Math 233 Sample Quiz 1

Problem 1. Let $\vec{\mathbf{u}} = (4, 4, 5)$ and $\vec{\mathbf{v}} = (2, -1, 1)$.

(a) Find a unit vector in the direction of $\vec{\mathbf{v}}$.

(b) Find $||\text{proj}_{\vec{\mathbf{v}}}\vec{\mathbf{u}}||$.

(c) Express $\vec{\mathbf{u}}$ as the sum $\vec{\mathbf{u}} = \vec{\mathbf{m}} + \vec{\mathbf{n}}$,
where $\vec{\mathbf{m}} = \vec{\mathbf{u}}_{||}$ is parallel to $\vec{\mathbf{v}}$, and $\vec{\mathbf{n}} = \vec{\mathbf{u}}_{\perp}$ is orthogonal to $\vec{\mathbf{v}}$.

Problem 2. Consider three points $P(2, -1, 0)$, $Q(0, -2, 1)$ and $R(3, 0, -1)$.

(a) Find a parametric equation of the line through Q and R.

(b) Find the equation of the plane passing through P, Q, and R.

(c) Find the area of triangle $\triangle PQR$.

Problem 3.

(a) Find the angle between the planes $x - y = 3$ and $-y + z = 1$.
(Hint: The angle between planes equals the angle between their normal vectors.)

(b) Find the equation of a plane containing the line $\ell(t) = (2 + 3t,\ -t,\ 4 + t)$ and passing through the point $P(0, 2, -1)$.

(c) Find the equation of the plane that passes through the point $(1, 2, -1)$ and is perpendicular to the line through the two points $E(1, 0, 1)$, $F(-3, 2, 3)$.