Linear progress in the complex of curves

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Σ closed orientable surface

Def: \( G = \text{MCG}(\Sigma) = \text{Diff}^+(\Sigma)/\text{Diff}_0(\Sigma) \)

Finitely generated

Let \( \Gamma \) be a Cayley graph for \( G \),
consider nearest neighbour random walk on \( \Gamma \)

[Kesten et al.] \(|w_n|_\Gamma \) grows linearly.

i.e. \( \mathbb{P}(\frac{n}{E} \leq |w_n|_\Gamma \leq nE) \to 1, \text{ as } n \to \infty. \)
[Masur-Minsky] $G$ is weakly relatively hyperbolic.

Given $H < G$, the relative space $\hat{\Gamma}$ is $\Gamma$, with each coset $gH$ coned off to a vertex $v_{gH}$.

\[
G = F_2 = \langle a, b \mid \rangle \\
H = \langle a \rangle
\]

If $\hat{\Gamma}$ is $\delta$-hyperbolic then we say $G$ is weakly relatively hyperbolic

[Masur-Minsky] $\hat{\Gamma} \sim_{QI} C(\Sigma)$ complex of curves.
[M] $|w_n|_\Gamma$ grows linearly

[Klarreich] $\partial \hat{\Gamma} = \text{minimal foliations} \subset \mathcal{PML}$

[Kaimanovich-Masur] $w_n \to \lambda \in \mathcal{PML}$ a.s.

This gives a measure $\nu$ on $\mathcal{PML}$,
$\nu(X) =$ probability $w_n$ converges to $\lambda \in X$

Half space $H(1, x) = \{y \in \hat{\Gamma} | \hat{d}(x, y) \leq \hat{d}(y, 1)\}$. 
Lemma: $\nu(H(1, x)) \leq L|x|_{\widehat{\Gamma}}$, for some constant $L < 1$ independent of $x$

Proof: $\nu(H(1, x)) \leq 1 - \epsilon$ for all $|x|_{\widehat{\Gamma}} \geq K$, for constants $\epsilon > 0$ and $K$

Nested half spaces, conditional probability.
Linear progress:

For large enough $m$, $-mL^m + (1 - L) \geq \epsilon > 0$

So $\mathbb{E}(w_{n+m}) \geq \mathbb{E}(w_n) + \epsilon$

use: Kingman’s subadditive ergodic theorem