Linear Algebra (Math 338) Midterm Exam 1

Date: February 25, 2009 Professor Ilya Kofman

Justify answers and show all work for full credit. No *symbolic* calculators.

NAME: ______________________________

Problem 1. *30 pts.*

(a) Let $A = \begin{bmatrix} 1 & 2 & -1 \\ 0 & 3 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 1 & 0 & 1 \\ -1 & 1 & 1 & 0 \\ 3 & 0 & -1 & 1 \end{bmatrix}$.

If possible, compute the matrix. Otherwise, write "Impossible".

$AB =$ $BA =$ $B^T =$

(b) Reduced row-echelon forms of the augmented matrices of four systems are shown. Below each matrix, write how many solutions each system has.

$$\left[\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{array}\right] \quad \left[\begin{array}{ccc|c} 1 & 0 & 2 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right] \quad \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{array}\right] \quad \left[\begin{array}{ccc|c} 1 & 0 & 2 & 0 \\ 0 & 1 & 2 & 0 \end{array}\right]$$

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(c) Find one non-trivial solution for $A\mathbf{x} = \mathbf{0}$, where $A = \begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix}$.

(d) Solve the linear system: $\begin{bmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} 6 \\ 0 \\ -5 \end{bmatrix}$.

Problem 2. *24 pts.* Justify with a general argument for any $n \times n$ matrices A and B.

(a) If P is an invertible matrix such that $B = P^{-1} A\, P$, then $\det A = \det B$.

(b) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ cannot be written in the form $A\, A^T$ for any 2×2 matrix A.

(c) If $A\mathbf{x} = \mathbf{0}$ has only the trivial solution, then $A^5\mathbf{x} = \mathbf{0}$ has only the trivial solution.

Problem 3. *22 pts.* Consider the following linear system:

$$\begin{cases} x_1 + x_2 - x_3 + x_4 = -2 \\ x_1 + 2x_2 + 2x_4 = 0 \\ x_2 + x_3 - x_4 = -4 \end{cases} .$$

(a) Write its associated augmented matrix.

(b) Reduce the matrix to its <u>reduced row-echelon form (rref)</u>.

(c) Use this procedure to solve the system.

Problem 4. *24 pts.*

$$A = \begin{bmatrix} 1 & -1 & 1 \\ -1 & 0 & a \\ 0 & 1 & 2 \end{bmatrix}$$

(a) If $\begin{bmatrix} 4 \\ 5 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 6 \\ 9 \\ -2 \end{bmatrix}$ are solutions to $A\mathbf{x} = \mathbf{b}$, find another solution. Justify.

(b) For which values of a is A invertible? (Hint: $\det A$.)

(c) Use elementary operations to find the inverse of A when $a = -2$.