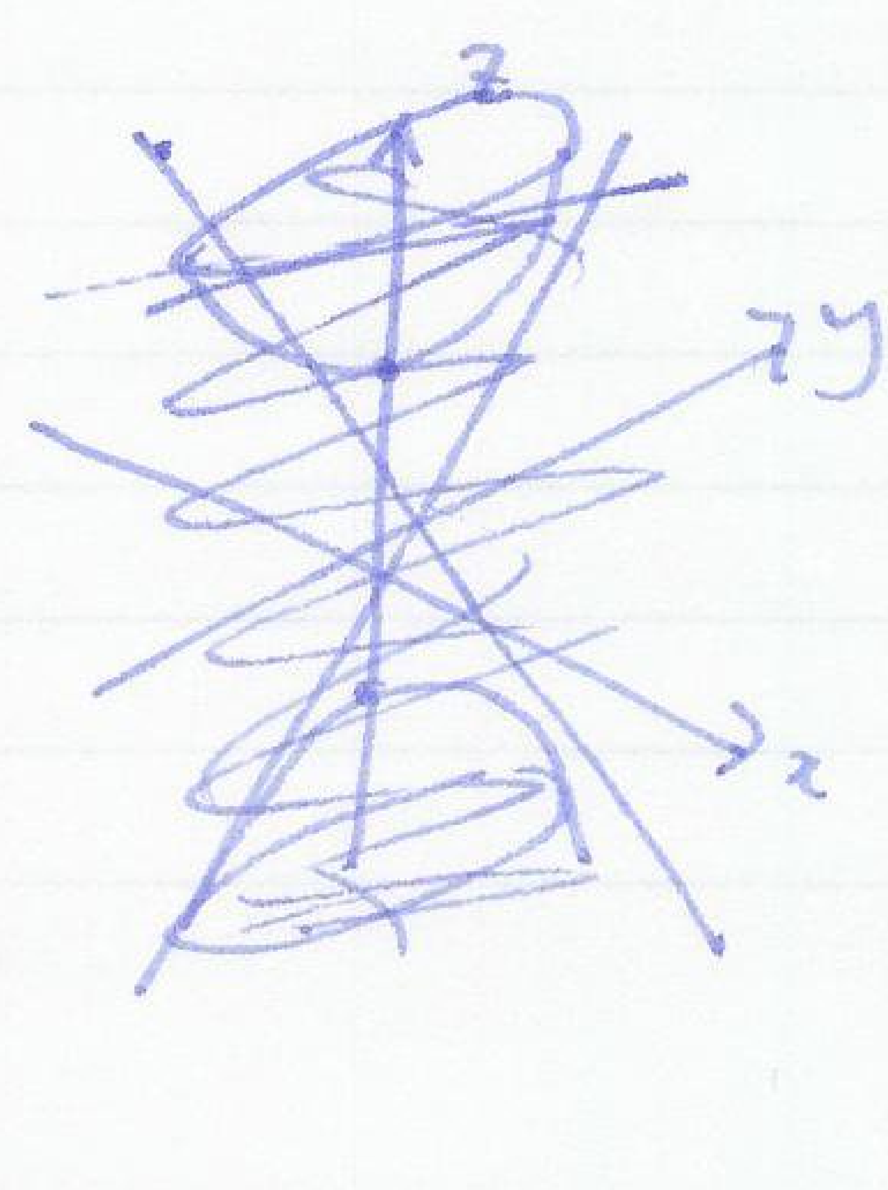
$\frac{x^2}{8} + \frac{y^2}{18} - \frac{z^2}{36} = 1$



f) 

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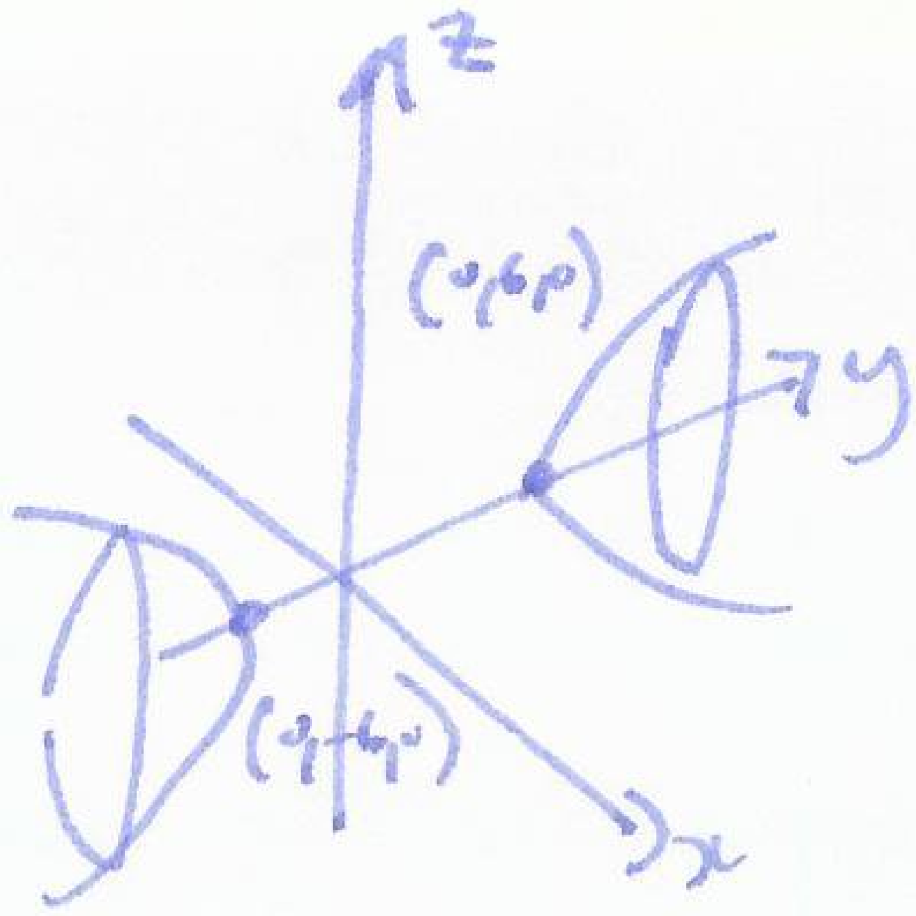
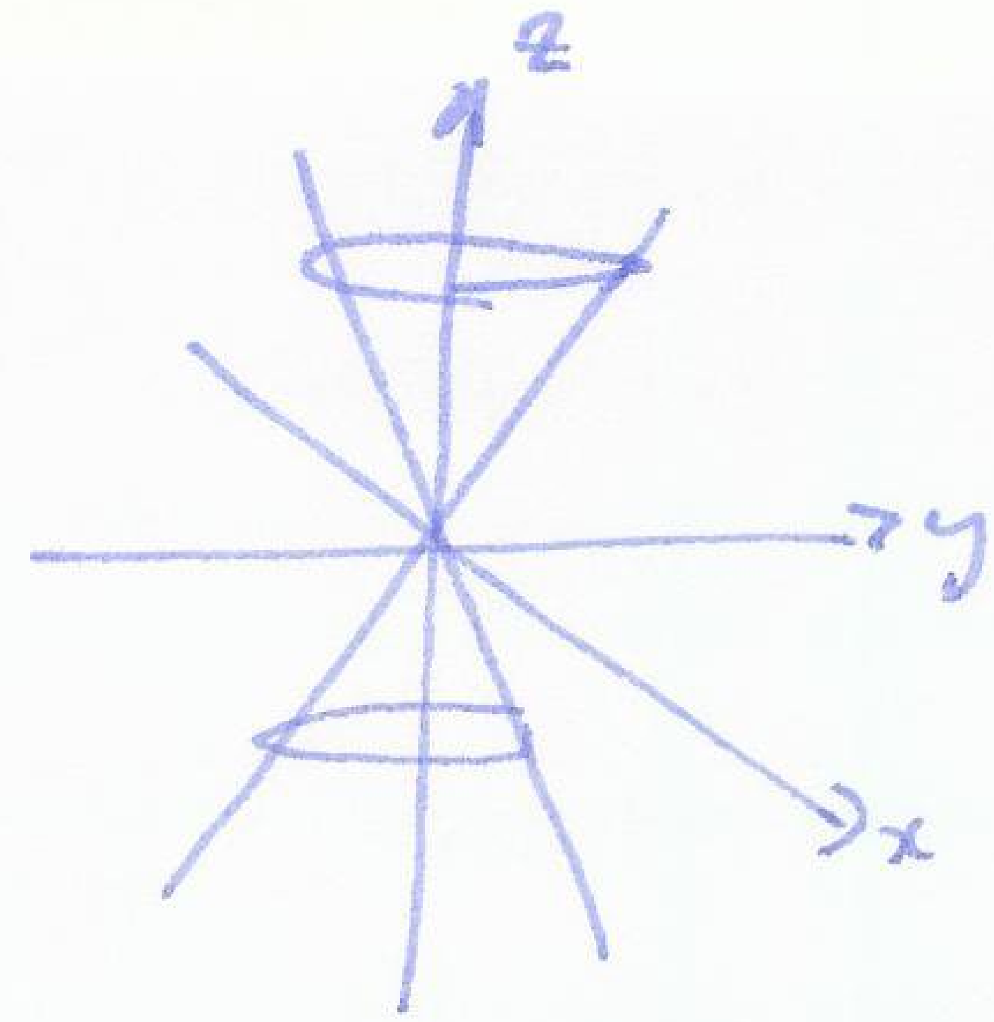
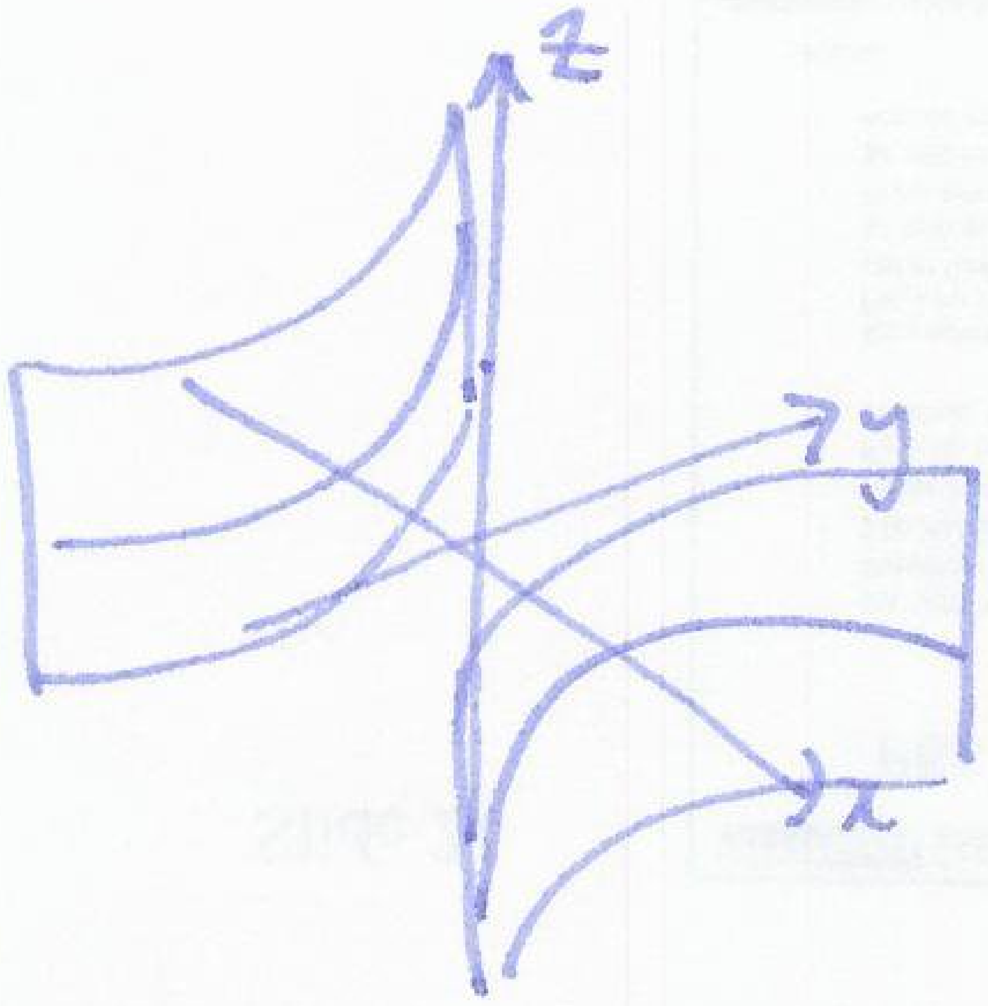
f)



g)



h)



$9x^2 = 72 + 4y^2$