Homework 1

Topology I, Math 70700, Fall 2015

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http://www.math.csi.cuny.edu/∼ikofman/topology.html

Due: Thursday, September 3, before class

Problems

1. Let $f : X \to Y$ be a function of sets. Then for any $A, B \subset X$ and $V, W \subset Y$,

$$\begin{aligned} f(A \cup B) &= f(A) \cup f(B) & \qquad f^{-1}(V \cup W) &= f^{-1}(V) \cup f^{-1}(W) \\ f(A \cap B) &\subset f(A) \cap f(B) & \qquad f^{-1}(V \cap W) &= f^{-1}(V) \cap f^{-1}(W) \\ f(A) - f(B) &\subset f(A - B) & \qquad f^{-1}(V - W) &= f^{-1}(V) - f^{-1}(W) \end{aligned}$$

   Give examples where equality fails to hold on the LHS, and verify that equality still holds on the RHS.

2. Let $X = \{\frac{1}{n} \,|\, n \in \mathbb{N}\} \subset \mathbb{R}$ and $Y = X \cup \{0\} \subset \mathbb{R}$. Show that X is discrete but Y is not.

3. Show that the collection $\{\{a\} \times (b, c) \subset \mathbb{R}^2 \,|\, a, b, c \in \mathbb{R}\}$ of vertical intervals in the plane is a basis for a topology on $\mathbb{R}^2$, which is called the *vertical interval topology* on $\mathbb{R}^2$. Compare this topology with the standard topology on $\mathbb{R}^2$.

4. Problems $\#\, 2,\ 5,\ 9 - 13$ of Hatcher's notes.

5. State the universal properties and draw the appropriate commutative diagrams related to Prop. 2.3 and 2.5 of J.P. May's notes.