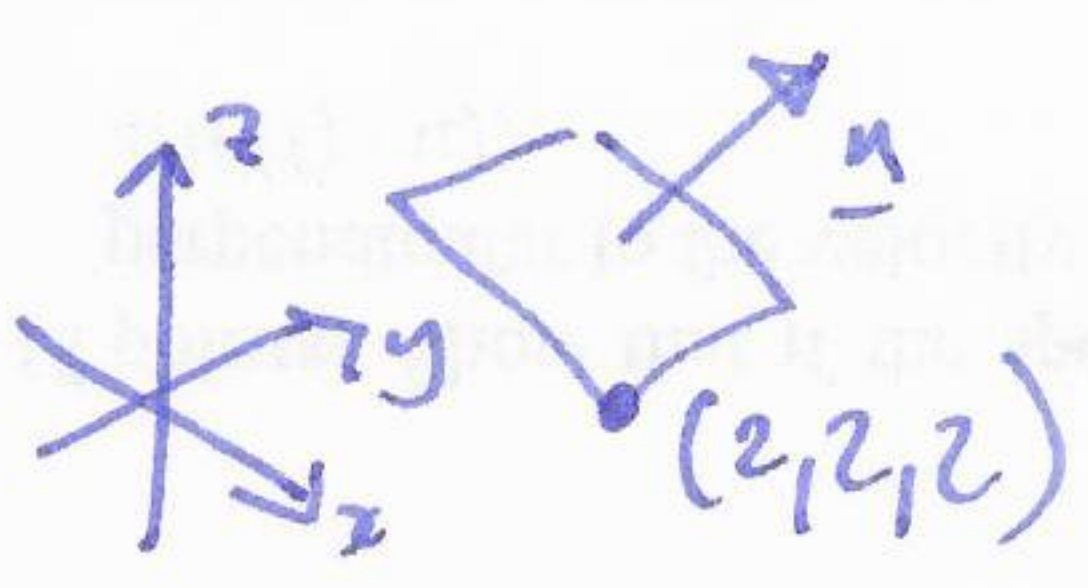


Sample final solutions

①

Q1 a) $\underline{n} = \langle 1, 1, -1 \rangle \times \langle 1, 0, 0 \rangle = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 1 & 1 & -1 \\ 1 & 0 & 0 \end{vmatrix}$
 $= \langle 0, -1, -1 \rangle$

b) $0(x-2) - (y-2) - (z-2) = 0$

Q2 a)  distance to origin = length of projection of $\langle 2, 2, 2 \rangle - \langle 0, 1, 1 \rangle$ onto $\langle 0, 1, 1 \rangle$

$$= \frac{\langle 2, 2, 2 \rangle \cdot \langle 0, 1, 1 \rangle}{\sqrt{2}} = \frac{1}{\sqrt{2}} \cdot 4 = 2\sqrt{2}$$

b) distance from origin $d^2 = \frac{x^2 + y^2 + z^2}{F}$ minimize this, subject to

$$\frac{y+z}{G} = 4$$

$$\nabla F = \langle 2x, 2y, 2z \rangle \quad \nabla G = \langle 0, 1, 1 \rangle$$

$$\nabla F = \lambda \nabla G : \quad \left. \begin{array}{l} 2x = 0 \\ 2y = \lambda \\ 2z = \lambda \end{array} \right\}$$

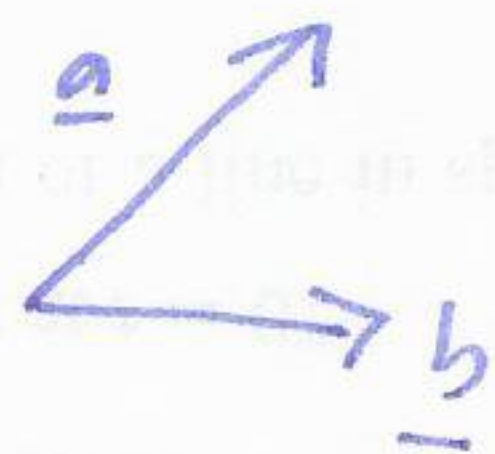
$$x=0, y=2, z=2$$

$$\text{distance} = \sqrt{4+4} = 2\sqrt{2}$$

$$y+z=4$$

Q3 $\underline{a} = \langle -2, 4, 2 \rangle$

$$\underline{b} = \langle 1, 0, -1 \rangle$$



projection of \underline{a} to \underline{b} : $\frac{\underline{a} \cdot \underline{b}}{|\underline{b}|^2} \underline{b} = \frac{(-2-2)}{2} \langle 1, 0, -1 \rangle$

$$= \langle -2, 0, 2 \rangle$$

so $\underline{a} = \langle -2, 0, 2 \rangle + \langle 0, 4, 0 \rangle$

Q4 $\underline{x}(t) = \langle \cos t + t \sin t, \sin t - t \cos t, t^2 \rangle$

arc length = $\int_0^{\pi/2} \|\underline{x}'(t)\| dt$

$\underline{x}'(t) = \langle -\sin t + \sin t + t \cos t, \cos t - \cos t + t \sin t, 2t \rangle$

$\int_0^{\pi/2} \sqrt{t^2 \cos^2 t + t^2 \sin^2 t + 4t^2} dt = \int_0^{\pi/2} \sqrt{5} t dt = \left[\frac{\sqrt{5}}{2} t^2 \right]_0^{\pi/2} = \frac{\sqrt{5} \pi^2}{8}$

Q5 $\underline{v}(t) = \langle 1, 3t^2, 2t^3 \rangle$

$\underline{r}(t) = \langle t, t^3, \frac{1}{2}t^4 \rangle + \underline{c}(t)$

$\underline{r}(0) = \langle 0, 0, 0 \rangle + \underline{c}(t) = \langle 7, 5, 0 \rangle$

so $\underline{v}(t) = \langle t+7, t^3+5, \frac{1}{2}t^4 \rangle$

$\underline{v}(2) = \langle 9, 13, 8 \rangle$

Q6 $P = IV$ $v = IR$ so $P = IV = I^2 R = \frac{V^2}{R}$

$V = 110, P = 200 \Rightarrow I = \frac{200/110}{1} \approx 1.82$ $R = \frac{110^2}{200} = 60.5$

① $\Delta P \approx V \Delta I + I \Delta V \approx 110 (0.152) + (1.82) 110 \approx 40$

② $\Delta P \approx 2IR \Delta I + I^2 \Delta R \approx 2 (1.82) 60.5 (0.152) + (1.82)^2 6.05 \approx 60$

③ $\Delta P \approx \frac{2V}{R} \Delta V + -\frac{V^2}{R^2} \Delta R \approx \frac{220}{60.5} 11 - \frac{(110)^2}{(60.5)^2} 6.05 \approx 20$

measure V and R.

Q7 $f(x,y,z) = e^{2x-y} + \tan(yz)$

a) $\nabla f = \langle 2e^{2x-y}, -e^{2x-y} + z \sec^2(yz), y \sec^2(yz) \rangle$

$\nabla f(1,2,0) = \langle 2, -1, 2 \rangle$

b) $2(x-1) - (y-2) + 2(z-0) = 0$

Q8 $f(x,y) = 3xy - x^2y - xy^2$

$$\left. \begin{aligned} f_x &= 3y - 2xy - y^2 = 0 \\ f_y &= 3x - x^2 - 2xy = 0 \end{aligned} \right\} \begin{aligned} y(3 - 2x - y) &= 0 \\ x(3 - x - 2y) &= 0 \end{aligned}$$

x=0 : $y(3-y)=0$ $(0,0)$ $(0,3)$

y=0 : $x(3-x)=0$ $(0,0)$ $(3,0)$

x≠0, y≠0 : $\left. \begin{aligned} \textcircled{1} \quad 3 - 2x - y &= 0 \\ \textcircled{2} \quad 3 - x - 2y &= 0 \end{aligned} \right\} \begin{aligned} \textcircled{1} - 2\textcircled{2} : -3 + 3y &= 0 & y &= 1 \\ & & x &= 1 \end{aligned} \quad (1,1)$

$f_{xx} = -2y$

$f_{xy} = 3 - 2x - 2y$

$f_{yy} = -2x$

$D = (f_{xx})(f_{yy}) - (f_{xy})^2$

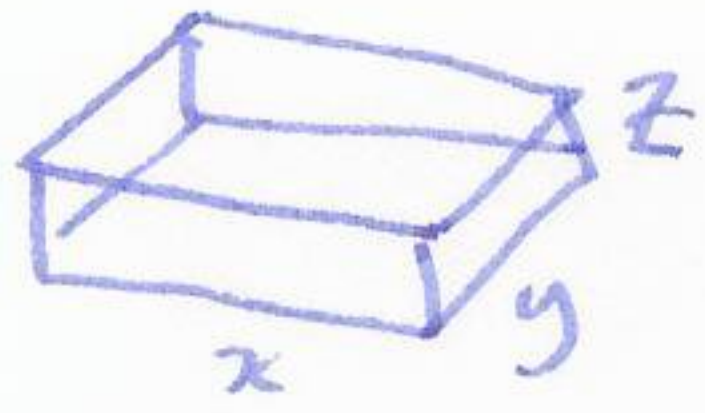
$D(0,0) = 0 \cdot 0 - 9 < 0$ saddle.

$D(0,3) = -6 \cdot 0 - (-3)^2 < 0$ saddle.

$D(3,0) = 0 \cdot -6 - (-3)^2 < 0$ saddle.

$D(1,1) = (-2)(-2) - (-1)^2 > 0$ } maximum
 $f_{xx} < 0$

Q9



$$V = xyz = 4000$$

$$H = xy + 2yz + 2xz \leftarrow \text{minimize}$$

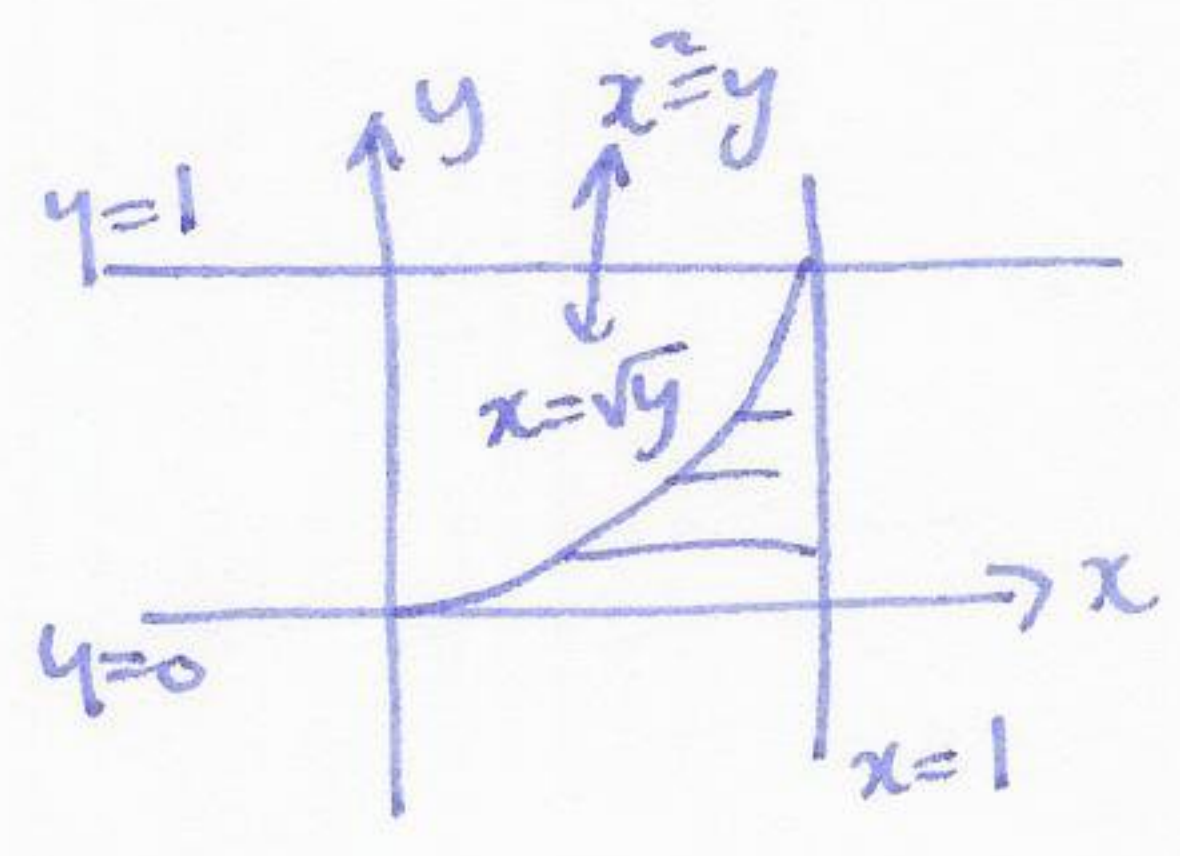
$$\nabla H = \lambda \nabla V : \quad \nabla H = \langle y + 2z, x + 2z, 2x + 2y \rangle$$

$$xyz = 4000 \quad \nabla V = \langle yz, xz, xy \rangle$$

$$\left. \begin{aligned} y + 2z &= \lambda yz \\ x + 2z &= \lambda xz \\ 2x + 2y &= \lambda xy \end{aligned} \right\} \left. \begin{aligned} xyz + 2xz &= \lambda xyz = \lambda 4000 \\ xyz + 2yz &= \lambda xyz = \lambda V \\ 2xz + 2yz &= \lambda xyz = \lambda V \end{aligned} \right\} \begin{aligned} 2xz &= 2yz \Rightarrow x = y \\ xy &= 2xz \Rightarrow y = 2z \end{aligned}$$

$$2z^3 = 4000 \text{ so } x = \sqrt[3]{2000} = y \quad z = \frac{1}{2} \sqrt[3]{2000}$$

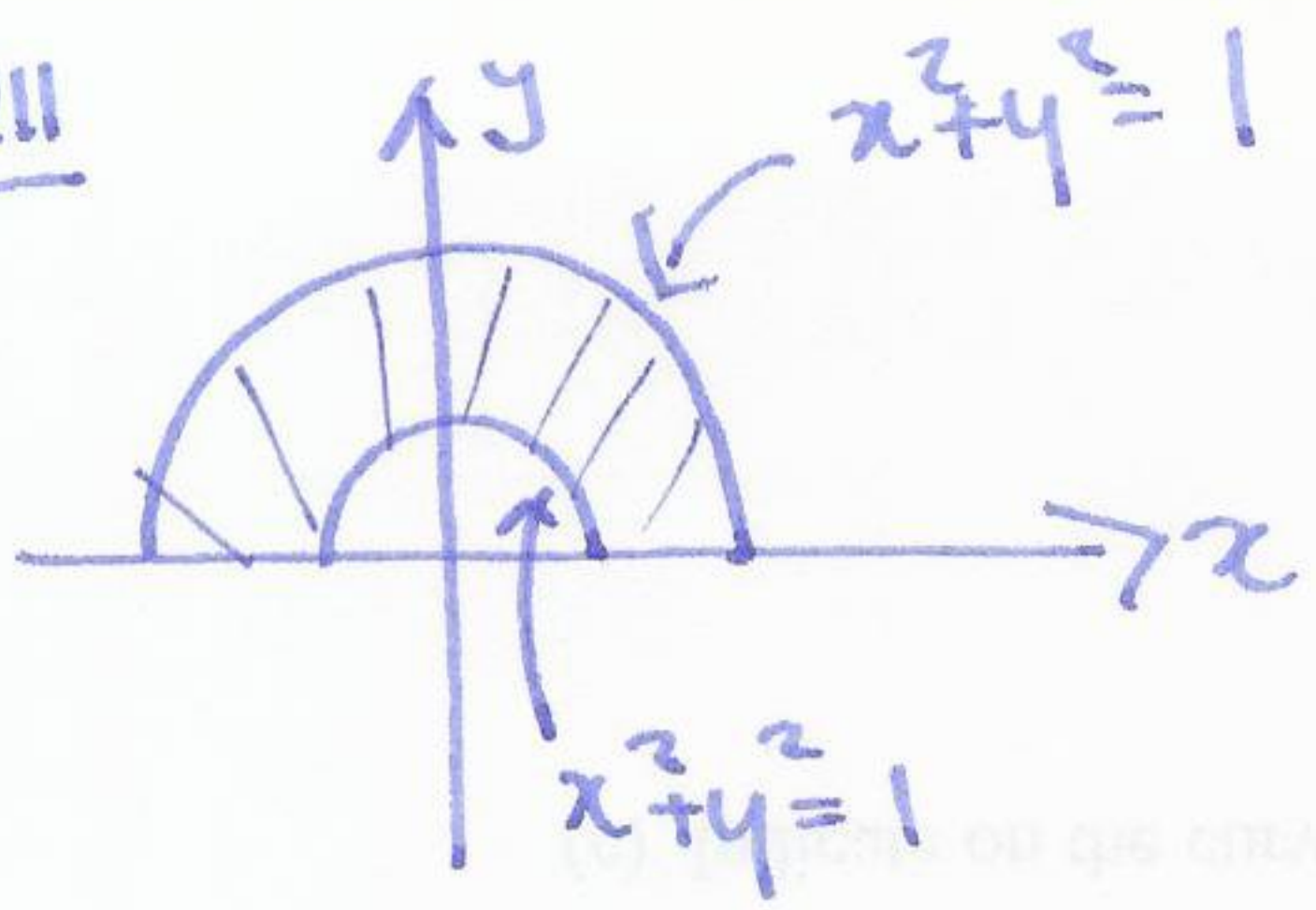
Q10 $\int_0^1 \int_{\sqrt{y}}^1 \frac{ye^{x^2}}{x^3} dx dy$



$$\int_0^1 \int_0^{x^2} \frac{ye^{x^2}}{x^3} dy dx = \int_0^1 \left[\frac{1}{2} y^2 \frac{e^{x^2}}{x^3} \right]_0^{x^2} dx$$

$$= \int_0^1 \frac{1}{2} x e^{x^2} dx = \left[\frac{1}{4} e^{x^2} \right]_0^1 = \frac{1}{4} (e - 1)$$

Q11



$$\rho(x,y) = x^2 + y^2$$

use polar.

$$x = r \cos \theta$$

$$y = r \sin \theta$$

$$\text{mass} = \iint_D \rho \, dA = \int_0^\pi \int_1^2 r^2 \cdot r \, dr \, d\theta = 2\pi \left[\frac{1}{4} r^4 \right]_1^2 = 2\pi \left(4 - \frac{1}{4} \right) = \frac{30\pi}{4} = 15\pi/4$$

$$M_x = \iint_D y \rho \, dA = \int_0^\pi \int_1^2 r^4 \sin \theta \, dr \, d\theta$$

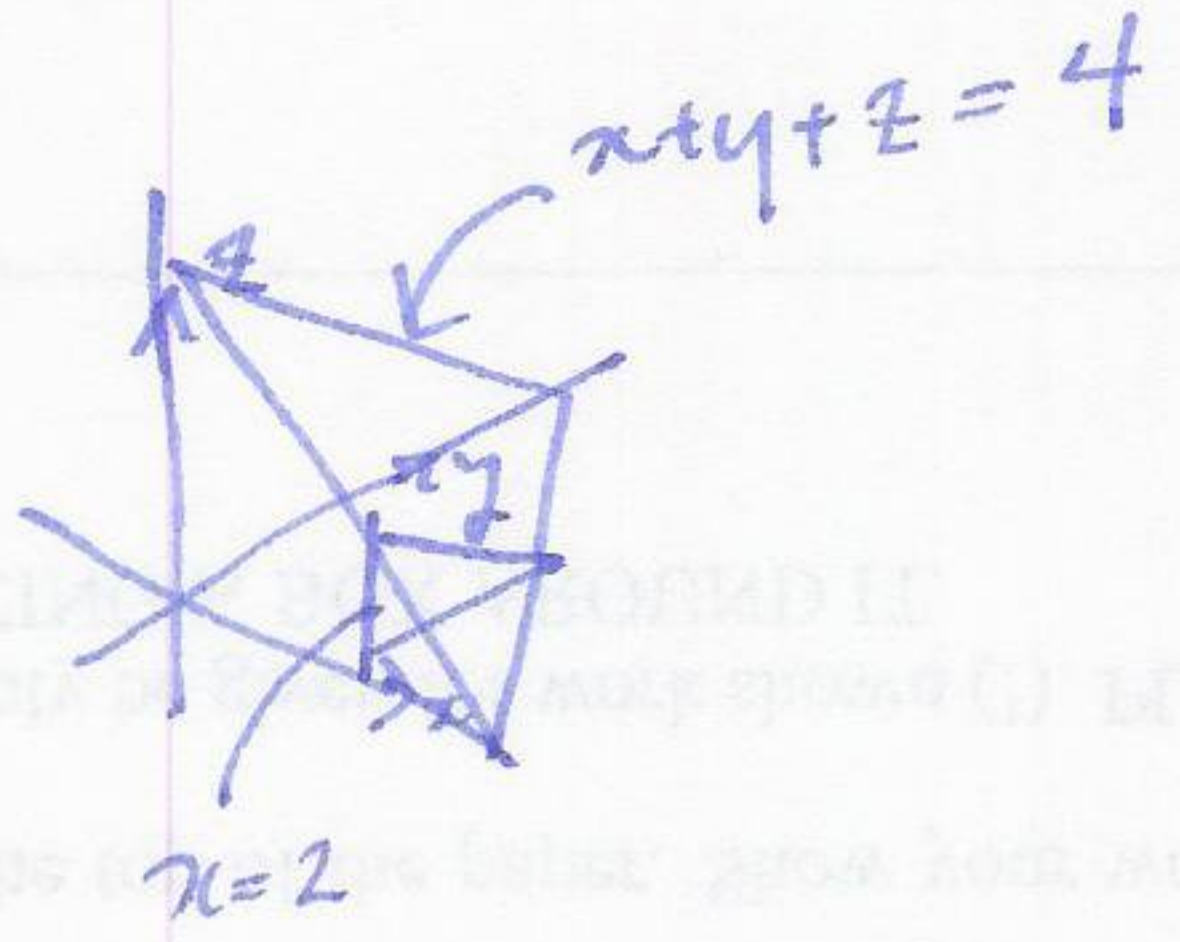
$$\int_0^\pi \sin \theta \, d\theta = \left[-\cos \theta \right]_0^\pi = -(-1) - (-1) = 2$$

$$\int_1^2 r^4 \, dr = \left[\frac{1}{5} r^5 \right]_1^2 = \frac{1}{2} (32 - 1) = \frac{31}{2}$$

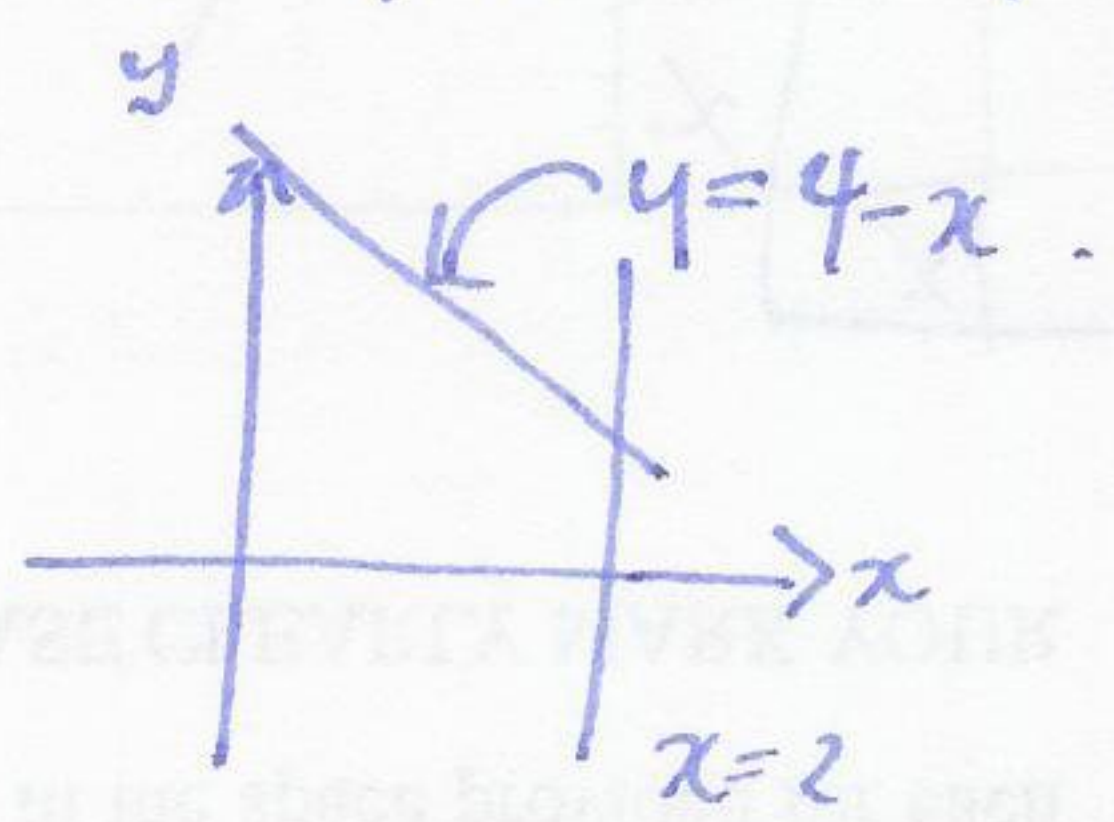
$$(\bar{x}, \bar{y}) = \left(0, \frac{12}{15\pi} \right)$$

Q12

$$\iiint_E dV$$



projection into xy-plane



$$\int_0^2 \int_0^{4-x} \int_0^{4-x-y} dz \, dy \, dx$$

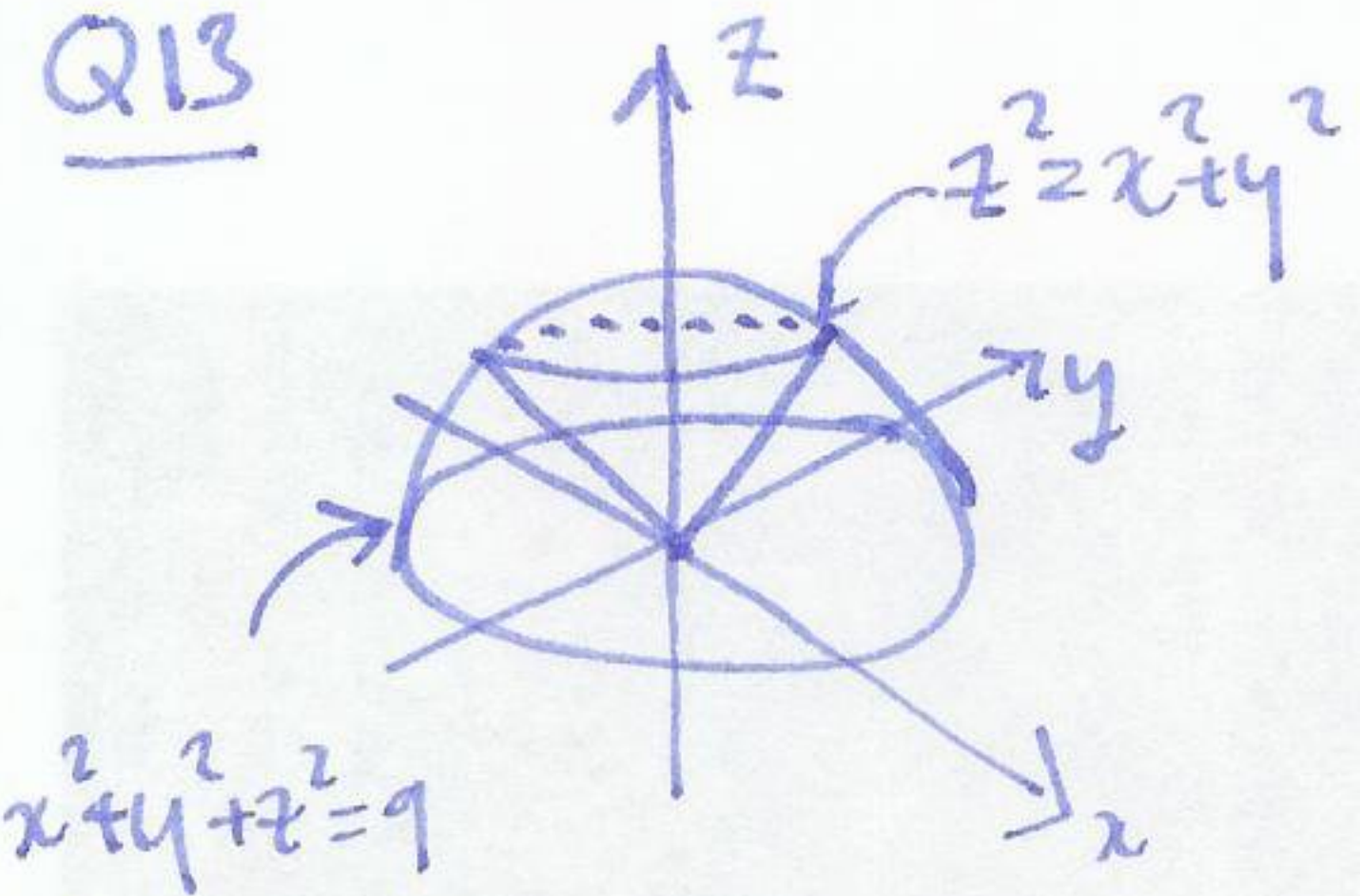
$$\left[z \right]_0^{4-x-y} = 4-x-y$$

$$\int_0^{4-x} (4-x-y) \, dy$$

$$\left[4y - xy - \frac{1}{2}y^2 \right]_0^{4-x} = (4-x)^2 - \frac{1}{2}(4-x)^2 = \frac{1}{2}(4-x)^2$$

$$\int_0^2 \frac{1}{2}(4-x)^2 dx = \left[-\frac{1}{6}(4-x)^3 \right]_0^2 = 0 + \frac{1}{6}4^3 = \frac{32}{3}$$

Q13



use spherical coordinates

$$\begin{aligned} x &= \rho \sin\phi \cos\theta \\ y &= \rho \sin\phi \sin\theta \\ z &= \rho \cos\phi \end{aligned}$$

$$z^2 = x^2 + y^2$$

$$\rho^2 \cos^2\phi = \rho^2 \sin^2\phi$$

$$\tan\phi = 1$$

$$\phi = \pi/4$$

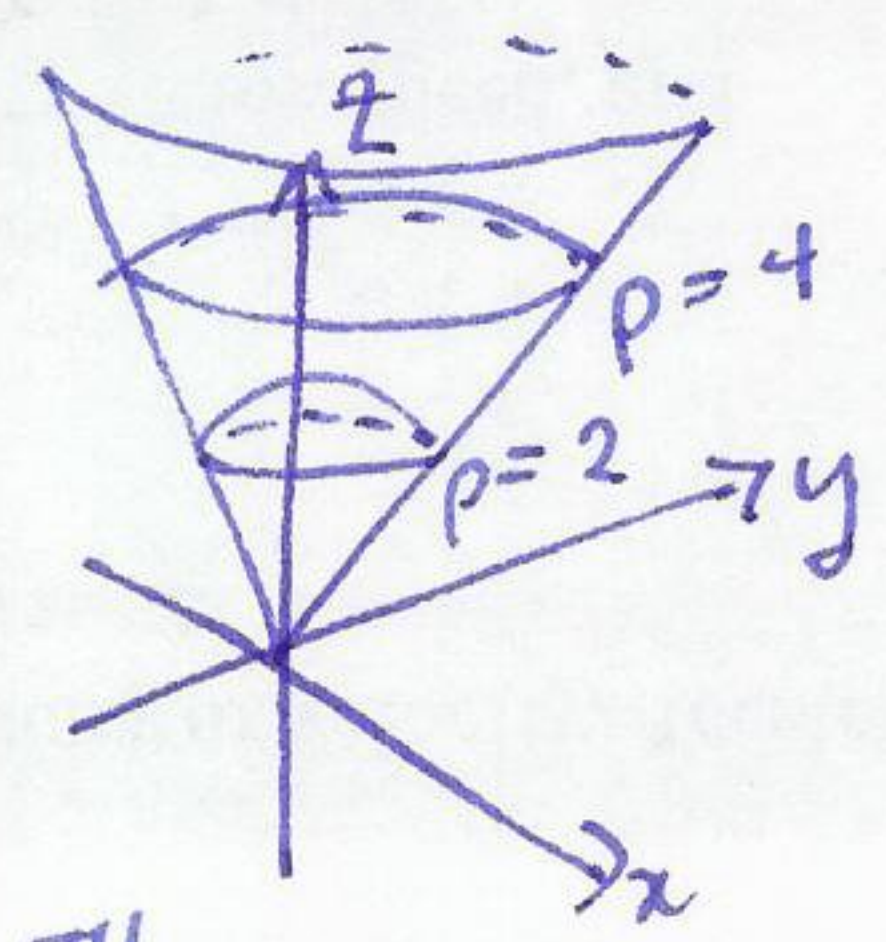
$$\int_0^{\pi/4} \int_0^{2\pi} \int_0^3 \rho^2 \sin\phi \, d\rho \, d\theta \, d\phi$$

$$\int_0^3 \rho^2 \, d\rho = \left[\frac{1}{3}\rho^3 \right]_0^3 = 9$$

$$\int_0^{2\pi} 9 \sin\phi \, d\theta = 18\pi \sin\phi$$

$$\int_0^{\pi/4} 18\pi \sin\phi \, d\phi = 18\pi \left[-\cos\phi \right]_0^{\pi/4} = 18\pi (1 - \sqrt{2}/2)$$

Q14



$$\begin{aligned} x &= \rho \cos\theta \sin\phi \\ y &= \rho \sin\theta \sin\phi \\ z &= \rho \cos\phi \end{aligned}$$

$$\int_0^{\pi/3} \int_0^{2\pi} \int_2^4 \rho^3 \cos\theta \sin\theta \sin^2\phi \cos\phi \rho^2 \sin\phi \, d\rho \, d\theta \, d\phi$$

$$\int_2^4 \rho^5 d\rho = \left[\frac{1}{6} \rho^6 \right]_2^4 = \frac{1}{6} (4^6 - 2^6)$$

$$\int_0^{2\pi} \cos\theta \sin\theta d\theta = \int_0^{2\pi} \frac{1}{2} \sin 2\theta d\theta = \left[-\frac{1}{4} \cos 2\theta \right]_0^{2\pi} = 0$$

$$\int_0^{\pi/3} \sin^3 \phi \cos \phi d\phi = \left[\frac{1}{4} \sin^4 \phi \right]_0^{\pi/3} = \frac{1}{4} \left(\frac{9}{16} - 0 \right) = \frac{9}{64}$$

final answer : $\frac{1}{6} (4^6 - 2^6) \cdot 0 \cdot \frac{9}{64} = 0$

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