Sample problems for Exam 1 for Math 233

This sample exam has many more questions than the actual exam will have.

1. Let \( \vec{a} = \langle 4, -3, -1 \rangle \) and \( \vec{b} = \langle -2, -3, 5 \rangle \). Find (a) \( \vec{a} + \vec{b} \) (b) \( 2\vec{a} - 3\vec{b} \) (c) \( |\vec{a}| \) (d) \( |\vec{b}| \) (e) \( |\vec{a} + \vec{b}| \) (f) \( \vec{a} \cdot \vec{b} \) (g) \( \vec{a} \times \vec{b} \) (h) \( \text{proj}_\vec{a} \vec{b} \) (i) \( \text{proj}_\vec{b} \vec{a} \)

2. Find the angle between the vectors \( \vec{a} = -4\vec{i} + 2\vec{j} + 3\vec{k} \) and \( \vec{b} = 2\vec{i} + \vec{j} + 5\vec{k} \).
Find the angle that \( \vec{a} \) makes with the \( z \)-axis. (Note this is its \( \phi \) in spherical coordinates.)

3. Express the vector \( \vec{u} = \langle 2, 4, 5 \rangle \) as a sum of vector \( \vec{m} \) parallel to \( \vec{v} = \langle 2, -1, -2 \rangle \), and a vector \( \vec{n} \) perpendicular to \( \vec{v} \).

4. Find the area of the triangle with vertices \( (1, 2, 3), \ (3, 1, 5) \) and \( (4, 5, 6) \).

5. Find the area of the parallelogram with vertices \( (-1, 2, 0), \ (1, 0, 1), \ (-2, 0, 0) \) and \( (0, -2, 1) \).

6. Find unit vectors perpendicular to the plane passing through the points \( (1, 3, 5), \ (3, -1, 2) \) and \( (4, 0, 1) \).

7. Find the parametric and symmetric equations of the line passing through the points \( (4, 5, 6) \) and \( (3, 2, 1) \).

8. Find the parametric and symmetric equations of the line at the intersection of the planes, \( 3x - 3y - 7z = -4 \) and \( x - y + 2z = 3 \).

9. Find the equation of the plane passing through the points \( (1, 3, 2), \ (0, 3, 0) \) and \( (2, 4, 3) \).

10. Find the equation of a plane passing through point \( (2, 1, 0) \) and containing the line \( \frac{x-3}{2} = y = \frac{-8-z}{1} \).

11. Find the equation of a plane perpendicular to the line \( x = 2 + 3t, \ y = -t, \ z = -1 + t \) and passing through point \( (0, 1, -1) \).

12. Determine whether the points \( (1, 1, 2), \ (0, 1, 0) \) and \( (1, 2, 3) \) are collinear.

13. Determine whether the points \( (0, 2, 1), \ (0, 1, 0), \ (1, 1, 0) \) and \( (1, 2, 3) \) are coplanar.

14. Find the equation of the plane perpendicular to the line \( x = 1 + 2t, \ y = -3 + 4t, \ z = -1 - t \) and passing through the point \( (-1, -1, 5) \).

15. Find the distance between the parallel planes \( 3x + 3y - 9z = 4 \) and \( x + y - 3z = 1 \).

16. Describe and sketch the surface \( z = x^2 - 4y^2 \).

17. Describe and sketch the surface \( x^2 - y^2 - z^2 - 4 = 0 \).

18. Find the cylindrical and spherical coordinates for the point \( P(\sqrt{3}, \ 1, \ 2\sqrt{3}) \).

19. Describe and sketch the surface given in spherical coordinates by \( \phi = \pi/3 \). Find the equations in cylindrical and cartesian coordinates for this surface.